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Measurement and Prediction of Free-Free Natural Frequencies and Mode Shapes of a Commercial Vehicle Turbocharger

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ABSTRACT

When designing a turbocharger, certain properties must be known to ensure a successful design. These properties include the free-free natural frequencies and the turbocharger's diametric and mass moments of inertia. The following report outlines simple methods to determine these vital properties, and provides a comparison to computational data obtained from XLTRC^2. The measured and predicted first natural frequencies were 834 and 850 Hz respectively, yielding a 2% error, while the measured and predicted second natural frequencies were 2,394 and 2,737 Hz respectively, yielding a 14% error. The measured and predicted diametric moments of inertia were 5.98E-3 and 5.92E-3 kg-m^2 respectively, with an error of 1%. The measured and predicted mass moments of inertia were 2.90E-4 and 2.71E-4 kg-m^2 respectively, with an error of 7%.

NOMENCLATURE

L_D = length of strings in diametric moment of inertia setup
r_D = distance of string from center of gravity in diametric moment of inertia setup
L_P = length of strings in polar moment of inertia setup
r_P = distance of string from center of gravity in polar moment of inertia setup
f = frequency of rotation
g = acceleration due to gravity
I = mass moment of inertia
σ = standard deviation

INTRODUCTION

In the design of a turbocharger, it is important to know the free-free natural frequencies, for they determine the critical speeds of a turbocharger. Most turbochargers operate above their first flexural mode to avoid large vibrations. This report outlines a simple method of measuring the free-free natural frequencies and mode shapes of a commercial vehicle (CV) turbocharger. As computer models become more prevalent, it is necessary to have a method to experimentally verify the model. The main goal of the experiment is to compare measured natural frequencies and mode shapes with predicted values.

DESCRIPTION OF EXPERIMENTAL APPARATUS

For the experiment, two strings 5 ft. in length suspend a CV turbocharger. A reference and a roaming accelerometer are placed at particular stations along the length of the turbocharger, as seen in Figure 1. The reference accelerometer is affixed at station 1, located at the compressor end, while the roaming accelerometer traverses the length to all stations.

XLTRC^2, a rotordynamic analysis program developed by the Texas A&M University Turbomachinary Lab, was used to generate the computer model and predicted responses [1]. Figure 2 illustrates the finite element geometry plot of the CV turbocharger used to predict the free-free natural frequencies.

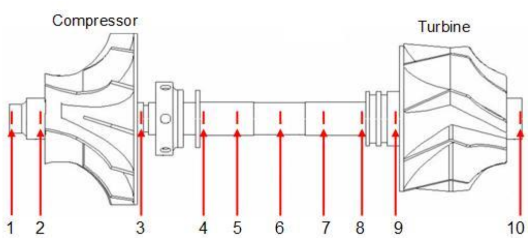

Figure 1. Volvo TC Accelerometer Positions

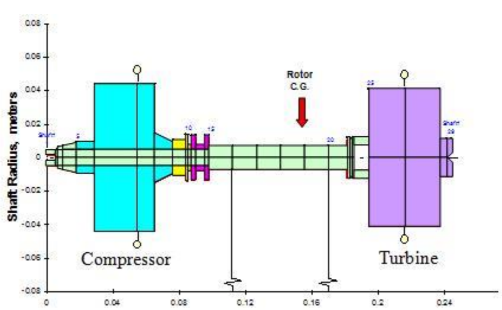

Figure 2. Volvo TC XLTRC^2 Geometry Plot

It is also of interest to measure the mass moments of inertia of the turbocharger as a secondary means of verifying the inputs to the computational model. Figure 3 shows the setup for determining the TC diametric mass moment of inertia. The TC is suspended from two equal length strings equidistant from the center of gravity. The center of gravity is determined by balancing the TC on a knife edge.

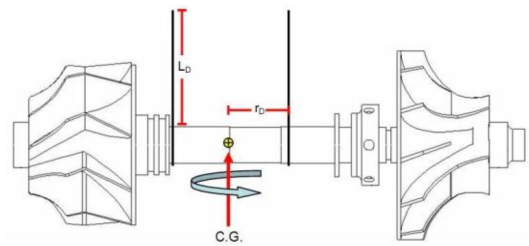

Figure 3. Diametric Moment of Inertia Setup

Figure 4 shows the setup for determining the polar mass moment of inertia. The TC is suspended vertically by two equal length strings equidistant from the polar center of gravity. The polar center of gravity is assumed to be the center of the rotor.

