Example 1 – Zero Gravity flight

Luis San Andrés, 2010

During a *zero gravity* flight, an aircraft engine thrust is set to equal drag force while lift is removed. Hence, weight is the only force acting on the plane and, like any unpowered object fired into the air, it will move in a parabolic curve as **it freefalls.**

The experience starts at **7,700 m** while the AIRBUS A300 cruises at **500 miles/hour**. Engines are *shut off*[*]! An instant later, the world inside the plane becomes chaos: people screaming as they are lifted, as if they are weightless. **20 seconds later**, the engines roar back to life. The people inside, some pinned to the cabin roof, slam back against the floor.

Visit http://gozerog.com (weightless flight experience)

Develop mathematical analysis, based on physical principles and detailing assumptions, towards prediction of the aircraft motion.

At the end of the *weightless* experience, at $t=20$ sec,

a)   what is the aircraft speed (km/hour) and its acceleration (g's)? How many meter did the aircraft fall? How many km did the aircraft travel along the horizontal direction?

b)   Demonstrate that the relationship between Y and X is indeed a parabola, i.e. $Y \sim X^2$
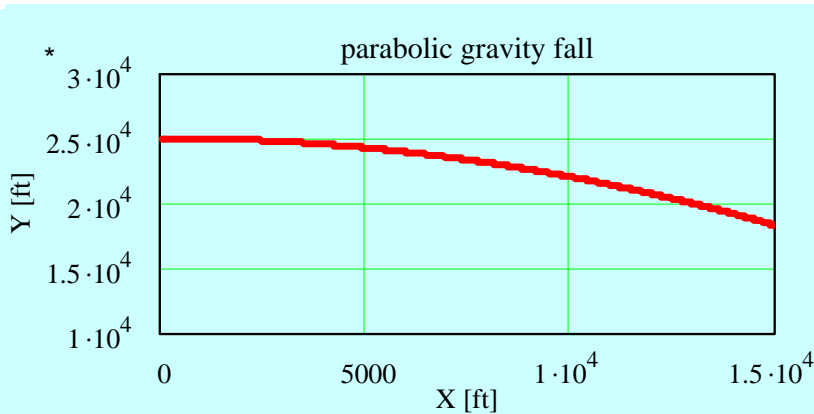
c)   At a price tag of $4,950 USD, would you pay for the weightless experience?

d)   Tell me about other examples where you already felt this experience?

parabolic gravity fall
Y [ft] vs X [ft]

Note: jotting formulas w/o explanatory sentences counts very little

Airplanes fly by forcing air over their wings in such a way that there is an excess of the upwards force, lift. While in the air, the plane stays up thanks to lift and moves forward because of the forward thrust provided by the engines. The aircraft battles against two forces trying to slow it down and pull it out of the sky - air friction and gravity. On a normal flight, the amount of thrust will be greater than the friction and the lift will equal the weight so that the plane stays up.

[*] During a parabolic flight, thrust is set to equal air friction while lift is simultaneously removed.

Example. Zero gravity(?) parabolic flight   Luis San Andres (2011)

Some useful conversions

$$\text{hour} := 3600 \cdot s$$
$$\text{mile} = 1.609 \times 10^3 \, m$$

DATA:

$$Y_o := 7700 \cdot m \qquad \text{start altitude for weightless experience}$$

$$V_o := 500 \cdot \frac{\text{mile}}{\text{hour}} \qquad \text{AIRBUS speed (constant throught fall)}$$

ANALYSIS:

A. Set the stage (nomenclature)

<u>Define coordinate system</u>

X: Horizontal direction

Y: Vertical direction

V_X: **speed** $V_o = 223.52 \cdot \dfrac{m}{s}$

<u>Define forces:</u>
T: Thrust, **D**: Drag, **L**: Lift, **W**: Plane weight = **Mg**,

