LECTURE 12. STEADY-STATE RESPONSE DUE TO ROTATING IMBALANCE

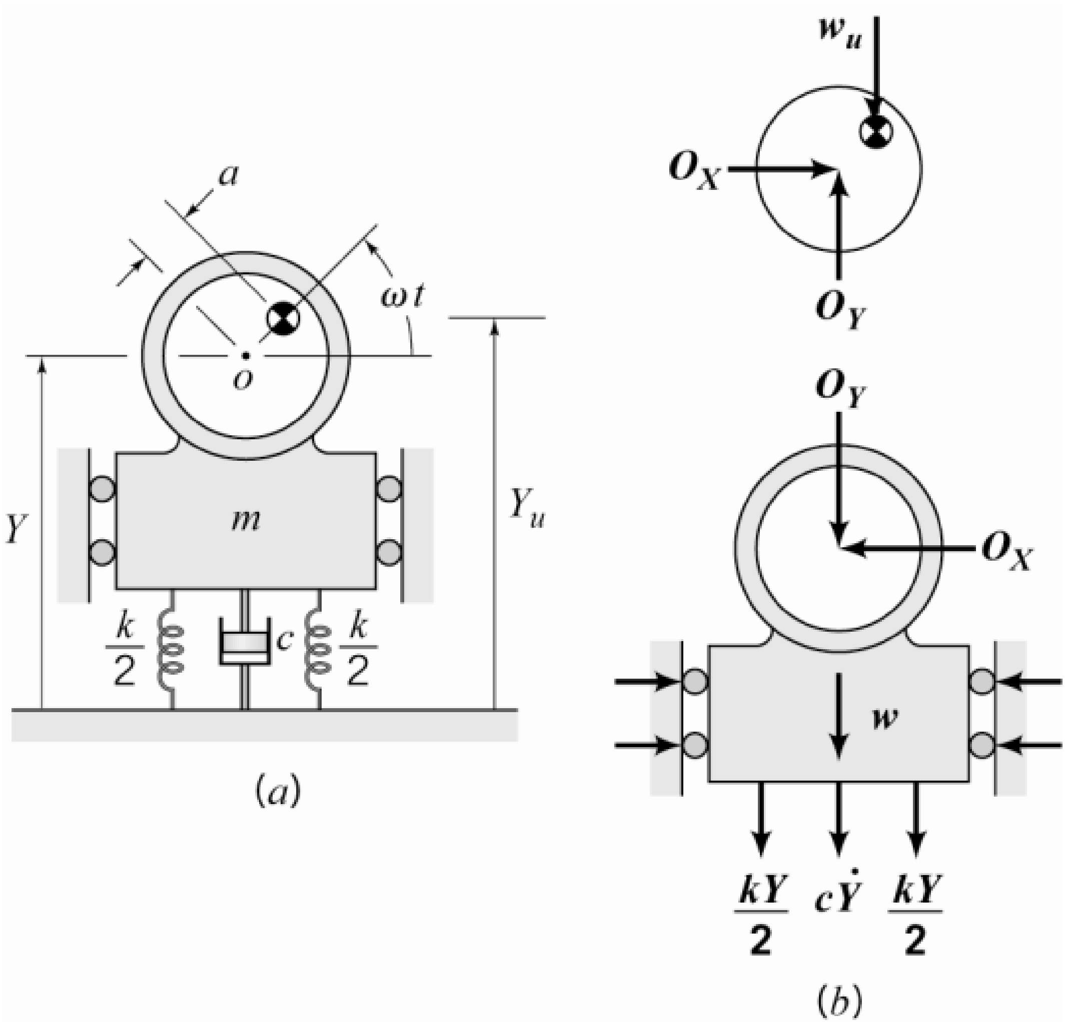

Figure 3.18 (a) Imbalanced motor with mass m_u supported by a housing mass m, (b) Free-body diagram for $Y>0$, $\dot{Y}>0$

The product $m_u \boldsymbol{a}$ is called the "imbalance vector." Y defines m's vertical position with respect to ground. From figure 3.17A, m_u's vertical position with respect to ground is

$$Y_u = Y + a \sin \omega t \;\Rightarrow\; \ddot{Y}_u = \ddot{Y} - a\,\omega^2 \sin \omega t \; . \qquad (3.46)$$

The free-body diagram of figure 3.17B for the support mass applies for upwards motion of the support housing that causes tension in the support $(Y>0)$ springs and the support damper $(\dot{Y}>0)$. The internal reaction force (acting at the motor's bearings) between the motor and the support mass is defined by the components (O_X, O_Y).

