Chapter 2 Problems

Problem 2.1 Figure P2.1 illustrates a point P located in the X-Y coordinate system by polar coordinates r and θ.

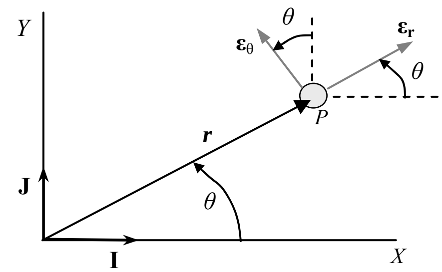

Figure P2.1

Task: Derive the polar-coordinate expressions for the velocity and acceleration components of the point P with respect to the X-Y coordinate system; i.e. derive expressions for v_r, v_θ, a_r, a_θ.

Problem 2.2 A particle is following the path shown in figure P2.2. By definition, its velocity vector $\mathbf{v} = v\ \boldsymbol{\varepsilon}_t$ is directed tangent to the path. The normal vector, $\boldsymbol{\varepsilon}_n$ is perpendicular to $\boldsymbol{\varepsilon}_t$. The radius of curvature is ρ.

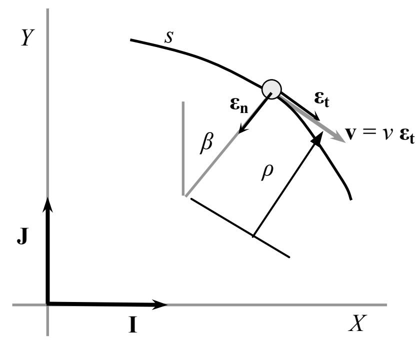

Figure P2.2

Task: Show that the acceleration vector can be expressed as

$$\mathbf{a} = \dot{v}\boldsymbol{\varepsilon}_t + \frac{v^2}{\rho}\boldsymbol{\varepsilon}_n = \ddot{s}\boldsymbol{\varepsilon}_t + \frac{v^2}{\rho}\boldsymbol{\varepsilon}_n \ .$$

Problem 2.3 Use the information provided in the figure P2.3 to derive expressions for the normal and tangential components of acceleration: i.e. find a_t and a_n.

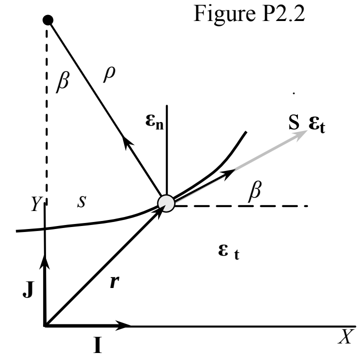

Figure P2.3

Problem 2.4 The X and Y motions of the guides A and B, figure P2.4, with right angle slots control the curvilinear motion of connecting pin P, which slides in both slots. For a short interval, the motion is governed by $X_p = 20 + \frac{1}{4} t^2$ and $Y_p = 15 - 1/6\ t^3$, where X and Y are in millimeters and t is in seconds.

Task: At $t = 2$ s, perform the following:

(a). Determine the velocity and acceleration components in Cartesian coordinates.
(b). Determine the velocity and acceleration components in path coordinates.
(c). Determine the velocity and acceleration components in polar Coordinates.
(d). Draw the resulting velocity and acceleration vectors with components for each coordinate system.

Problem 2.5 In problem 2.4, the position of the pin P is controlled by the guided slots such that $X_p = 0.2 + 0.1 \cos(\omega t)\ m$ and $Y_p = 0.6 + 0.3 \sin(2\ \omega t)\ \ m$.

Tasks: For $\omega t = 22.5°$, $\omega = 10$ *rad/sec (constant)*, perform the following:

(a). Determine the velocity and acceleration components in Cartesian coordinates.
(b). Determine the velocity and acceleration components in path coordinates.
(c). Determine the velocity and acceleration components in polar Coordinates.
(d). Draw the resulting velocity and acceleration vectors with components for each coordinate system.

Problem 2.6 In Problem 2.4, point P's coordinates are defined $X_p = D \cos(\omega t)$, $Y_p = 2 D \sin(\omega t)$, where D, and ω are constants.