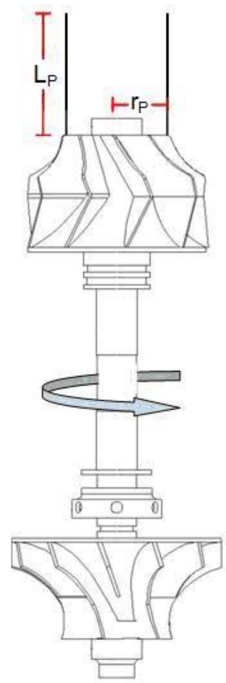

Figure 4. Polar Moment of Inertia Setup

PROCEDURE

Free-Free Natural Frequencies and Mode Shapes

First, the reference accelerometer was attached to Station 1 using wax, where it remained for the duration of the experiment. The roaming accelerometer was attached to Station 1 on the opposite side of the turbocharger. The turbocharger was excited with a strike from a steel bolt that was isolated from damping by a string tied to the bold head. The amplitude of acceleration, frequency, and phase angle of acceleration was recorded from the HP FFT for both the first and second modes. Five excitations were performed, and measurements were taken each time. The roaming accelerometer was moved to the next measurement station and the above was repeated, and so on for the remaining stations. This procedure allowed the relative dynamic response of the reference and roaming accelerometer to be mapped along the length of the turbocharger.

Mass Moments of Inertia

To find the diametric and polar mass moments of inertia, the turbocharger was set up as shown in Figures 3 and 4 respectively. The procedure to determine mass moment of inertia is described in reference [2]. Equation 1 is used to calculate the mass moment of inertia for each setup.

$$I = \left(\frac{r}{2\pi f} \sqrt{\frac{g}{L}} \right)^2 \qquad (1)$$

In the above equation, r is the distance from the string to the center of gravity, as shown in Figures 3 and 4, and L is the length of the strings for each setup. The frequency was determined by giving the turbocharger an angular displacement to initiate rotation about the appropriate axis and timing how long it takes for the turbocharger to complete 6 cycles. Ten trials were conducted and an average frequency was determined.

RESULTS

Free-Free Natural Frequencies and Mode Shapes

Both the ratio of the roaming to reference accelerometer amplitude and the difference in phase angles was found for each station. Appendix A contains the values for the first and second natural bending frequencies. The data was then plotted on the following figures along with the XLTRC[2] predictions. Figure 5 shows the comparison of the predicted and experimental first free-free mode shape, and Figure 6 shows the comparison of the predicted and experimental second free-free mode shape. The measured first natural frequency was 834 Hz, and the measured second natural frequency was 2394 Hz. Table 1 shows the comparison between the predicted and measured natural frequencies along with the percent differences. Additionally, the measured and predicted free-free mode shapes are superimposed above the turbocharger geometry plot in Figures 7 and 8 in Appendix B.

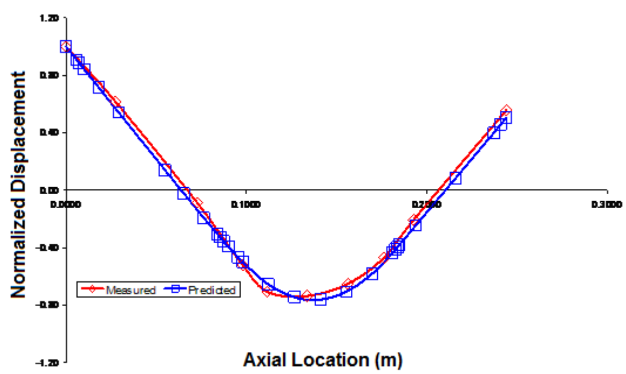

Figure 5. First Free-Free Mode Shape Comparison

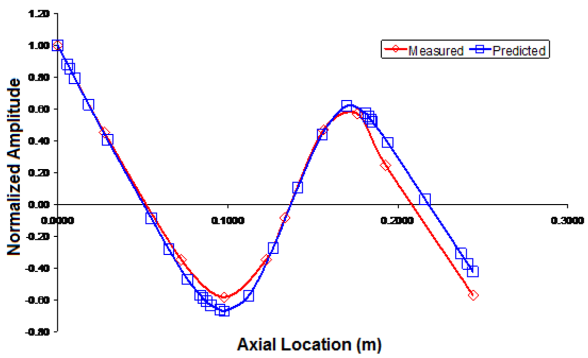

Figure 6. Second Free-Free Mode Shape Comparison

Table 1. Natural Frequency Comparison

Natural Frequency Comparison			
	Measured (Hz)	Predicted (Hz)	% Error
First	834	850	1.97
Second	2394	2373	14.32

<u>Mass Moments of Inertia</u>

Table 2 Mass Properties Comparison of TC

Mass Properties Comparison of TC			
Property	**Predicted**	**Measured**	**Percent Difference**
I_d (kg-m^2)	5.92E-03	5.98E-03	1.08
I_p (kg-m^2)	2.71E-04	2.90E-04	7.01
C.G. Location (mm)	156	153	1.85

The mass of the turbocharger is 1.34 kg. As stated previously, the measured mass moments of inertia are used as a secondary means to verify the accuracy of the computational model. As shown in Table 2, the predicted moments of inertia match very well with the measured values. The measured diametric moment of inertia differs by 1.08% while the polar moment of inertia differs by 7.01%.

