A Simple Way to Measure Mass Moments of Inertia

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A simple rotational pendulum method to measure the radii of gyration or mass moments of inertia of a rotor and other assemblies is described.

In mechanical dynamic problems, it is often necessary to know the radii of gyration or equivalent mass moments of inertia for components and assemblies. Using the rotational pendulum technique described here, one can easily measure the radii of gyration about the polar and diametric axes of any rigid rotor without requiring a special fixture. The principles employed are also applicable to more complicated assemblies such as aircraft, boats and cars, where the radius of gyration and vehicle maneuverability are of interest. This description focuses on rotors.

The relative values of polar and diametric radii of gyration characterize some dynamic behavior and stability of spinning rotors. When the ratio of polar to diametric radii of gyration approaches unity, the spinning rotor may exhibit undesirable dynamic behavior. Consequently, prior to high-speed spin testing the rotor or otherwise operating the assembly, it is desirable to have a simple and inexpensive procedure which directly measures the radii of gyration of existing hardware. These data permit the technician to estimate the rotor dynamic behavior or to identify potential problems before committing to operation.

If sufficient part information is available, such as: dimensions, geometry and material density, one can calculate the radii of gyration. For complicated parts, this can be time consuming. Often, the technician does not have access to the rotor's dimensional details in order to make the calculations. Hence, an inexpensive empirical technique such as the one described is valuable.

The procedure makes use of measuring the natural frequency and key dimensions of a rotational pendulum formed by hanging the rotor on wires which have negligible mass. From these measured parameters, the radii of gyration are computed using the simple natural frequency formula described below.

A typical rotational pendulum arrangement for measuring the polar radius of gyration is shown in Figure 1a. Typical arrangements for measuring the diametric radius of gyration are shown in Figures 1b and 1c. Either diametric arrangement (Figures 1b or 1c) can be used. The choice is a matter of convenience depending upon the rotor configuration. To calculate high-speed rotor behavior, it is necessary to know both the polar and diametric radii of gyration.

In all cases, the pendulum wires should be parallel, have equal length L, be equidistant r from the rotor center of gravity and should share the rotor weight equally. Often, the center of gravity can be determined by geometric symmetry or by balancing the rotor on a knife edge or from a wire. The lengths L and r should be chosen such that errors in measuring length are negligible and that the pendulum natural frequency oscillations can be timed with a stop watch. Usually oscillations with natural frequency in the 0.2 to 2 Hz range can easily be counted and timed.

Pendulum oscillations are started with an initial rotational displacement (up to 10° single amplitude) then released such that the small-angle linearization assumption is valid. Since damping is usually small, tens of oscillations of approximately equal amplitude can be counted and timed giving an accurate and repeatable estimate of pendulum natural frequency.

The measured natural frequency f in Hz is determined as:

$$f = \frac{n}{t}$$

where: n = number of cycles
t = time for n cycles, sec.

The linearized differential equation of motion for the rotational pendulum is given by

$$\frac{d^2\theta}{dt^2} + \frac{r^2 g}{k^2 L}\,\theta = 0$$

where: θ = angular displacement the wire makes with the ver tical, rad
L = length of pendulum wire, in.
r = distance of the rotor center of gravity to the wire, in.
g = local gravitational acceleration, (~ 386.4 in./sec^2)
k = radius of gyration, in.

