

The DFT and its usage - Example

ORIGIN := 1

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Build

$$X_{(t)} := \sum_{i=1}^n \left(C_i \cos(\omega_i \cdot t) + S_i \cdot \sin(\omega_i t) \right) + X_0$$

$$X_0 := 0$$

$$n := 3$$

Input frequencies and amplitudes

$$f_1 := 22 \cdot \text{Hz}$$

$$C_1 := 0$$

$$S_1 := 1$$

$$f_2 := 118 \cdot \text{Hz}$$

$$C_2 := 0$$

$$S_2 := 0$$

$$f_3 := 200 \cdot \text{Hz}$$

$$C_3 := 0$$

$$S_3 := 0$$

ALIASING

Select sampling rate

$$\Delta \text{rate} := 30 \cdot \frac{1}{s}$$

samples/sec or Hz in DAQ jargon

$$\text{Sampling rate} = \text{MIN} = 2 \times f_{\text{max}}$$

Number of samples

$$N_P := 2^8 \cdot 1$$

$$N_P = 256$$

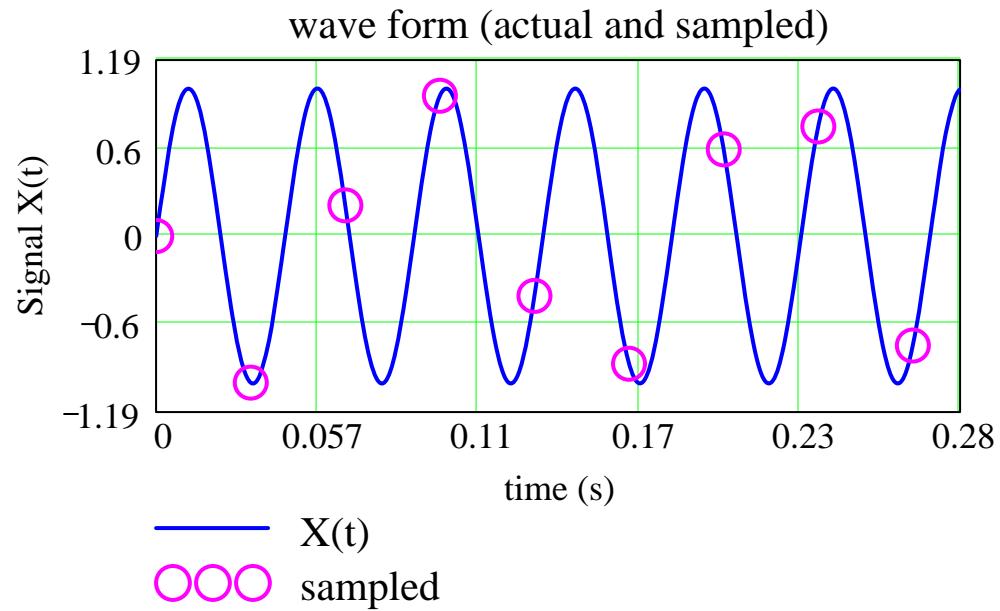
▶ window

$$T := \frac{1}{f}$$

$$T^T = \left(0.045 \quad 8.475 \times 10^{-3} \quad 5 \times 10^{-3} \right) s$$

