TO:	**Interested students**
FROM:	**Luis San Andres**
DATE:	**10/25/00**
RE.:	**Measurement ot free-free mode shapes**

EXPERIMENT

The procedure to obtain the free-free modes of the T-2 turbocharger rotor is the suspended bump test. The rotor was suspended from a hoist structure. The length of the suspension was five feet. This ensured that a low stiffness was obtained in the direction orthogonal to the nylon suspension strings. A one-gram accelerometer was wax-fixed to the center of the bearing span to provide a reference signal. A second one-gram acclerometer was used as the roaming acclerometer from which relative vibration was measured along the axis of the rotor. Nine positions of the rotor were measured for both the first and second modes, see Figure 1.

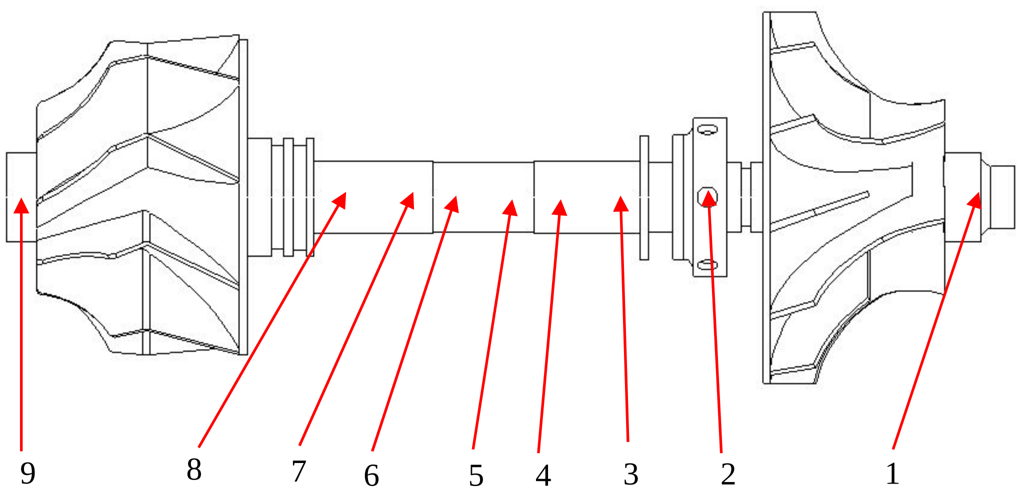

Figure 1. Axial locations of roaming acclerometer.

The transfer function of channel B to A is taken as the relation of vibration of the roaming accelerometer (B) to the reference accelerometer (A). The phase is measured to give the appropriate phase relation between both accelerometers. The sensitivity ratio is the ratio of the calibrated sensitivities of the two accelerometers. The sensitivity of the two accelerometers does not effect the overall modeshapes of the two natural frequencies. Table 1 lists the results of the measurements for the first mode while Table 2 lists the results for mode 2.

The Amplitude presented in Table 2 and 3 is computed from the measurements of the transfer function (B/A), Sesitivity Ratio, and Phase angle. Equation 4 shows the relationship between the amplitude and these measured parameters.

$$A = \left(\frac{B}{A}\right) \cdot (\mu) \cdot \cos(\phi) \tag{4}$$

Table 1. Modal measurements of T-2 rotor (1st Mode).

Axial Position	*Frequency (f)*	*TrFunx (B/A)*	*Amplitude (A)*	*Phase (ϕ)*	*Coherence*	*Sensitivity Ratio (μ)*
0.364	2000	1.09	0.994	-3	1.0	0.913
1.585	2016	0.491	-0.447	175	1.0	0.913
2.168	2012	1.06	-0.967	-182	1.0	0.913
2.505	2008	1.09	-0.995	-181	1.0	0.913
2.776	2008	1.12	-1.022	-182	1.0	0.913
3.064	2000	1.04	-0.949	-179	1.0	0.913
3.281	2012	0.974	-0.889	180	1.0	0.913
3.510	2017	0.855	-0.780	182	1.0	0.913
5.087	2005	1.09	0.994	3	1.0	0.913

Table 2. Modal measurements of T-2 rotor (2nd Mode).

Axial Position	*Frequency (f)*	*TrFunx (B/A)*	*Amplitude (A)*	*Phase (ϕ)*	*Coherence*	*Sensitivity Ratio (μ)*
0.364	5600	1.95	1.776	-4	1.0	0.913
1.585	5600	1.62	-1.479	-180	1.0	0.913
2.168	5600	0.966	-0.881	-178	1.0	0.913
2.505	5600	0.0412	-0.038	-179	1.0	0.913
2.776	5600	1.02	0.931	1	1.0	0.913
3.064	5600	1.8	1.641	3	1.0	0.913
3.281	5600	2.1	1.916	2	1.0	0.913
3.510	5600	2.25	2.051	3	1.0	0.913
5.087	5600	1.98	-1.801	-185	1.0	0.913

The two experimentally measured mode-shapes are presented in Figure 2.