Applying $\Sigma \boldsymbol{f} = m\ddot{\boldsymbol{r}}$ to the individual masses of figure 3.17B gives:

$$\Sigma f_Y = -w_u + O_Y = m_u \ddot{Y}_u = m_u(\ddot{Y} - a\omega^2 \sin\omega t)$$

$$\Sigma f_Y = -w - O_Y - kY - c\dot{Y} = m\ddot{Y} \ ,$$

where Eq.(3.46) has been used to eliminate $\ddot{Y}_u$. Adding these equations eliminates the vertical reaction force O_Y and gives the single equation of motion,

$$(m + m_u)\ddot{Y} + c\dot{Y} + kY = -(w + w_u) + m_u a\omega^2 \sin\omega t \ . \qquad (3.47)$$

This equation resembles Eq.(3.32),

$$m\ddot{Y} + c\dot{Y} + kY = f_o \sin\omega t \ . \qquad \textbf{(3.32)}$$

except $(m + m_u)$ and $m_u a\omega^2$ have replaced m and f_0, respectively. Dividing Eq.(3.47) by $(m + m_u)$ gives

$$\ddot{Y} + 2\zeta\omega_n\dot{Y} + \omega_n^2 Y = -g + \frac{m_u a\omega^2}{M}\sin\omega t \ , \qquad (3.48a)$$

where,

$$M = (m + m_u) \ , \quad 2\zeta\omega_n = \frac{c}{M} \ , \quad \omega_n^2 = \frac{k}{M} \ . \qquad (3.48b)$$

We want the steady-state solution to Eq.(3.48a) due to the rotating-imbalance excitation term $[(m_u a\omega^2)/M] \ \sin(\omega t)$ and are not interested in either the homogeneous solution due to initial conditions or the particular solution due to weight. Eq.(3.48a)

$$Y_{op} = \frac{f_o}{m} \cdot \frac{1}{\left[(\omega_n^2 - \omega^2)^2 + 4\zeta^2 \omega_n^2 \omega^2 \right]^{1/2}} \ , \qquad (3.37)$$

has the same form as Eq.(3.32) except $m_u a\omega^2/M$ has replaced f_0/m. Hence, by comparison to Eq.(3.37),

the steady-state response amplitude due to rotating imbalance is

$$Y_{op} = \frac{m_u a\omega^2}{M} \cdot \frac{1}{\left[(\omega_n^2 - \omega^2)^2 + 4\zeta^2 \omega_n^2 \omega^2 \right]^{1/2}}$$

$$= \frac{m_u a\omega^2}{M\omega_n^2} \cdot \frac{1}{\left\{ [1 - (\omega/\omega_n)^2]^2 + 4\zeta^2 (\omega/\omega_n)^2 \right\}^{1/2}} \ ,$$

and the amplification factor due to the rotating imbalance is

$$Y_{op}/a\left(\frac{m_u}{M}\right) = \frac{r^2}{\left[(1 - r^2)^2 + 4\zeta^2 r^2\right]^{1/2}} = J(r) . \qquad (3.49)$$