<u>Define variables:</u>   **g**: gravitational constant
a: acceleration = d**V**/dt
V: velociy

$$g = 9.807 \frac{m}{s^2}$$

<u>Other</u>
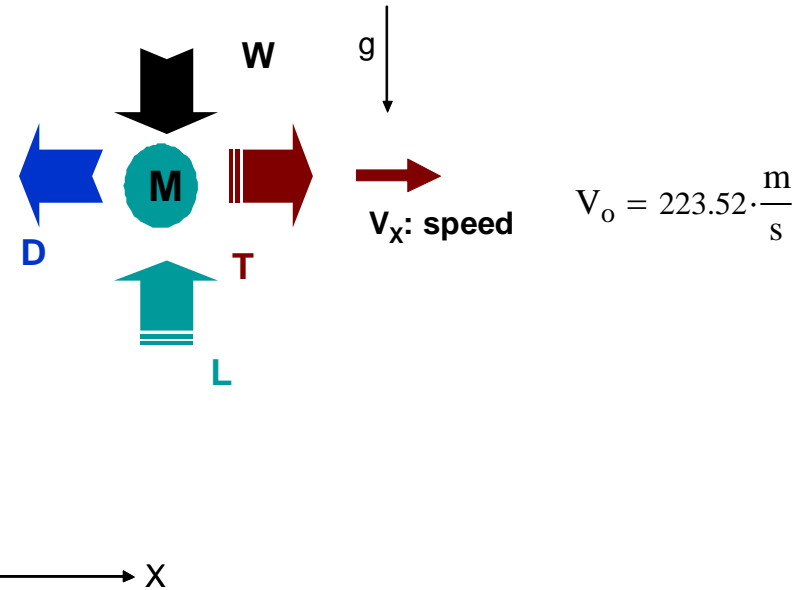M = mas of plane
t = time

Figure 1. Coordinate system and free body diagram

B. Apply fundamental principles

The equations of MOTION (Newton's law) for the plane (considered as a point mass) are

$$M \cdot a_X = T - D$$
$$M \cdot a_Y = L - W \qquad (1) \qquad\qquad W = M \cdot g$$

where a_X and a_Y are the plane components of acceleration along the X and Y directions

C. State and apply assumptions

(From problem statement)
At t=0, Lift is removed and engines keep enough thrust to overcome drag. That is; **T=D** and **L=0**. Hence, gravity is the only external agent causing the plane motion.

With the known assumptions and knowledge, the equations of motion (1) simplify to

$$a_X(t) := 0$$

$$a_Y(t) := -g$$

(2)   which are well kown equations for free fall.

D. Apply mathematics to obtain solution (prediction of motion)

Integration of the accelerations in Eqs. (2) gives

$$V_X(t) := V_o$$

(3)   Note: Eq (3b) assumes vertical velocity is nil at t=0 s

$$V_Y(t) := -g \cdot t$$

That is, the plane keeps its horizontal speed while it free falls, its vertical speed increases with time

The plane speed is $V(t) := \left(V_Y(t)^2 + V_X(t)^2\right)^{0.5}$ (4)

Integration of the velocity Eqs. (3) gives

the horizontal and vertical displacements as

$$X(t) := V_o \cdot t \quad \textbf{(5a)}$$

$$Y(t) := Y_o - \frac{g}{2} \cdot t^2 \quad \textbf{(5b)}$$

$$Y_o = 7.7 \times 10^3 \, m$$

Thus, the plane treveled distance along the horizontal axis increases with time, while the vertical displacement is proportional to the square of time.

From Eq, (5a), extract time (t), t=X/Vo, and replace it into Eq. (5b), to obtain $y(x) := Y_o - ss \cdot x^2$ (6)   where $ss := \left(\dfrac{g}{V_o^2}\right) \cdot \dfrac{1}{2}$ (7)

$$ss = 9.814 \times 10^{-5}\,\frac{1}{m}$$

Eq. (6) describes a typical parabola. Figure 2 describes the parabolic flight of the plane as it freefalls.

parabolic gravity fall

Y [m]