Tasks:

(a). Obtain the general expressions (variables only) for the velocity and acceleration components in Cartesian form.
(b). Given the following: $D = 62.1\ mm$, and $\omega = 12.56$ rad/sec. At $\omega t = \pi/4\ rad$, determine P's velocity and acceleration vectors (with respect to the X, Y system) in terms of components in the Cartesian, polar, and path coordinate systems.
(c). How would you describe the path that P makes?
(d). Draw the resulting velocity and acceleration vectors with components for each coordinate system.

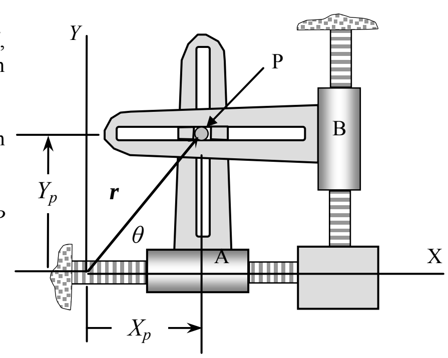

Figure P2.4

Problem 2.7 The mechanism of problem 2.4 is used to control the position of point P such that the components of P's position vector are:

$$X_p = 10 + 5t + \frac{t^2}{5}\ mm, \qquad Y_p = 10 - 5t - \frac{t^2}{8}\ \ mm$$

Tasks: at $t = 1$ sec, for point P, perform the following:

(a). Determine the velocity and acceleration components in Cartesian coordinates.
(b). Determine the velocity and acceleration components in polar coordinates.
(c). Determine the velocity and acceleration components in path coordinates.
(d). Draw the resulting velocity and acceleration vectors with components for each coordinate system.

Problem 2.8 The guide with the vertical slot is given a horizontal oscillatory motion according to $X = 4\sin 2t$, where X is in inches and t is in seconds. The oscillation causes the pin P to move in the fixed parabolic slot whose shape is given by $Y = X^2/4$, with Y also in inches, figure P2.8.

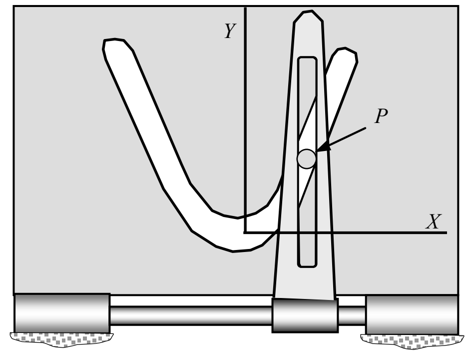

Figure P2.8

Tasks: At $t = \pi/12$ seconds perform the following:

(a). Find the components of **v** (velocity) and **a** (acceleration) in Cartesian coordinates.
(b). Find the components of **v** (velocity) and **a** (acceleration) in Polar coordinates.
(c). Find the components of **v** (velocity) and **a** (acceleration) in Path coordinates.
(d). Draw the resulting velocity and acceleration vectors with components for each coordinate system.

Problem 2.9 A projectile is fired with an initial velocity of 150 mph at an angle of 35˚ above the horizontal plane, figure P2.9. The projectile impacts an inclined plane that is at an angle of 30˚ from the vertical 7.6 seconds after being launched.

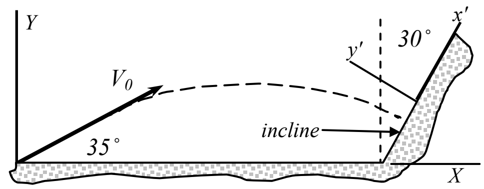

Figure P2.9

Tasks:

(a). Determine the X and Y components of the velocity of the projectile at the point of impact.
(b). Transform the velocity components into components parallel and perpendicular to the inclined plane.
(c). Draw a diagram showing the velocity components at the time of impact for both reference systems.
(d). Draw the resulting velocity and acceleration vectors with components at the time of impact for each coordinate system.

Problem 2.10 For the system illustrated in figure P2.10 r and θ are given by the following:

$$r = r_0 + \frac{r_0}{2}\cos(\omega t) \qquad \theta = \frac{\pi}{6}\sin(\omega t),$$

where r_0 and ω are constant.