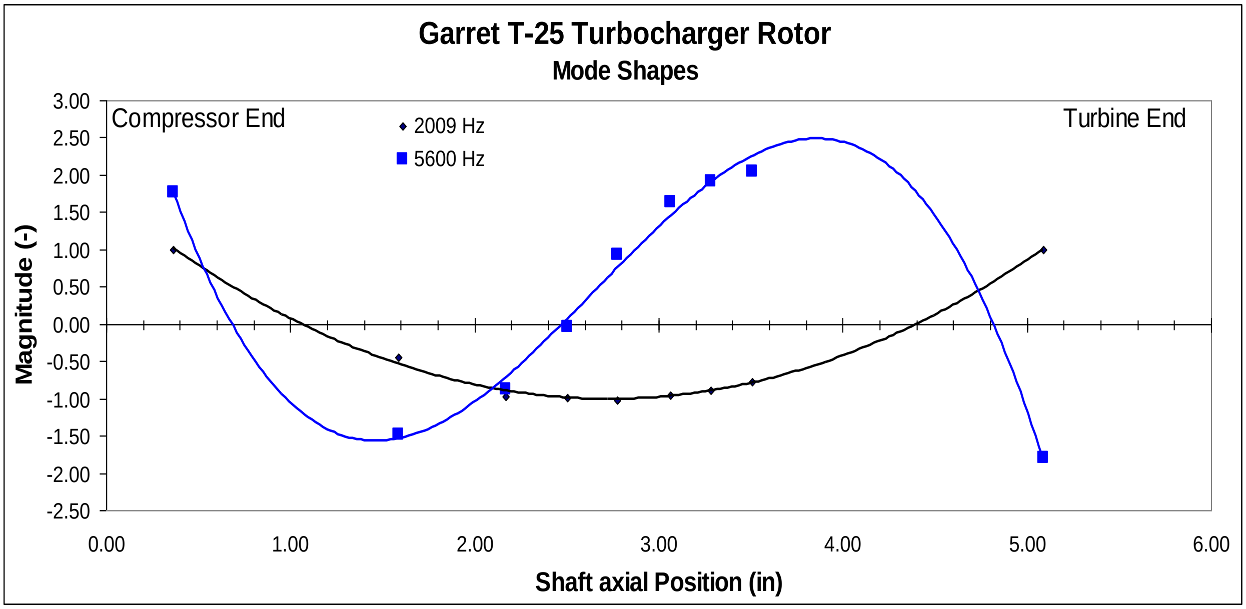

Figure 2. First and second mode-shapes.

UNCERTAINTY ANALYSIS OF MEASUREMENTS

The Kline-McClintock second power uncertainty analysis is not employed to determine uncertainty of the measurements of the natural frquencies. There are no equations developed to produce the natural frequency. It is simply measured with a signal analyzer. The uncertainty of the measurement of the natural frequency is simply half the scale of the frequency spectrum used to obtain the natural frequency. However, due to the added 1 gram masses of the two accelerometers to the rotor, the natural frequency of the rotor is modified. With the total mass of the rotor being 135.00 grams. The effect of two added accelerometers gives an uncertainty of.

$$\omega_n = \sqrt{\frac{k}{m + \Delta m}} \tag{1}$$

In obtaining mode-shapes one of the acclerometers is stationary while the second acclerometer is used to traverse the axial direction of the rotor. Due to the changing position of the second acclerometer on the rotor, the natural frequencies of the rotor change. These changes in frequency are accounted by a statistical method of averaging (eq. 2) the natural frequency at all measurements and obtaining the standard deviation (3).

$$\omega_{avg} = \frac{1}{N}\sum_{i=1}^{N}\omega_i \tag{2}$$

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\omega_i - \omega_{avg}\right)^2} \tag{3}$$

The half scale uncertainty of the frequency measurement is 2 Hz. To obtain the high end of uncertainty of a measurement, add the 2 Hz half scale to each individual measurement and obtain the average and standard deviation. Do the same for the low end of the uncertainty of the measurement. Depending on desired accuracy, apply one standard deviation for 67%, two standard deviaitions for 95%, and three standard deviations for 99.99%.

The uncertainty in measurement of the first mode of the rotor with a 99.99% confidence level is ± 20.5 Hz. This gives an error of 1.02 %. For the higher 5,600 Hz measured natural frequency, there are no deviations in the measured frequencies. This is primarily due to the decreased resolution of measurement from the frequency spectrum. The bandwidth is now 40 Hz. This gives an uncertainty in the measurement of ± 20 Hz and a possible error in measurement of 0.36%.