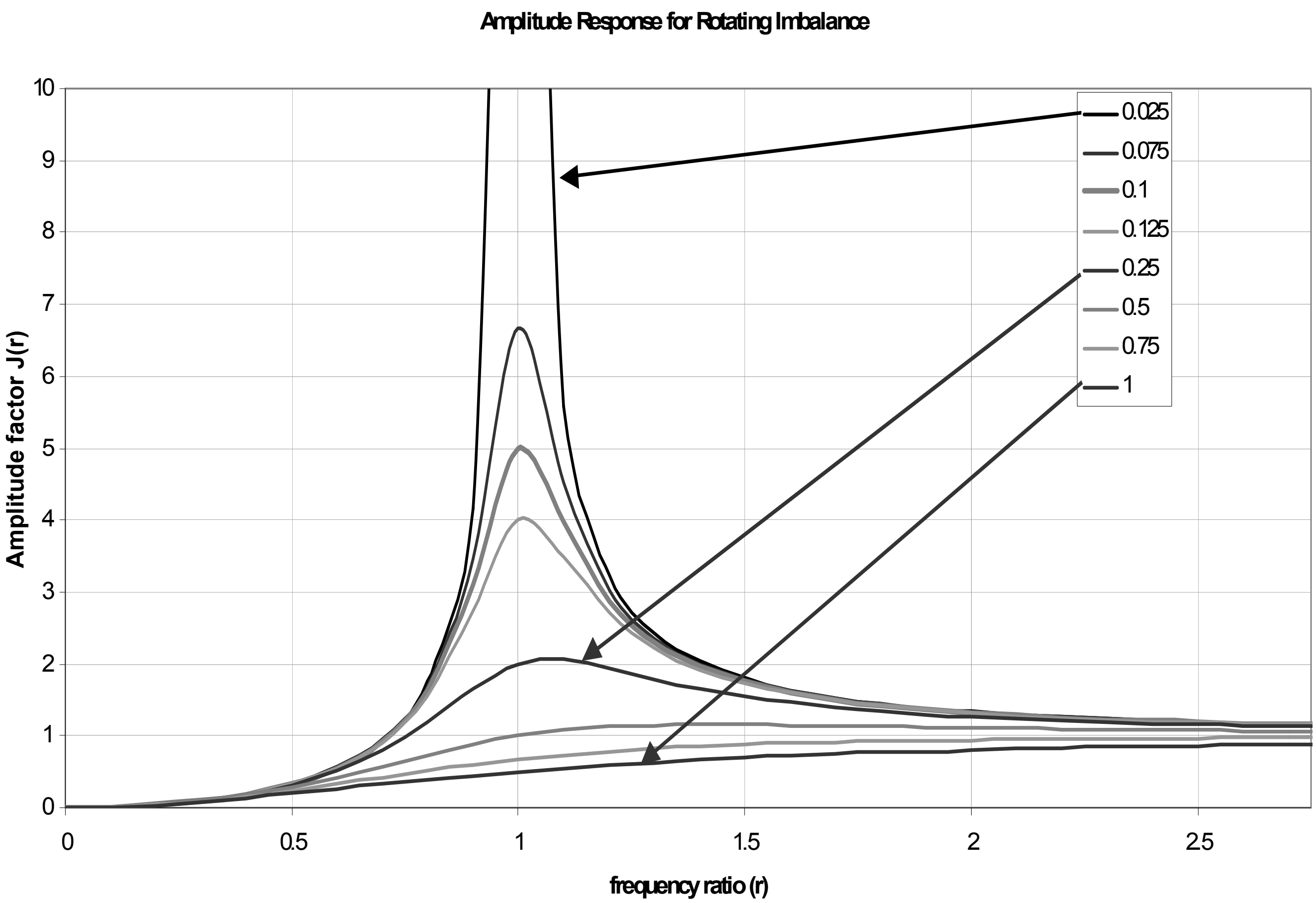

Figure 3.18. $J(r)$ versus frequency ratio $r = \omega/\omega_n$ from Eq.(3.49) for a range of damping ratio values.