<u>Uncertainty Analysis</u>

The placement of the accelerometers at various positions along the length of the turbocharger affects its natural frequencies. These changes in frequency are accounted for by averaging the frequencies and obtaining a standard deviation from Equation (2). From this, a high and low end confidence interval is constructed.

$$\sigma = \sum_{i=1}^{N} \frac{1}{N} (\omega_i - \omega_{avg})^2 \qquad (2)$$

The half scale uncertainty of the frequency measurement is ±2 Hz, so a high and low end uncertainty is calculated by adding or subtracting 2 Hz to all the measurements and finding the average and standard deviation. The uncertainty in the measurement of frequency for the first mode with a 99.9% confidence interval is ±12 Hz

with an error of 2.9%. The uncertainty in the measurement of frequency for the second mode with a 99.9% confidence interval is ±30.72 Hz with an error of 1.3%. The error in the mass moments of inertia is due to the uncertainty in measurement of mass of the TC, length of the strings, and the timing of the cycles. The uncertainty of the scale used to measure the mass is ±0.005 kg. The string length was measured by a ruler with an uncertainty of ±0.0625 in. The stopwatch used to measure the time of cycles is accurate to ±0.05 s.

Conclusions

As shown in Figures 5, 6, 7, and 8, the general trend of the experimental results agrees with the predicted values. The first free-free mode shape is a ½ sine curve, while the second free-free mode shape is a sine curve. The predicted natural frequencies were 850 Hz and 2,737 Hz, yielding a 2% error for the first natural frequency and a 14% error for the second natural frequency. The measured mass moments of inertia also match well with the predicted values. The diametric mass moment of inertia differs by 1% while the polar mass moment of inertia differs by 7%. The main source of error arises from the inaccuracies in material properties used in the computer analysis program. Also there exist inaccuracies in the modeling of the TC in the finite element geometry plot. Other possible sources of this error include voltage losses in the wiring, rounding error, miscalibration, and human error in the physical striking of the bar or the positioning of the roaming accelerometer. Overall, considering the crudeness of the experimental procedure and all of the sources of error, this simple method for measuring the free-free natural frequencies and mode shapes serves as a useful tool in verifying and validating the predictive model.

References

[1] Andriulli, John B., 1997, "A simple way to measure mass moments of inertia", *Sound and Vibration,* Nov, p. 18 -19.

[2] San Andres, L., 2008, "Measurement of Free-Free Mode Shapes," Personal Communication.

Appendix A: Raw Data

Table 3. First Mode Average Data

First Mode =	0.835	kHz		
Station #	Position	Node/Ref. amp.	Node-Ref. phase	Linearized Rel. amp.
	[m]	[mV]	[deg]	
1	0.0000	0.63	361	1.00
2	0.0274	0.39	180	0.62
3	0.0727	0.05	2	-0.09
4	0.0979	0.33	0	-0.52
5	0.1116	0.44	-2	-0.71
6	0.1336	0.46	-2	-0.73
7	0.1562	0.41	-2	-0.65
8	0.1763	0.29	2	-0.47
9	0.1927	0.13	2	-0.21
10	0.2441	0.35	-174	0.56

Table 4. Second Mode Average Data

Second Mode =	2.394	kHz		
Station #	Position	Node/Ref. amp.	Node-Ref. phase	Linearized Rel. amp.
	[m]	[mV]	[deg]	
1	0.0000	0.67	2	1.00
2	0.0274	0.30	-180	0.45
3	0.0727	0.23	-2	-0.35
4	0.0979	0.39	-1	-0.58
5	0.1225	0.23	-1	-0.35
6	0.1336	0.06	-180	-0.08
7	0.1562	0.31	-172	0.47
8	0.1763	0.38	-94	0.57
9	0.1927	0.17	2	0.25
10	0.2441	0.38	-1	-0.57

