MEEN 459/659– SPRING 2019 Group Homework 1 due February 5, 2019

Figure 1 depicts a schematic view of a seal test rig with the instrumentation for parameter identification. The rig consists of a massive cartridge and seal (BC) supported on four structural rods that act as elastic elements. Towards identifying the test rig structural parameters (K, C, M), impact loads along the X direction were exerted on the BC. An eddy current (REBAM) sensor and an accelerometer recorded the displacement (x) and acceleration (a) of the BC resulting from the applied impact, respectively.

Assume that the test rig is a single degree of freedom mechanical system without structural coupling, i.e., its equation of motion for $x_{(t)}$ is

$$[\, M\,a + C\,v + K\,x = f_{(t)}];\quad v=dx/dt,\; a=dv/dt \qquad (1)$$

where M, C, K are the system equivalent mass, (viscous) damping and stiffness coefficients, respectively and f is an externally applied force. Presently, the physical parameters, hereby renamed as force coefficients, are unknown.

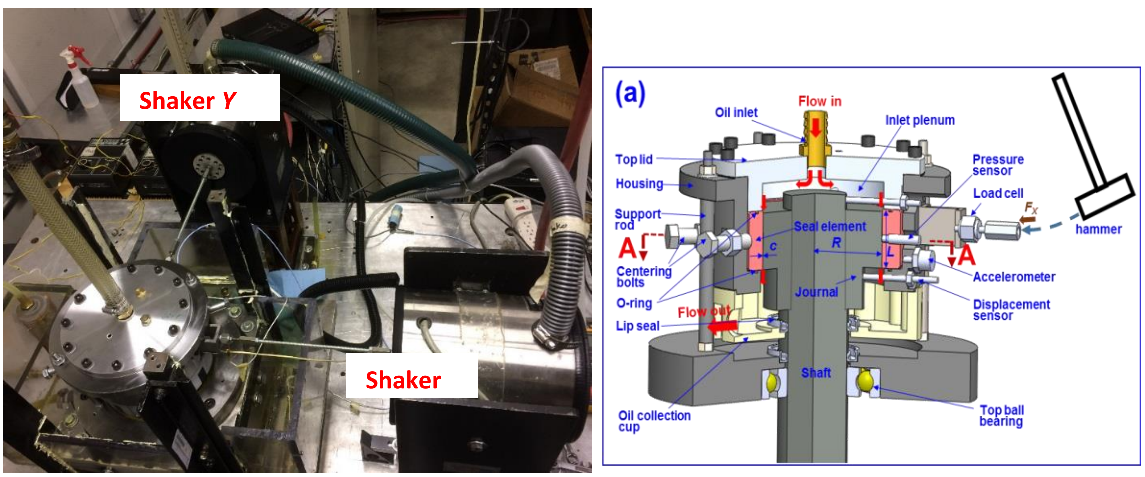

Figure 1. Photograph of seal test rig and schematic view of test rig and instrumentation for an impact load test

The sampling rate and sensors used are

sampling rate	points		Time_step
5120	2048	sec	0.000195

Sensor		Conversion	Gain
displacement	Bently Nevada 3300 XL 5 mm	V/mm	7.87
acceleration	PCB 353 B33	mV/(m/s^2)	10.19
Force	PCB 208 C02	mV/kN	11241

The attached Excel data file contains the data in columns. impact force (*f*), displacement (*x*) and acceleration (*a*) recorded in VOLTS. The file contains a worksheet that will calculate the discrete Fourier transform for a selected time series, as noted. You can edit the VB code[1] as needed (if you wish).

Convert the columns of the data file to appropriate physical units (Force: N, displacement: m, acceleration: m/s^2). The acceleration shows a non-zero mean value – *remove it!*

Your task is to determine/estimate/identify the system damping ratio and natural frequency, and the system physical parameters (stiffness *K*, damping *C*, and inertia *M*) using various methods and correlating (amongst each other) the various physical results obtained. Also provide estimations on the uncertainty of the identified parameters.