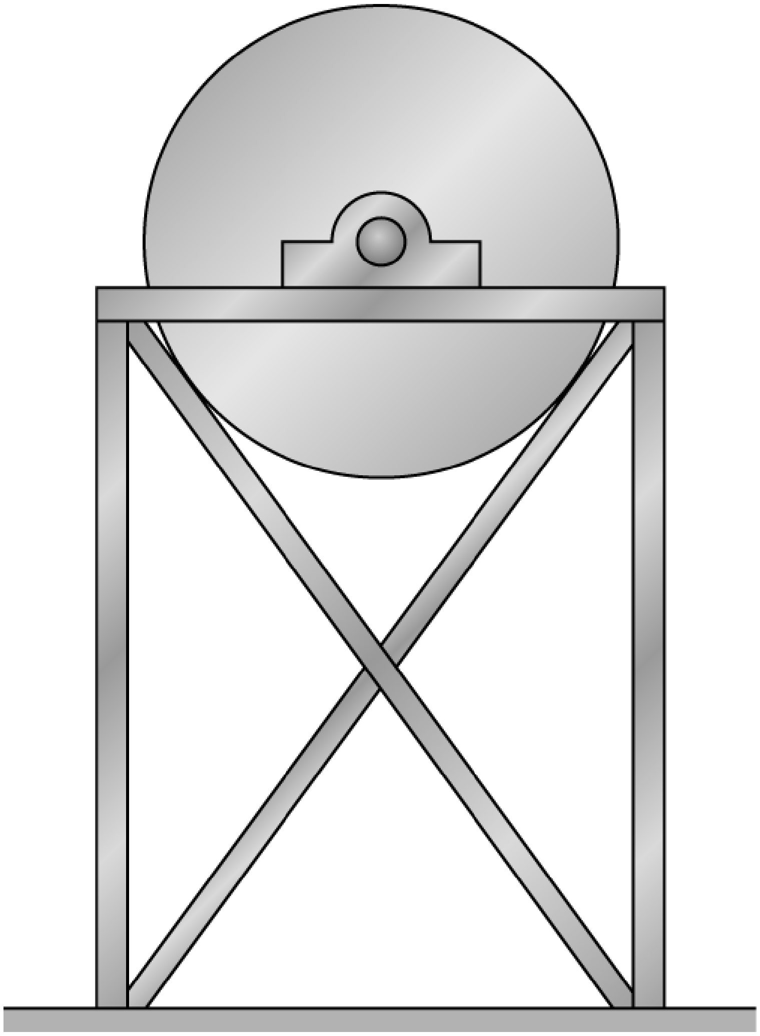

Figure XP3.4 Industrial blower supported by a welded-frame structure.

The support structure is much stiffer in the vertical direction than in the horizontal; hence, the model of Eq.(3.48a) , without the weight, holds for horizontal motion. This unit runs at 500 rpm, and has been running at "high" vibration levels. A "rap" test has been performed by mounting an accelerometer to the fan base, hitting the support structure below the fan with a large hammer, and recording the output of the accelerometer. This test shows a natural frequency of 10.4 Hz = 625 rpm with very little damping ($\zeta \cong .005$). The rotating mass of the fan is 114 kg. The total weight of the fan (including the rotor) and its base plate was stated to weigh 1784 N according to the manufacturer. Answer the following questions:

a. Assuming that the vibration level on the fan is to be less than $0.1g$, how well should the fan be balanced; i.e., to what value should *a* be reduced?

b. Assuming that the support structure could be stiffened laterally by approximately 50%, how well should the fan be balanced?

Solution. In terms of the steady-state amplitude of the housing, the housing acceleration magnitude is $|a_p| = \omega^2 Y_{op} = 0.1g$, where $g = 9.81\,m^2/\sec$. Hence, at

$$\omega = 500(\frac{rev}{\min}) \times (\frac{1\min}{60\sec}) \times (\frac{2\pi\, rad}{1\, rev}) = 52.36\,\frac{rad}{\sec}\ ,$$

the housing-amplitude specification is:

$$Y_{op} \le 0.1 \times 9.81/52.36^2 = 3.578 \times 10^{-4} m = .358\,mm\ .$$

Applying the notation of Eq.(3.49) gives

$$M = W/g = 1784./9.81 = 181.9\,kg\ ;\quad m_u = 114\,kg\ .$$

and $r = \omega/\omega_n = 500/625 = 0.8$. Applying Eq.(3.49) gives

$$Y_{op}/a(\frac{m_u}{M}) = \frac{0.64}{[(1-0.64)^2+4(.005)^2(.64)]^{1/2}} = 1.77 \ .$$

Hence, the imbalance vector magnitude *a* should be no more than

$$a = Y_{op}/(\frac{m_u}{M})/1.77 = .358\,mm/(114/182.)/1.77 = .323\,mm \ ,$$

which concludes *Task a*. Balancing and maintaining the rotor such that $a \le .323\,mm(.013\,in)$ for a 114 kg fan rotor is not easy.