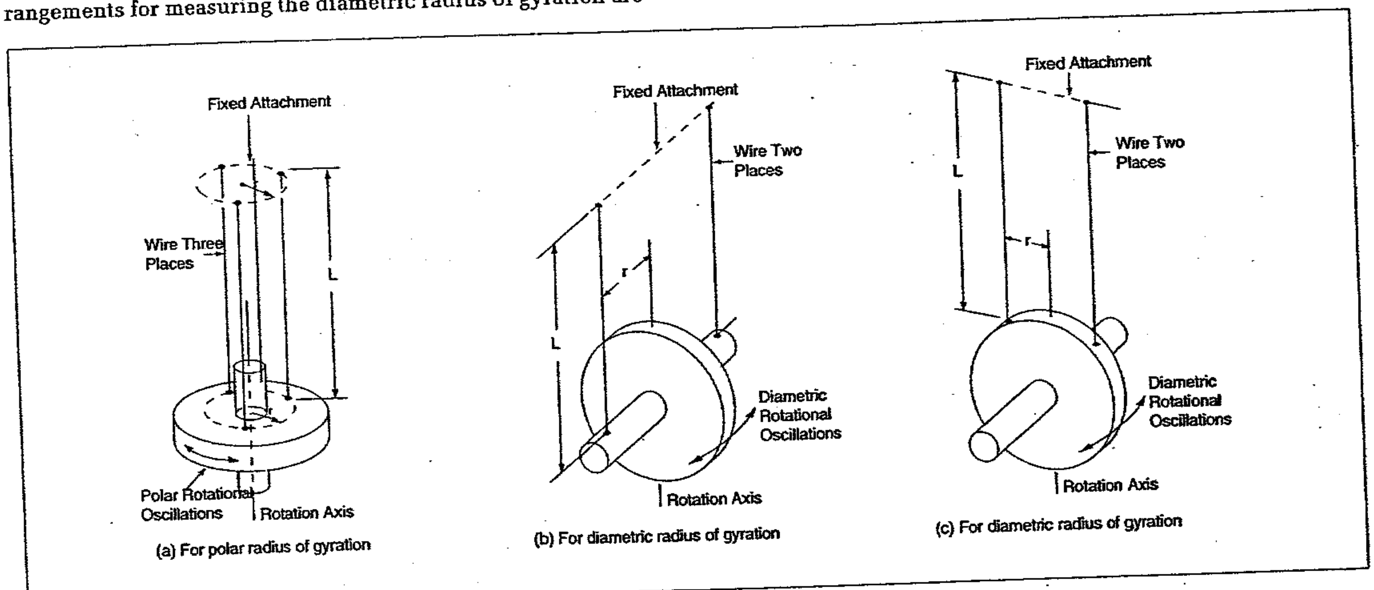

Figure 1. Typical rotational pendulum arrangements for measuring the radii of gyration of a rotor.

The following example is a practical application of the rotational pendulum method for measuring the mass moments of inertia in the field or laboratory.

A customer requests a spin test laboratory to proof-spin to 16,000 RPM a custom-made rotor for a radial-gap permanent magnet (PM) electric motor. The test purpose is to demonstrate that a hoop-wound fiberglass composit g overwrap will safely support the permanent ma s mounted on the outside diameter of a 6-in.-diameter by 2-in. thick steel hub. The hub has a 1-in. slightly tapered bore for a shaft that was unavailable for this test. An adapting arbor had to be fabricated for the spin test to support the 5.9 lb rotor assembly on a vertical cantilever quill shaft of an air turbine. The addition of the arbor adaptor reduced the ratio of polar-to-diametric mass moments of inertia of the assembly from 1.74 (without the adaptor) to 1.17 (with adaptor) as measured by the described pendulum method. The reduced ratio alerted test personnel that the rotor assembly has become more susceptible to disturbances and imbalances that can induce whirl. As a guideline for rigid rotor spin stability especially at speeds above the first rigid body mode, it is desirable to have the ratio of polar-to-diametric inertias of the rotor/adaptor assembly less than 0.8 (long cylinder-like) or greater than 1.2 (disk-like). The described method to measure mass moments of inertia is a quick way for the technician to check if the polar-to-diametric inertia ratio falls into the 0.8 to 1.2 warning range so that appropriate test precaution or test fixture redesign can be considered. In this rotor example where the 1.17 ratio is just inside the caution range, testing was pursued cautiously and asynchronous whirl was experienced above 12,000 RPM well above the 2000 RPM first rigid body mode. Fortunately, the whirl amplitude was low enough to complete the test without rebalance or adaptor redesign.

The solution to the differential equation yields th tu frequency f in Hz of the rotational pendulum as

$$f = \frac{1}{2\pi}\sqrt{\frac{r^2 g}{k^2 L}}$$

from which the radius of gyration can be calculated as

$$k = \frac{r}{2\pi f}\sqrt{\frac{g}{L}}$$

The mass moment of inertia I is obtained from the measu rotor weight w, using the classical definition of mass mom of inertia.

$$I = \frac{w}{g}k^2$$

As an example, for the measured parameters $L = 30$ in., r in., $n = 10$, $t = 10$ sec and $w = 10$ lb, the natural frequency f Hz, the radius of gyration $k = 2.856$ in., and the mass mom of inertia $I = 0.211$ lb-in.-sec^2. A time saver – to ensure reas able oscillation count and timing for a specific rotor, estim a value for radius of gyration k and use the frequency equat to determine trial values for lengths L and r before initiat the test setup.

In summary, the rotational-pendulum technique is an in pensive and accurate method to determine radii of gyratio rotors. The advantages of the rotational pendulum method a) no rotor-specific adapters or fixtures, b) no special ins mentation, c) quick and simple setup and measurements, d) good accuracy depending upon length and frequency m surements. The method is also applicable to other comp structures as long as they can be appropriately susp ed rotational pendulum.