Task: For $\omega t = \pi/2$ perform the following for point P:

(a). Find the velocity and acceleration components in polar coordinates.
(b). Find the velocity and acceleration components in Cartesian coordinates.
(c). Find the velocity and acceleration components in path coordinates.
(d). Draw the resulting velocity and acceleration vectors with components for each coordinate system.

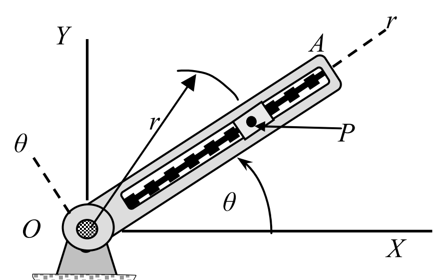

Figure P2.10

Problem 2.11 The position of the slider P in the rotating slotted arm OA, figure P2.10, is controlled by a power screw as shown. At the instant represented, $\dot{\theta}$ = 8 rad/s and $\ddot{\theta}$ = -20 rad/s^2. Also, at this same instant r = 200 mm, $\dot{r}$ = -300 mm/s, and $\ddot{r}$ = 0.

Tasks: For θ = 30 º, perform the following for point P:

(a). Find the velocity and acceleration components in polar coordinates.
(b). Find the velocity and acceleration components in Cartesian coordinates.
(c). Find the velocity and acceleration components in path coordinates.
(d). Draw the resulting velocity and acceleration vectors with components for each coordinate system.

Problem 2.12 The rotation of rod OA about O is defined by $\theta = 2t^2$ where θ is in radians, and t is in seconds, figure P2.12. The distance from collar B to O is defined by $r = 60\,t^2 - 0.9t^3$, with r in meters and t in seconds.

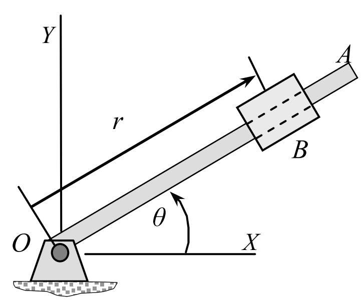

Figure P2.12

Tasks: At t = 1 second, perform the following for the collar B:

(a). Find the velocity and acceleration components in polar coordinates.
(b). Find the velocity and acceleration components in Cartesian coordinates.
(c). Find the velocity and acceleration components in path coordinates.
(d). Draw the resulting velocity and acceleration vectors with components for each coordinate system.

Problem 2.13 For the mechanism of problem 2.12, the motion is defined by :

$$\theta = \frac{\pi}{4}\cos(\omega t) \text{ rad}, \quad r = 10 + 5\cos(2\omega t) \text{ mm}$$

where ω = 60 cycles/sec (*constant*), and ωt = 30 ° .
Tasks: Perform the following for point *B*:
(a). Find the velocity and acceleration components in polar coordinates.
(b). Find the velocity and acceleration components in Cartesian coordinates.
(c). Find the velocity and acceleration components in path coordinates.
(a). Draw the resulting velocity and acceleration vectors with components for each coordinate system.

Problem 2.14 For the rocket system illustrated in figure P2.14,

$$r = 2200 \text{ m} \qquad \dot{r} = 500 \text{ m/s} \qquad \ddot{r} = 5 \text{ m/s}^2$$
$$\theta = 45^o \qquad \dot{\theta} = 0.1 \text{ rad/s} \qquad \ddot{\theta} = -0.01 \text{rad/s}^2$$

Tasks: Perform the following for the given state of the rocket:
(a). Find the velocity and acceleration components in polar coordinates.
(b). Find the velocity and acceleration components in Cartesian coordinates.
(c). Find the velocity and acceleration components in path coordinates.
(a). Draw the resulting velocity and acceleration vectors with components for each coordinate system.

