$$B_1 = \frac{1.14}{K_0}\left[\frac{\rho_1 C_1 U_1 h}{2K_0}\right]^{0.013}\left[\frac{2L}{h}\right]^{-1.003}$$

$$\left[\frac{K_1}{K_0}\right]^{0.013} e^{-900\times 10^{-6}\left[\frac{U_1\rho_0 c_0 h}{2K_0}\right]} \quad \text{(A-5)}$$

$$B_2 = \frac{1.14}{K_0}\left[\frac{\rho_2 C_2 U_2 h}{2K_0}\right]^{0.013}\left[\frac{2L}{h}\right]^{-1.003}$$

$$\left[\frac{K_2}{K_0}\right]^{0.013} e^{-900\times 10^{-6}\left[\frac{U_2\rho_0 c_0 h}{2K_0}\right]} \quad \text{(A-6)}$$

$$q_t = |U_1 - U_2|\, Wf \quad \text{(A-7)}$$

where all the variables are defined in the notation except the film thickness h. A iteration process is needed to decide f from equation (1) and the temperature rise in this Appendix.

$$h = \epsilon h_0 \quad \text{(A-8)}$$

ϵ is a factor proposed by W. R. Wilson and S. Shen [17]:

$$\epsilon = \frac{1}{1 + 0.241[(1 + 14.8z^{0.83})\delta^{0.64}]} \quad \text{(A-9)}$$

where

z — is the sliding/rolling ratio

$$\delta = \eta_0 \gamma U^2/K \quad \text{(A-10)}$$

where

η_0 — is the lubricant viscosity at the entry condition
λ — is the temperature viscosity coefficient of the lubricant
U — is the mean rolling velocity
K — is the heat conductivity of the lubricant

Coating Thickness Effects on Temperature Rise

It should be noted here that the Formula (A-1) is derived for the case when the two entire disks have homogeneous properties, i.e., K_1, C_1, ρ_1, E_1 for disk 1 and K_2, C_2, ρ_2, E_2 for disk 2. In order to use the formula to calculate the temperature rise for coated surfaces, the temperature penetration depth D is calculated as:

$$D = \sqrt{\frac{5KL}{\rho CU}} \quad \text{(A-11)}$$

The temperature penetration depth in most cases is much bigger than the coating thickness. This means that both the coating material properties and the base material properties must be considered during applying Formula (A-1). Therefore, a coating thickness factor β is used to modify the temperature rise calculated with the coating material properties:

$$\beta = 1 - e^{\frac{h_c/D}{\tau}} \quad \text{(A-12)}$$

where

h_c — coating thickness
D — temperature penetration depth for steel under the corresponding conditions
τ — is a constant with a value of 0.033

Effective Viscosity

Using the notation:

T_b = absolute bulk disk temperature (for example, $T_b = 273.16 + °C$)

ΔT_s = temperature rise for steel-steel contact from formula (A-1)

ΔT_c = temperature rise from formula (A-1) using the material properties of the contacting surfaces for steel coating contact

$\Delta T = \Delta T_c - \Delta T_s$ temperature rise difference between the steel-coating contact and the steel-steel contact

ΔT_e = effective temperature rise difference between the steel-coating contact and the steel-steel contact

Then,

$$\Delta T_e = \Delta T \beta \quad \text{(A-13)}$$

where

β — is the coating thickness factor from the previous section.

Then, $T_e = T_b + \Delta T_e$ is used to calculate the viscosity for that coating condition.

Approximate Design of Statically Loaded Cylindrical Journal Bearings

Luis A. San Andres[1]

This technical note presents an approximate solution for the fluid film forces and design coefficents for statically loaded cylindrical journal bearings. The solution is an improved correction to the infinitely long journal bearing model. The formulas derived are of extreme simplicity and the results are excellent for most practical journal bearing configurations.