The sensors are quite exact (very little inaccuracy); however, the DAQ stores the data in Volts with just 4 digits (equivalent to 0.1 mV). From this information you could quantify the **uncertainty for each sensor (as recorded), and also perform an uncertainty analysis for the estimated parameters.**

This is an opportunity to use your master skills in computational software (VB. MATLAB, etc.)

a) **Use the time series**, and apply the concept of *log-dec* to estimate the test system damping ratio (ζ), the period of motion, the natural frequency; and also the (*K*,*C*,*M*) parameters. You MUST develop an algorithm that analyzes the transient response and captures more than just ONE peak or amplitude and that can actually count time and evaluate the period of motion.

   From the analysis of the captured peaks, perform a curve fit $y = m\ x + b$ where y=log(peak amplitude) and x=n (peak #). See Notes 2a – *the concept of log dec*.

b) **Calculate the DFT (Discrete Fourier Transform) of each signal, say $A_{(\omega)}$=DFT(*a*), $F_{(\omega)}$=DFT(*f*), $X_{(\omega)}$=DFT(x),** where ω is frequency.

   From the amplitude of the DFT find the damped natural frequency, and using the ½ power method estimate the system damping ratio (ζ). The method is explained in any vibrations book. Why do you think using the ½ power method is a good idea?

c) **Calculate the DFT (Discrete Fourier Transform) of each signal, say $A_{(\omega)}$=DFT(*a*), $F_{(\omega)}$=DFT(*f*), $X_{(\omega)}$=DFT(x).**

   To rind the transfer functions: *dynamic complex stiffness* (*F/X*) and the complex inertance (*F/A*).

[1] Please note that most (routine) engineering work in the US is conducted using MS Excel®. Hardly anyone in industry uses MATLAB, MATHCAD, and worse yet more advanced languages. Hence. It would be a good idea for you to learn a little about VB. However, you do NOT need to use the given worksheet FFT.

From the real and imaginary parts of the transfer functions, determine the system parameters. That is, for example

$$\mathrm{Re}(F/X) \rightarrow (K-\omega^2 M) \text{ and } \mathrm{Ima}(F/X) \rightarrow (\omega C). \qquad (1)$$

The procedure calls for a curve fit of Re(*) and Ima(*) to appropriate functions. Please also tell the correlation coefficient (R^2), i.e. how good does the curve reproduce the test data?

d) The amplitude of a linear *KCM* system compliance (or flexibility) and accelerance frequency functions are[2]

$$H(\omega) = \left| \frac{X(\omega)}{F(\omega)} \right| = \frac{1}{\sqrt{\left(K - M\omega^2\right)^2 + \left(C\,\omega\right)^2}} \qquad (2)$$

$$G(\omega) = \left| \frac{A(\omega)}{F(\omega)} \right| = \frac{\omega^2}{\sqrt{\left(K - M\omega^2\right)^2 + \left(C\,\omega\right)^2}} \qquad (3)$$

Use a MATLAB built-in function produce a **nonlinear curve fit** of (2) and (3) to the test results, *H* and *G*, so as to determine the system parameters (*K*, *C*, *M*). Overlay the curve fits (2) and (3) in graphs that also show the test data |*X/F*| and |*A/F*|.

e) Perform an uncertainty analysis for each of the estimated parameters (*K*, *C*, *M*). The analysis must include a description of the method and rationale used.

In (c) and (d) SELECTING the frequency range for validity of the identification process is crucial.

Discuss your findings. All methods should give SIMILAR (magnitude) parameters. Is the damping truly viscous in nature? Did you notice anything unusual with the time (or DFT) series data? Is the acceleration truly proportional and out of phase 180 degrees with respect to the displacement? How did you figure this out?

[2] Ginsberg, J.H., 2001, Mechanical and Structural Vibrations, John Wiley & Sons, Inc., New York, pp. 135-139.