Appendix B: Mode Shape Overlay on TC Geometry

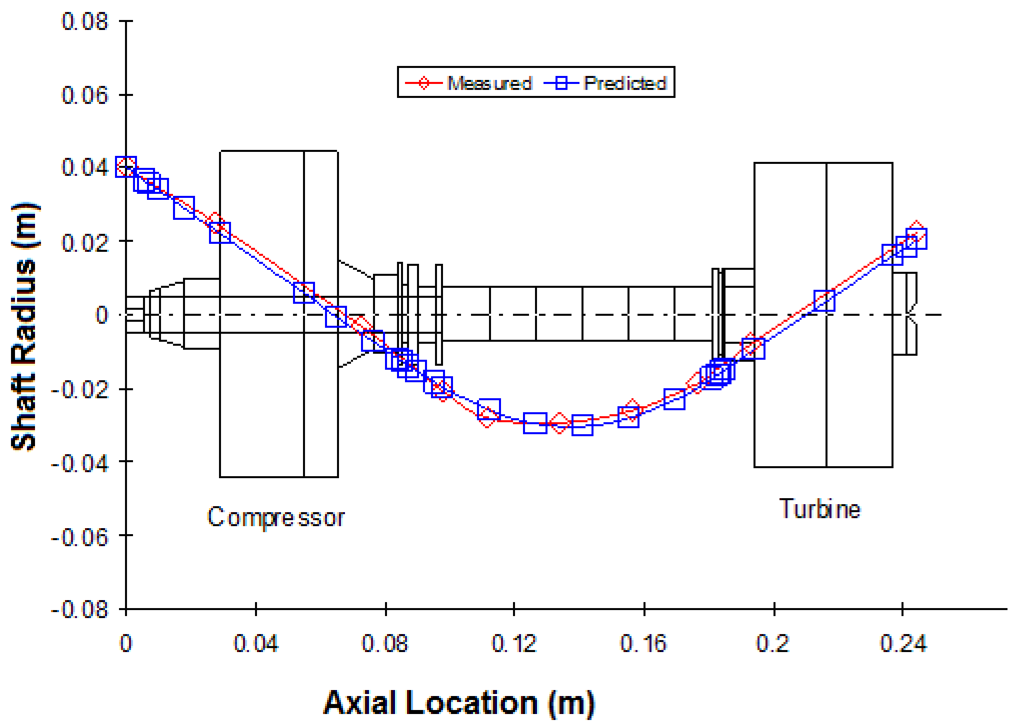

Figure 7. First Mode Shape Overlay on TC Geometry

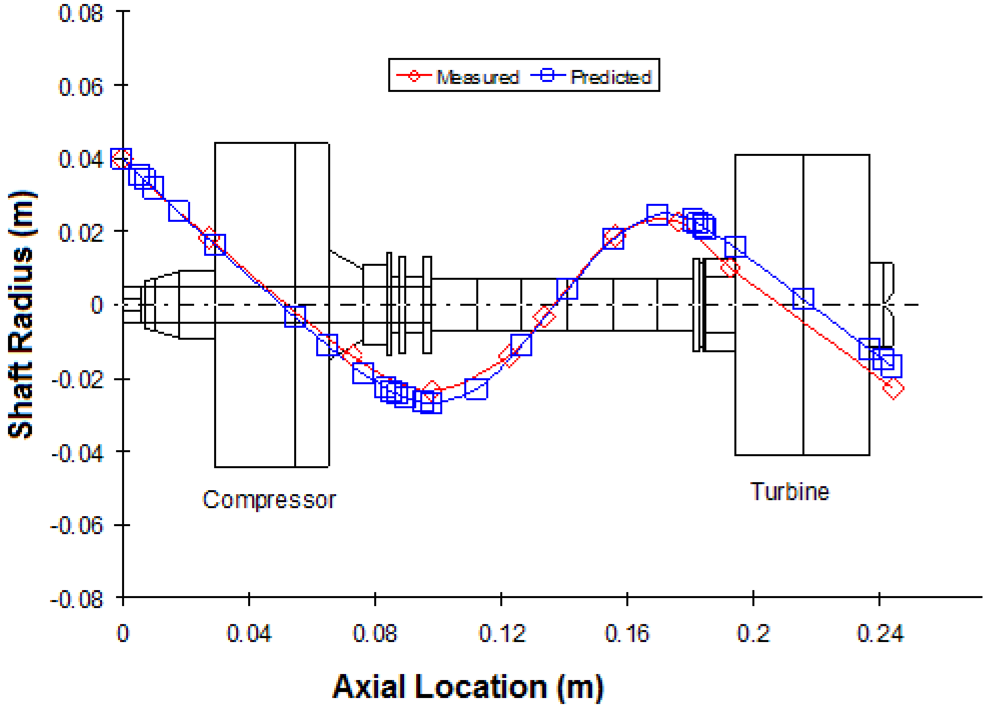

Figure 8 Second Mode Shape Overlay on TC Geometry