Nomenclature

c = journal bearing radial clearance
Cf = $\mu\omega R^3 L/c^2$, force conversion coefficient
Cp = lubricant specific heat
$Cpre$ = $\mu\omega R^2/c^2$, pressure conversion coefficient
Cqe = $Qe/RNcL$, entrance flow coefficient
Cqm = $Qm/RNcL$, flow coefficient at minimum gap
Cqo = $Qo/RNc1$, lateral flow coefficient
Ct = $J\rho C_p(T_2 - T_1)LD/W$, coefficient of temperature increment
$C\mu$ = $(F/W)\times(R/c)$: friction coefficient
D = $2\times R$ = journal bearing diameter
e = journal bearing static eccentricity
F = friction force on rotating journal
fr, ft = Fr/Cf, Ft/Cf: = dimensionless radial and tangential fluid film forces
fr = $\frac{-12\epsilon^2}{(2+\epsilon^2)\gamma^2}$ — radial film force for long bearing
ft = $\frac{6\pi\epsilon}{(2+\epsilon^2)\gamma}$ — tangential film force for long bearing
H = $1+\epsilon\cos\theta$: dimensionless film thickness $h = cH$
J = 4.1868 Nm/Cal: mechanical energy equivalent
L = journal bearing axial length
L/D = journal bearing slenderness ratio
N = $\omega/2\pi$: angular speed in rev/s
p = $P/Cpre$: dimensionless pressure field

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p_∞ = $\dfrac{6\epsilon \sin\theta (H+1)}{(2+\epsilon^2)H^2}$: pressure field for Sommerfeld solution
P_0 = $C\mu Wc\omega$: power loss
S = $\mu NLD(R/c)^2/W$: Sommerfeld number
T_0 = $C\mu Wc$: friction torque
$T1$, $T2$, Tm = lubricant temperatures at inlet, minimum gap and axial exit of bearing
W = static load on journal bearing = $\sqrt{Fr^2+Ft^2}$
Zi = axial correction function, equation (3)
β = $\tan^{-1}$ 1-ft/fr1: attitude angle
ξ = z/R = dimensionless axial coordinate
ϵ = e/c = static eccentricity ratio
γ = $(1-\epsilon^2)^{1/2}$
θ = circumferential coordinate
λ = eigenvalue for forces and coefficients calculations
ρ = lubricant density
μ = lubricant absolute viscosity
ω = journal angular speed <1/s>

Subscript

∞ = refers to the infinitely long journal bearing model

Introduction

Modern design of cylindrical journal bearings for rotating machinery support normally requires of long numerical codes and extensive use of computers with high costs of execution and time sharing. The ready availability of these complicated analyses is out of hand for field engineers and small companies dedicated to manufacture machinery like is the case in most underdeveloped countries. There then, it is necessary to resort to approximate analyses which could answer specific design questions in a minimum of time and without great expenses.

Several approximate analyses on journal bearing design have been reported in the literature and are still widely used. Most approximate analyses are based on the application of correction factors to the long (Sommerfeld) or short (Ocvirck) bearing models, and improve the limits of applicability of the original models. Relevant analyses currently used for finite length journal bearing design are given by Warner (1963), Rhode (1973), Barrett et al. (1980), and Reason (1981).

This note introduces an approximate analysis for the rapid calculation of bearing coefficients used in practice to design statically loaded journal bearings. The analysis is an extension of an original work developed by the author primarily for squeeze film damper design (San Andres, 1984, 1985, 1987a, 1987b). The solution presented is of extreme simplicity and can be implemented in a hand calculator.

Analysis

Most approximate solutions to the Reynolds equation in cylindrical journal bearings are based on the fundamental work of P. C. Warner (1963). This solution introduces an approximate pressure field as the product of the pressure (p_∞) for the long bearing model and an axial correction factor, i.e.,

$$p \cong p_\infty \{1-\cosh(\lambda\xi)/\cosh/\lambda L/D)\} \tag{1}$$

in the region $0 \le \theta \le \pi$, $-L/D \le \xi = z/R \le L/D$.
The parameter λ is the first eigenvalue of the resulting boundary value problem and it is a function of the eccentricity (ϵ) and slenderness (L/D) ratios. (Rhode, 1973).

Radial and Tangential fluid film forces, fr and ft, are given in dimensionless form by:

$$fr = Fr/Cf = \oint p \cos\theta d\theta d\xi \cong Zr \cdot fr$$
$$ft = Ft/Cf = \oint p \sin\theta d\theta d\xi \cong Zt \cdot ft \tag{2}$$

where, fr_∞, ft_∞ are the bearing forces for the long bearing model and Zi is an axial correction factor equal to (Warner, 1963, Rhode, 1973):

$$Zi = 1 - \tanh(\lambda iL/D)/(\lambda iL/D) \tag{3}$$

where $\lambda i = \lambda r$ or λt. The appropriate eigenvalues to be used for evaluation of forces are given by San Andres (1984, 1985, 1987a, 1987b) as equal to:

