MEEN 363. SPRING 06. Exam 1. Problem 3

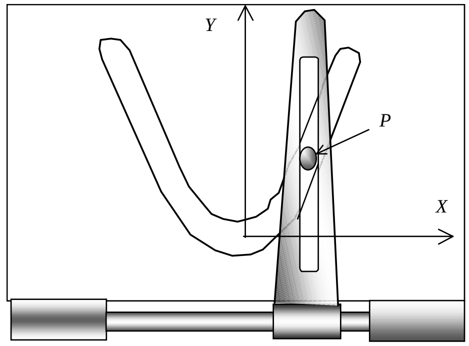
ORIGIN := 1

The guide with the vertical slot is given an oscillatory motion X(t). The oscillation causes pin P to move along the fixed parabolic slot whose shape Y(t) is given below.
At *t* = 2 seconds, find the components of **v** (velocity) and **a** (acceleration) in:
(a) Cartesian coordinate system, (b) Polar coordinate system, and (c) path coordinate system

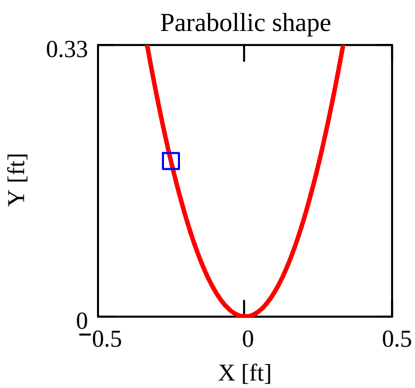
$A := 4 \cdot in \qquad B := 4 \cdot in \qquad \omega := 2 \cdot \frac{rad}{sec}$

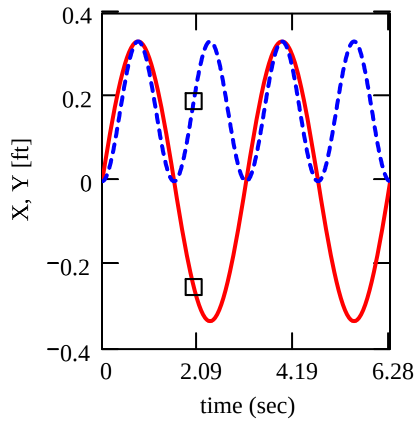
$X(t) := A \cdot \sin(\omega \cdot t)$

$Y(t) := \frac{1}{B} \cdot X(t)^2$



At: $t := 2 \cdot sec$





X solid, Y dash

Solution Key:

Task a: components of velocity and acceleration in coordinates (X,Y):

at $t = 2\,sec$

at desired time, position vector P has components: $X(t) = -3.027\,in \qquad Y(t) = 2.291\,in$

determine, first and second time derivatives of position vector P, i.e. velocities and accelerations:

$v_X := A \cdot \omega \cdot \cos(\omega \cdot t) \qquad a_X := -A \cdot \omega^2 \cdot \sin(\omega \cdot t)$

$v_Y := \frac{2}{B} \cdot X(t) \cdot v_X \qquad a_Y := \frac{2}{B} \cdot {v_X}^2 + \frac{2}{B} \cdot X(t) \cdot a_X$

analytical expressions for **v** and **a**

$v_X = -5.229 \frac{in}{sec} \qquad v_Y = 7.915 \frac{in}{sec} \qquad a_X = 12.109 \frac{in}{sec^2} \qquad a_Y = -4.656 \frac{in}{sec^2}$

Task b: determine the velocity and acceleration in polar (ρ,θ) coordinates

The angle θ of the position P is given by $\theta := \operatorname{atan}\left(\frac{Y(t)}{X(t)}\right) + \pi$

since X<0, Y>0 (II quadrant)

$\theta \cdot \frac{180}{\pi} = 142.881$ degrees

and the magnitude is $r_P := \left(X(t)^2 + Y(t)^2\right)^{.5} \qquad r_P = 3.796\,in$

The matrix for transformation of coordinates **from** (X,Y) **to** polar (ρ,θ) is: $A := \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}$

