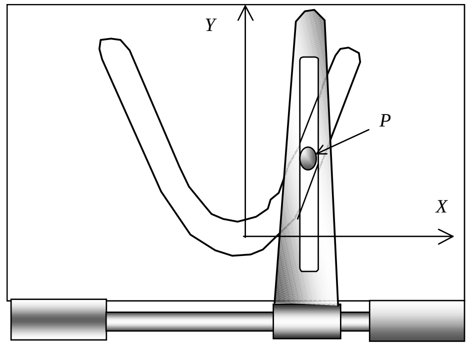
MEEN 363. SPRING 06. Exam 1. Problem 3

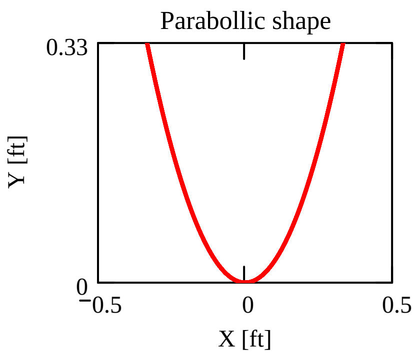
The guide with the vertical slot is given an oscillatory motion X(t). The oscillation causes pin P to move along the fixed parabolic slot whose shape Y(t) is given below.
At t = $\pi/12$ seconds, find the components of **v** (velocity) and **a** (acceleration) in:
(a) Cartesian coordinate system, (b) Polar coordinate system, and (c) path coordinate system

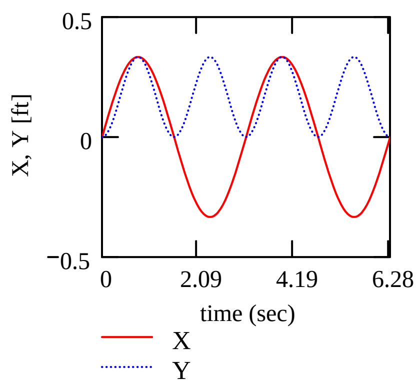
$A := 4\cdot in \qquad B := 4\cdot in \qquad \omega := 2\cdot\frac{rad}{sec}$



$X(t) := A\cdot\sin(\omega\cdot t)$

$Y(t) := \frac{1}{B}\cdot X(t)^2$





Solution Key:

At: $t := \frac{\pi}{12}\cdot sec$

Task a: components of velocity and acceleration in coordinates (X,Y):

at desired time, position vector P has components: $X(t) = 2\,in$ $\qquad Y(t) = 1\,in$

determine, first and second time derivatives of position vector P, i.e. velocities and accelerations:

$v_X := A\cdot\omega\cdot\cos(\omega\cdot t) \qquad a_X := -A\cdot\omega^2\cdot\sin(\omega\cdot t)$

$v_Y := \frac{2}{B}\cdot X(t)\cdot v_X \qquad a_Y := \frac{2}{B}\cdot v_X{}^2 + \frac{2}{B}\cdot X(t)\cdot a_X$

$v_X = 6.928\,\frac{in}{sec} \qquad v_Y = 6.928\,\frac{in}{sec} \qquad a_X = -8\,\frac{in}{sec^2} \qquad a_Y = 16\,\frac{in}{sec^2}$

Task b: determine the velocity and acceleration in polar (ρ,θ) coordinates

The angle θ of the position P is given by

$\theta := \operatorname{atan}\left(\frac{Y(t)}{X(t)}\right) \qquad \theta\cdot\frac{180}{\pi} = 26.565$ degrees

and the magnitude is $\qquad r_P := \left(X(t)^2 + Y(t)^2\right)^{.5} \qquad r_P = 2.236\,in$

The matrix for transformation of coordinates **from** (X,Y) **to** polar (ρ,θ) is: $A := \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}$

Thus, the components of velocity and acceleration vectors in polar coordinates (ρ,θ) are:

$$\begin{pmatrix} v_r \\ v_\theta \end{pmatrix} := A \cdot \begin{pmatrix} v_X \\ v_Y \end{pmatrix} \qquad \begin{pmatrix} a_r \\ a_\theta \end{pmatrix} := A \cdot \begin{pmatrix} a_X \\ a_Y \end{pmatrix} \qquad A = \begin{pmatrix} 0.894 & 0.447 \\ -0.447 & 0.894 \end{pmatrix}$$

$$\begin{pmatrix} v_r \\ v_\theta \end{pmatrix} = \begin{pmatrix} 9.295 \\ 3.098 \end{pmatrix} \frac{in}{sec} \qquad \begin{pmatrix} a_r \\ a_\theta \end{pmatrix} = \begin{pmatrix} 4.767 \times 10^{-15} \\ 17.889 \end{pmatrix} \frac{in}{sec^2}$$

Task c: determine the velocity and acceleration in path coordinates (t,n):

The velocity vector is along the path, i.e. parallel to the unit vector ε_t

Thus, the angle β is determined from:

$$\beta := \operatorname{atan}\left(\frac{v_Y}{v_X}\right) \qquad \beta \cdot \frac{180}{\pi} = 45$$

degrees (I quadrant since Vx, Vy>0)

The matrix for transformation of coordinates **from** cartesian (X,Y) **to** path (t,n) is: $A := \begin{pmatrix} \cos(\beta) & \sin(\beta) \\ -\sin(\beta) & \cos(\beta) \end{pmatrix}$

Thus, the components of the velocity and acceleration vectors in (t,n) path or trajectory system are:

$$\begin{pmatrix} v_t \\ v_n \end{pmatrix} := A \cdot \begin{pmatrix} v_X \\ v_Y \end{pmatrix} \qquad \begin{pmatrix} a_t \\ a_n \end{pmatrix} := A \cdot \begin{pmatrix} a_X \\ a_Y \end{pmatrix} \qquad A = \begin{pmatrix} 0.707 & 0.707 \\ -0.707 & 0.707 \end{pmatrix}$$

$$\begin{pmatrix} v_t \\ v_n \end{pmatrix} = \begin{pmatrix} 9.8 \\ 0 \end{pmatrix} \frac{in}{sec} \qquad \begin{pmatrix} a_t \\ a_n \end{pmatrix} = \begin{pmatrix} 5.657 \\ 16.971 \end{pmatrix} \frac{in}{sec^2}$$

$v_n = 0$ as it should