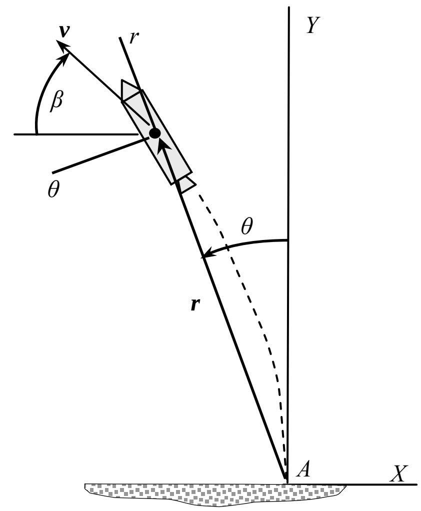

Figure P2.14

Problem 2.15 The cam is designed so that the center of the roller at *A* follows the contour and moves on a limacon defined by $r = b - c\cos\theta$, figure P2.15.

Tasks: For *b* = 1 *m*, *c* = 0.248 *m*, $\dot{\theta}$ = 1 *cycle/sec*, θ = 135 ° , and $\ddot{\theta} = 0$,perform the following:
(a). Find the velocity and acceleration components in polar coordinates.
(b). Find the velocity and acceleration components in Cartesian coordinates.
(c). Find the velocity and acceleration components in path coordinates.
(a). Draw the resulting velocity and acceleration vectors with components for each coordinate system.

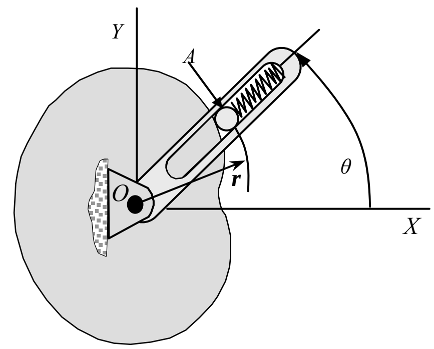

Figure P2.15

Problem 2.16 Illustrated in figure P2.16 is a bead sliding down a wire due to gravity and with no friction along the wire. The shape of the wire for a segment is defined by $Y = -X + \frac{1}{4} X^2$ *ft*. The velocity of the bead along the wire is defined by $v = (-2gY)^{1/2}$ *ft/sec*. The acceleration along the path is defined by $a_t = g\sin\beta$.

Task:
(a). Determine the velocity and acceleration components in path coordinates (in variable form).
(b). For *X* = 1 *ft*, what are the components of velocity and acceleration for path, Cartesian, polar coordinates? Solve using *g* = 32.17 *ft/sec²*.
(c). Draw the resulting velocity and acceleration vectors with components for each coordinate system.

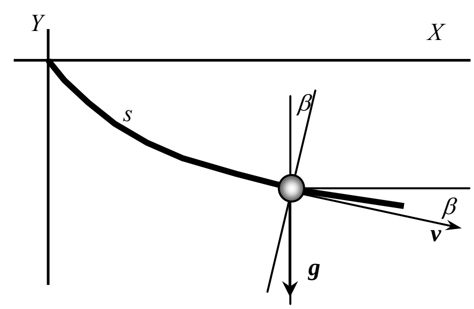

Figure P2.16

Problem 2.17 A vehicle is traveling in a vertical plane along a path defined by,

$$Y = A\sin\left(\frac{2\pi X}{L}\right), \quad A = 100\, m, \quad L = 2000\, m$$

At X = 1500 *m*, the velocity and acceleration of the vehicle along the path are *v* = 100 *km/hr*, and $a_t = 2$ m/sec^2. figure P2.17.

Task: For the given position of the car perform the following:
(a). Determine the velocity and acceleration components in Path coordinates.
(b). Determine the velocity and acceleration components in Cartesian coordinates.
(c). Determine the velocity and acceleration components in polar Coordinates.
(d). Draw the resulting velocity and acceleration vectors with components for each coordinate system.

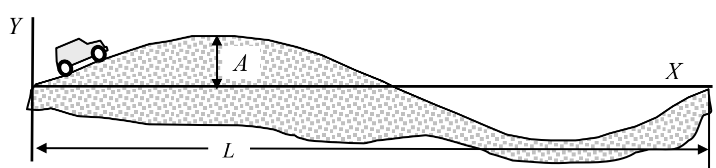

Figure P2.17

Problem 2.18 The jet plane travels along the vertical parabolic path, $Y = 0.4\,X^2$ (note that X and Y are in km, not m), figure P2.18. At point A $(5,10)$ *km*, the speed of the plane along the path is 200 *m/sec*, and it is decreasing at 1 *m/sec*^{2}.