Thus, the components of velocity and acceleration vectors in polar coordinates (ρ,θ) are:

$$\begin{pmatrix} v_r \\ v_\theta \end{pmatrix} := A\cdot\begin{pmatrix} v_X \\ v_Y \end{pmatrix} \qquad \begin{pmatrix} a_r \\ a_\theta \end{pmatrix} := A\cdot\begin{pmatrix} a_X \\ a_Y \end{pmatrix} \qquad A = \begin{pmatrix} -0.797 & 0.603 \\ -0.603 & -0.797 \end{pmatrix}$$

$$\begin{pmatrix} v_r \\ v_\theta \end{pmatrix} = \begin{pmatrix} 8.946 \\ -3.156 \end{pmatrix}\frac{in}{sec} \qquad \begin{pmatrix} a_r \\ a_\theta \end{pmatrix} = \begin{pmatrix} -12.465 \\ -3.595 \end{pmatrix}\frac{in}{sec^2}$$

recall:

$$v_Y = 0.66\,\frac{ft}{sec} \qquad v_X = -0.436\,\frac{ft}{sec}$$

Task c: determine the velocity and acceleration in path coordinates (t,n):

The velocity vector is along the path, i.e. parallel to the unit vector ε_t

Thus, the angle β is determined from:

$$\beta := \operatorname{atan}\left(\frac{v_Y}{v_X}\right) + \pi \qquad \beta\cdot\frac{180}{\pi} = 123.452 \text{ degrees}$$

(II quadrant since Vx<0, Vy>0)

The matrix for transformation of coordinates **from** cartesian (X,Y) **to** path (t,n) is: $A := \begin{pmatrix} \cos(\beta) & \sin(\beta) \\ -\sin(\beta) & \cos(\beta) \end{pmatrix}$

Thus, the components of the velocity and acceleration vectors in (t,n) path coordinate system are:

$$\begin{pmatrix} v_t \\ v_n \end{pmatrix} := A\cdot\begin{pmatrix} v_X \\ v_Y \end{pmatrix} \qquad \begin{pmatrix} a_t \\ a_n \end{pmatrix} := A\cdot\begin{pmatrix} a_X \\ a_Y \end{pmatrix} \qquad A = \begin{pmatrix} -0.551 & 0.834 \\ -0.834 & -0.551 \end{pmatrix} \qquad |A| = 1$$

$$\begin{pmatrix} v_t \\ v_n \end{pmatrix} = \begin{pmatrix} 9.49 \\ 0 \end{pmatrix}\frac{in}{sec} \qquad \begin{pmatrix} a_t \\ a_n \end{pmatrix} = \begin{pmatrix} -10.56 \\ -7.536 \end{pmatrix}\frac{in}{sec^2}$$

$v_n = 0$ as it should

One could also find the tang velocity easily from $v_t = \left(v_X^2 + v_Y^2\right)^{.5} = 9.486\,\frac{in}{sec}$

or $\left(v_r^2 + v_\theta^2\right)^{.5} = 9.486\,\frac{in}{sec}$

However, note that the normal acceleration is < 0. Hence, the coordinate transformation **is NOT correct.**

It means: ε_t and ε_n form a **Left handed coordinate system**.

Thus, let's change the second row of the transformation matrix $A_{2,1} := -A_{2,1} \quad A_{2,2} := -A_{2,2}$

$$A = \begin{pmatrix} -0.551 & 0.834 \\ 0.834 & 0.551 \end{pmatrix} \qquad |A| = -1$$

and rework: $\begin{pmatrix} a_t \\ a_n \end{pmatrix} := A\cdot\begin{pmatrix} a_X \\ a_Y \end{pmatrix}$ $\quad \begin{pmatrix} a_t \\ a_n \end{pmatrix} = \begin{pmatrix} -10.56 \\ 7.536 \end{pmatrix}\frac{in}{sec^2}$

$a_n > 0$ as it should

final check - not for exam

$$\left|\begin{pmatrix} a_t \\ a_n \end{pmatrix}\right| = 1.081\,\frac{ft}{sec^2} \qquad \left|\begin{pmatrix} a_r \\ a_\theta \end{pmatrix}\right| = 1.081\,\frac{ft}{sec^2} \qquad \left|\begin{pmatrix} a_X \\ a_Y \end{pmatrix}\right| = 1.081\,\frac{ft}{sec^2}$$