Period of motion for each component in signal

▶ Create function



$$X_{\max} = 0.995$$

$$T_M := \frac{T_{\max}}{30}$$

$$N_P = 256$$

$$T_{\max} = 8.5 \text{ s}$$

$$\Delta \text{rate} = 30 \text{ s}^{-1}$$

$$\Delta t = 0.033 \text{ s}$$

$$\frac{T_{\max}}{T_1} = 187$$

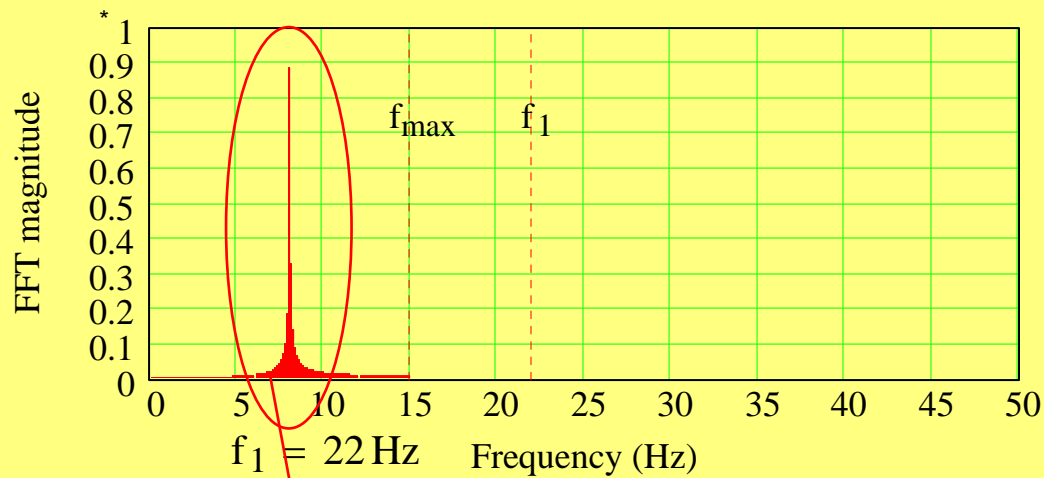
$$f_1 = 22 \text{ Hz}$$

$$\frac{1}{f_1} = 0.045 \text{ s}$$

for graphs

$$X_{\max} := 1$$

$$\text{freq}_{\max} := 50$$



$$f = \begin{pmatrix} 22 \\ 118 \\ 200 \end{pmatrix} \text{ Hz} \quad C = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad S = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\Delta \text{rate} = 30 \text{ Hz}$$

$$N_p = 256$$

$$T_{\max} = 8.5 \text{ s}$$

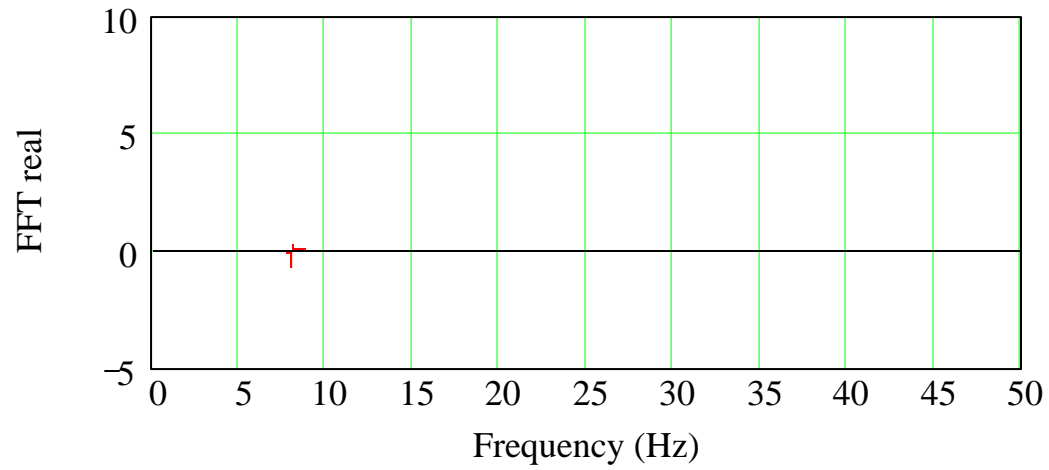
$$\Delta f = 0.117 \text{ Hz}$$

$$\max(f_{\text{req}}) = 14.883 \text{ Hz}$$

$$\max(A) = 0.884$$

$$\frac{f_{\max}}{\Delta f} = 128$$

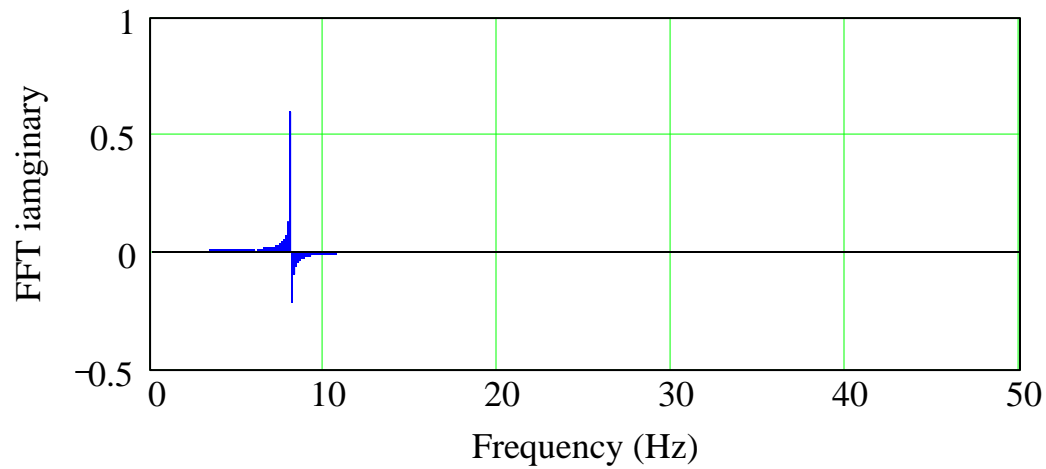
DFT Real and Imaginary parts



Real

$$\max(\text{Re_X}) = 0.244$$

$$\min(\text{Re_X}) = -0.655$$



Imag

$$\max(\text{Im_X}) = 0.594$$

$$\min(\text{Im_X}) = -0.216$$