8×10^3

7.5×10^3

7×10^3

6.5×10^3

6×10^3

*

0   1×10^3 2×10^3

X [m]
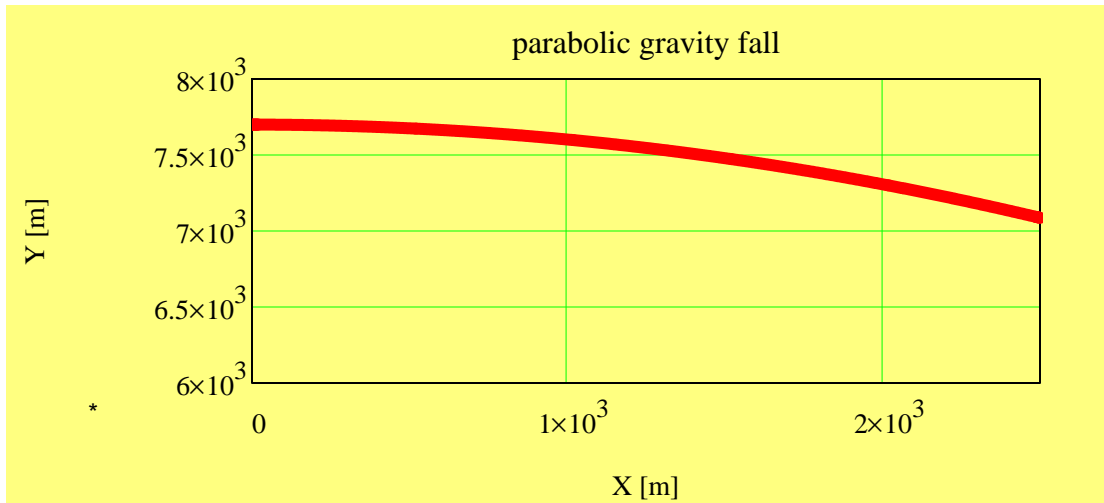
Note that the plane vertical velocity can be written as

$$v_Y(x) := \frac{-g}{V_o}\cdot x \quad (8)$$

Figure 2. Parabolic path of airplane during free-fall, Y(X)

After $t := 20\cdot s$

$$\left|Y(t) - Y_o\right| = 1.961 \times 10^3\,m \quad \text{height of fall}$$

$$X(t) = 4.47\cdot km \quad \text{horizontal distance traveled by plane } \textbf{with acceleration = g}$$

$$V(t) = 1.071 \times 10^3 \cdot \frac{km}{hour} \quad \textbf{speed of aircraft}$$

or $\quad V(t) = 665.199 \cdot \dfrac{mile}{hour}$

At 25,000ft, the pilot cuts power to the engines. For the briefest of moments, there is silence. Then, the world inside the plane become a phantasmagoria: people gasp and scream as they are lifted, limbs moving chaotically. Bottles filled with coins rise into the air and th contents begin a serene dance, as if held by an unknown force.

Finally, 20 seconds after the Airbus A300 begins plummeting towards the Mediterranean, the engines roar back to life. The people inside, some of whom are pinned to the roof of the cabin, slam back against the floor. The quiet that follows is broken by a sharp hiss. voice on the intercom crackles "Next parabola, one minute."

Thanks to these flights, scientists can, for 20 seconds at a time, test satellite components or predict problems with experiments destine for the International Space Station before they are sent into orbit. Falling out of the sky in an airplane is the only practical way to experience microgravity without leaving Earth.

At a safety briefing on the morning of my arrival, the pilot explains how the flight will proceed followed by a doctor who tells us how to survive the experience. Airplanes fly by forcing air over their wings in such a way that there is an excess of the upwards force, lift. Whi in the air, the plane stays up thanks to lift and moves forward because of the forward thrust provided by the engines. The aircraft battle against two forces trying to slow it down and pull it out of the sky - air friction and gravity. On a normal flight, the amount of thrust will b greater than the friction and the lift will equal the weight so that the plane stays up.

On a parabolic flight, thrust is set to equal air friction while lift is simultaneously removed. This leaves weight as the only force acting o the plane and, like any unpowered object fired into the air, it will move in a parabolic curve as it freefalls.
At this point, because everything inside the aircraft begins falling to Earth at exactly the same speed as the plane, the contents act as they are weightless.