$$\lambda t = (\lambda s - \lambda m)e^{-L/D} + \lambda m \tag{4a}$$

$$\lambda r = \sqrt{2}\{(\lambda s - \lambda m)e^{-L/D} + \lambda m\} \tag{4b}$$

where

$$\lambda_m^2 = \frac{2(2+\epsilon^2)(1+\gamma)}{4\gamma^2+4\gamma-\gamma\epsilon^2}, \text{ and } \lambda_s^2 = \frac{2+\epsilon^2}{2\gamma^2} \tag{4c}$$

with $\gamma = (1-\epsilon^2)^{1/2}$

The functional form of the eigenvalue derived makes the approximate forces given by equation (2) to be equal to the short bearing (Ocvirck) forces in the limit as L/D tends to zero. Thus, the force solution proposed is exact for the infinitely long and short bearing models.

On equilibrium, the resultant film force is equal to the applied external load (W) on the bearing. This leads to the definition of the Sommerfeld Number (S) and equilibrium attitude angle (β).

$$S = \frac{\mu NLD}{W}\left(\frac{R}{c}\right)^2 = \frac{1}{\pi\{Z_r^2 fr_\infty^2 + Z_t^2 ft_\infty^2\}^{1/2}} = S(\epsilon, L/D) \tag{5a}$$

$$\beta = \tan^{-1}\{-ft/fr\} \tag{5b}$$

The Sommerfeld Number (S) is a design parameter known a-priori, and (5a) is a nonlinear equation which when solved gives the journal bearing static eccentricity (ϵ) for a given L/D ratio.

The analysis is extended to derive simple formulae for the most important journal bearing design coefficients used in practice.

(a) *Friction Coefficient, $C\mu$* proportional to the surface friction force (F) and given by (Reason, 1981):

$$C\mu = \frac{F}{W}\frac{R}{c} = 2\pi^2 S/(1-\epsilon^2)^{1/2} + (\epsilon/2)\sin\beta \tag{6}$$

This cofficient is used to evaluate the friction torque (To) and power loss (Po) of the journal bearing.

(b) *Entrance Flow Coefficient, Cqe* is related to the circumferential feed flow (Qe) at the maximum gap ($\theta = 0$), defined by (Pinkus, 1961, Reason, 1981), and given here as:

$$Cqe = \frac{Qe}{RNcL} = \pi(1+\epsilon) - \pi\epsilon Ze(1+\epsilon)(2+\epsilon)/2+\epsilon^2) \tag{7}$$

(b) *Flow Coefficient at Minimum Gap, Cqm* is related to the circumferential flow (Qm) at the minimum gap ($\theta = \pi$). In the present analysis and following common practice, $\theta = \pi$ is considered as the onset of fluid cavitation. The coefficient of minimum flow is defined by (Reason, 1981) and given here by:

$$Cqm = \frac{Qm}{RNcL} = \pi(1-\epsilon) + \pi\epsilon \cdot Zm \cdot (\epsilon-1)(\epsilon-2)/(2+\epsilon^2) \tag{8}$$

(c) *Flow Ratio Coefficient, Qo/Qe* gives the ratio of the outlet axial flow (Qo) at the bearing ends to the feed flow (Qe). This relation is given by (Reason, 1981) as:

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$$Qo/Qe = Cqo/Cqe = 1 - Cqm/Cqe \tag{9}$$

In equations (7) and (8) the optimum eigenvalue to be used in the axial correction factors (Ze, Zm) is equal to:

$$\lambda = 1 - e^{-2L/D} \tag{10}$$

(*d*) *Coefficient of Temperature Increment, C_t* A simple balance of energy considers the conversion of mechanical energy into heat which is conveyed entirely to the lubricant and produces an increase in its temperature. As in the analyses of Raimondi (1958) and Lund (1966), the exit temperature is supposed to be equal to $Tm = 0.5(T1 + T2)$, where $T1$ and $T2$ are the fluid temperatures at the maximum and minimum gaps, respectively. With $\Delta T = T2 - T1$, the coefficient of temperature increment (Ct) is defined and given by (Reason, 1981) as:

$$Ct = \frac{J\rho Cp\Delta TLD}{W} = \frac{4\pi C\mu}{Cqe\left(1 - \frac{1}{2}\frac{Qo}{Qe}\right)} \tag{11}$$

It should be noted that these temperature considerations should be used with caution since the isothermal assumption is highly conservative and for improved results a thermohydrodynamic model should be employed.