Moving to Task b, and assuming that the structural stiffening does not appreciably increase the mass of the fan assembly, increasing the lateral stiffness by 50% would change the natural frequency to

$$\omega_n(new) = \sqrt{\frac{1.5\times k}{M}} = \sqrt{1.5}\times\omega_n(old) = 1.225\times 625\,rpm = 765\,rpm \ .$$

With a stiffened housing, $r_{new} = \omega/\omega_n = 500/765. = 0.653$, $r_{new}^2 = .427$. Since $2\zeta\omega_n = c/M \Rightarrow \zeta = c/2M\omega_n = c/2\sqrt{kM}$,

$$\zeta_{old} = \frac{c}{2\sqrt{M}}\times\frac{1}{\sqrt{k_{old}}} \ , \ \zeta_{new} = \frac{c}{2\sqrt{M}}\times\frac{1}{\sqrt{k_{new}}} \ .$$

Hence,

$$\zeta(new) = \zeta(old)\sqrt{\frac{k(old)}{k(new)}} = \frac{\zeta(old)}{\sqrt{1.5}} = \frac{.005}{1.225} = .004 \ .$$

Further,

$$Y_{op}/a(\frac{m_u}{M}) = \frac{0.427}{[(1-0.427)^2 + 4(.004)^2(.427)]^{1/2}} = .745 \ ,$$

and

$$\begin{aligned} a &= Y_{op}/(m_u/M)/.745 = .358\,mm/(114/182.)/.745 \\ &= .767\,mm \ \ (.030\,in) \end{aligned} .$$

Hence, by elevating the system natural frequency, the imbalance-vector magnitude can be $.767/.323 = 2.4$ times greater without exceeding the vibration limit. Stated differently, the fan can tolerate a much higher imbalance when its operating speed ω is further away from resonance.

Typically, appreciable damping is very difficult to introduce into this type of system; moreover, increasing the damping factor to $\zeta = .05$ in *Task a* reduces only slightly the required value for *a* to meet the housing-acceleration level specification. Damping would be more effective for $r = \omega/\omega_n \cong 1$. Of course, the vibration amplitudes would also be much higher.

Table 3.2 Forced-excitation results where $r = \omega/\omega_n$.

Application	Transfer Function	Amplitude Ratio	r for maximum response
Forced Response	$\frac{Y_{op}}{f_o/k} = H(r)$	$\frac{1}{[(1 - r^2)^2 + 4\zeta^2 r^2]^{1/2}}$	$r_{\max} = \sqrt{1 - 2\zeta^2}$
Base Excitation	$\frac{Y_{op}}{A} = G(r)$	$\frac{[1 + 4\zeta^2 r^2]^{1/2}}{[(1 - r^2)^2 + 4\zeta^2 r^2]^{1/2}}$	$r_{\max} = \frac{1}{2\zeta}\sqrt{(1 + 8\zeta^2)^{1/2} - 1}$
Relative Deflection with Base Excitation	$\frac{\Delta}{A} = J(r)$	$\frac{r^2}{\left[(1 - r^2)^2 + 4\zeta^2 r^2\right]^{1/2}}$	$r_{\max} = \frac{1}{\sqrt{1 - 2\zeta^2}}$
Rotating Imbalance	$Y_{op}/a(\frac{m_u}{M}) = J(r)$	$\frac{r^2}{\left[(1 - r^2)^2 + 4\zeta^2 r^2\right]^{1/2}}$	$r_{\max} = \frac{1}{\sqrt{1 - 2\zeta^2}}$