PROBLEM 1: Kinematics of rectilinear motion

Two drag race cars bth have a constant accceleration (a) over a quarter mile. Car A records an elapsed time of 5.0 sec. Car B records a terminal speed of 300 miles/hour. Find
a) which car will win the race?
b) which car accelerates faster? Specify physical values in units of "g"
c) which car has the fastest terminal velocity? Specify physical values in units of mph (miles/hour)
d) Is it possible for the car with the fastest terminal velocity to loose?

KEY: motion with constant acceleration

$mile := 5276 \cdot ft$

$hour := 3600 \cdot sec$

Car A:

$d_A := 0.25 \cdot mile$

$t_A := 5 \cdot sec$

from integration of constant acceleration with zero initial velocity and origin of distance from start line

$v(t) = a \cdot t$ (1)

$x(t) = 0.5 \cdot a \cdot t^2$ (2)

from formulas, the acceleration and terminal velocity of car A are

$$a_A := \frac{d_A}{0.5 \cdot t_A^{\,2}} \qquad a_A = 105.52 \frac{ft}{sec^2} \qquad g = 32.174 \frac{ft}{sec^2}$$

$$v_A := a_A \cdot t_A \qquad v_A = 359.972 \frac{mile}{hour} \qquad \frac{a_A}{g} = 3.28 \qquad t_A = 5\,sec$$

Car B:

$$v_B := 300 \cdot \frac{mile}{hour}$$

to find the time for car B to reach the end line, combine equations (2) and (1) by removing for the acceleration

$$v(t) = \frac{x}{0.5 \cdot t^2} \cdot t = 2 \cdot \frac{x}{t} \qquad d_B := d_A$$

same distance travelled

$d_B = 0.25\,mile$

hence, $t_B := \dfrac{2 \cdot d_B}{v_B}$ $\qquad t_B = 6\,sec$

and thus

$$a_B := \frac{v_B}{t_B} \qquad a_B = 73.278 \frac{ft}{sec^2}$$

$$v_B = 299.977 \frac{mile}{hour} \qquad \frac{a_B}{g} = 2.278 \qquad t_B = 6\,sec$$

Thus, car A wins the race by a full second!

d) Will car with fastest velocity to loose?

for case of constant acceleration, the terminal velocity is linear in time. So if two cars are to reach end line at least at the same time, then the velocities should be identical.

$$2 \cdot \frac{d_B}{t_A} = 359.727\,mph$$

As long as the acceleration remains constant, it is impossible for car with fastest velocity to loose. Not so if the acceleration increaseslinearly with time

NOT for exam:

Let's assume the acceleration of car C varies linearly with time $a_C = \kappa \cdot t$

then the velocity and displacements as functions of time are $v_C = \kappa \cdot \frac{t^2}{2}$

$x_C = \kappa \cdot \frac{t^3}{6}$

Let's find the acceleration rate κ assuming that car C and car A reach the end line at the same time. Hence, let

$d_C := d_A \qquad t_C := t_A \qquad t_C = 5\,\text{sec}$

$\kappa := d_B \cdot \frac{6}{t_B^{\,3}} \qquad \kappa = 36.639\,\frac{\text{ft}}{\text{sec}^3}$

Now, let's find the terminal speed of car C

$v_C := \kappa \cdot \frac{t_C^{\,2}}{2} \qquad v_C = 312.263\,\text{mph} \quad < \quad v_A = 359.727\,\text{mph}$

and car C final acceleration is $a_C := \kappa \cdot t_C \qquad \frac{a_C}{g} = 5.694 \quad > \quad \frac{a_A}{g} = 3.28$