Task: At the given instant:
(a). Determine the velocity and acceleration components in path coordinates.
(b). Determine the velocity and acceleration components in Cartesian coordinates.
(c). Determine the velocity and acceleration components in polar Coordinates.
(d). Draw the resulting velocity and acceleration vectors with components for each coordinate system.

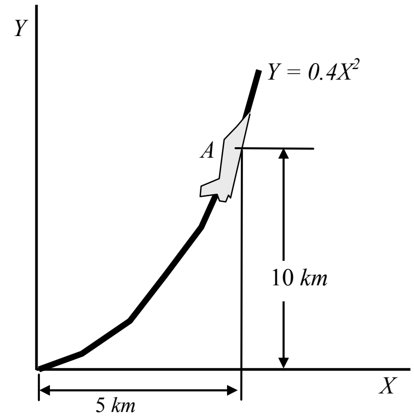

Figure P2.18

Problem 2.19 The track illustrated in figure P2.19 lies in a horizontal plane and is defined by $Y = \mathrm{k}\,X^2$ for x and y in meters with $k = 1/400\ m^{-1}$. At $X = 100\ m$, the velocity and acceleration components along the path are 20 *m/sec* and 2 *m/sec*^{2}, respectively.

Task: At the given instant:
(a). Determine the velocity and acceleration components in path coordinates.
(b). Determine the velocity and acceleration components in Cartesian coordinates.
(c). Determine the velocity and acceleration components in polar Coordinates.
(d). Draw the resulting velocity and acceleration vectors with components for each coordinate system.

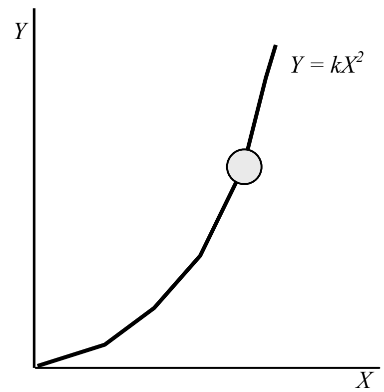

Figure P2.19

Problem 2.20 A portion of a roller coaster's path is defined by the parabola $Y = 0.05(X-X_O)^2 + 0.5(X-X_O) + 17$ with X and Y in feet. When $X = 30\ ft$ and $X_O = 50\ ft$ the lead car on roller coaster is moving at 40 feet per second relative to the track and accelerating at 0.32 *ft/sec*^{2} in the direction of motion, figure P2.20.

Tasks: At the given instant:
(a). Determine the velocity and acceleration components in path coordinates.
(b). Determine the velocity and acceleration components in Cartesian coordinates.
(c). Determine the velocity and acceleration components in polar Coordinates.
(d). Draw the resulting velocity and acceleration vectors with components for each coordinate system.

Problem 2.21 A particle travels along a path in a vertical plane defined by:

$$Y = -0.3X^3 - 0.7X^2 + 5X$$

where Y and X are in meters. At $X = 2.25\ m$, the velocity and acceleration of the particle along the path are $v = 5 m/sec$, and $a_t = -1.5\ m/sec^2$. Assume the particle is moving from the left to the right. Hint: Y'' defines the curvature of the path.

Tasks: At the given instant:
(a). Determine the velocity and acceleration components in path coordinates.
(b). Determine the velocity and acceleration components in Cartesian coordinates.
(c). Determine the velocity and acceleration components in polar Coordinates.
(d). Draw the resulting velocity and acceleration vectors with components for each coordinate system.

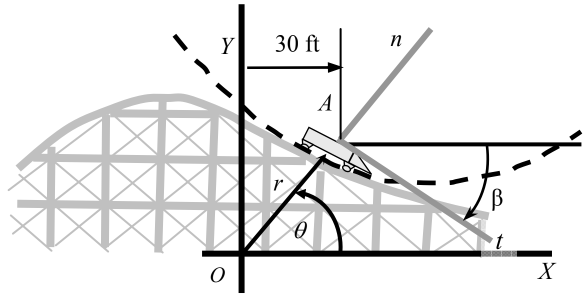

Figure P2.20