Results

Figures 1 and 2 depict the static eccentricity ratio (ϵ) and attitude angle (β) as functions of the Sommerfeld Number (S) for increasing L/D ratios. The continuous lines show the results of the present approximate analysis while symbols represent numerical results from Lund (1966) and a full numerical solution developed by the author. A comparison of results reveals the accuracy of the approximate solution except for low Sommerfeld numbers (high eccentricities). The error of the approximate solution is no greater than 5 percent for the highest eccentricity numerical results ($\epsilon = 0.9$). A notable fact of the present solution shows that its accuracy is excellent for journal bearings with small L/D ratios, a condition which not only is ever present in practice but can not be well predicted by the traditional analyses of Warner (1963) and Rhode (1973).

Comparison of the present solution with other numerical and approximate solutions, such as from Raimondi and Boyd

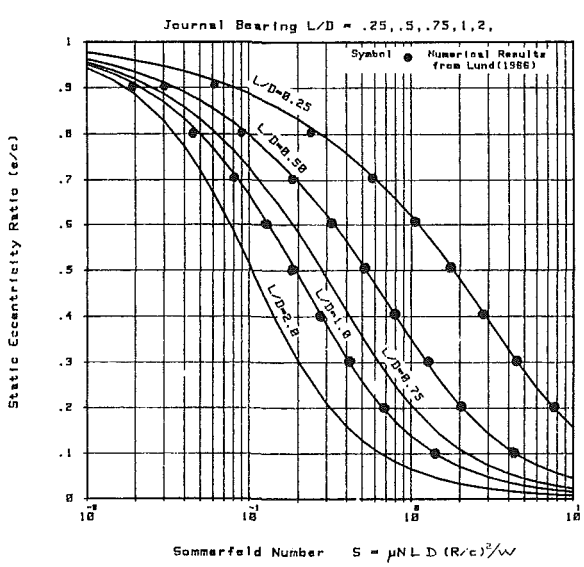

Fig. 1 Approximate eccentricity ratio (ϵ) versus Sommerfeld number. *L/D* ratio varies. Comparison with numerical results.

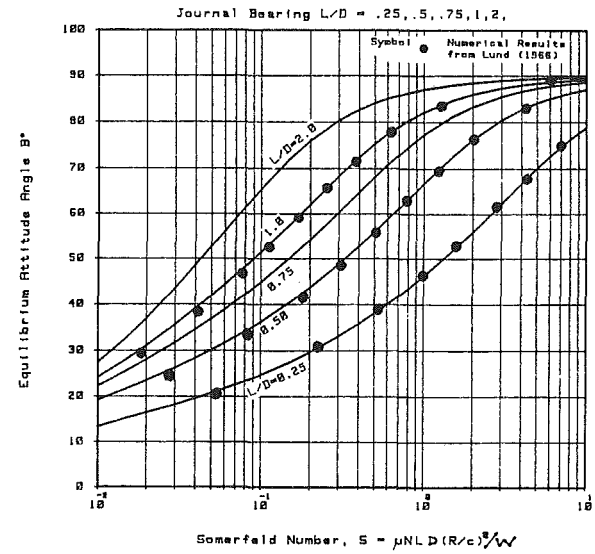

Fig. 2 Approximate attitude angle (β) versus Sommerfeld number. *L/D* ratio varies. Comparison with numerical results.

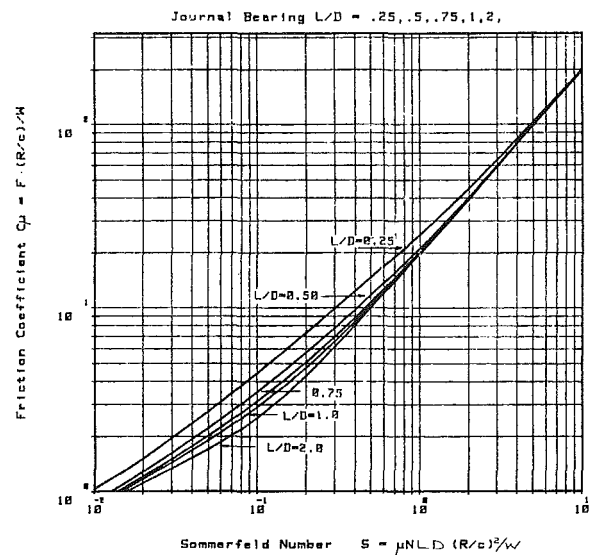

Fig. 3 Approximate friction coefficient ($C\mu$) versus Sommerfeld number. *L/D* ratio varies.

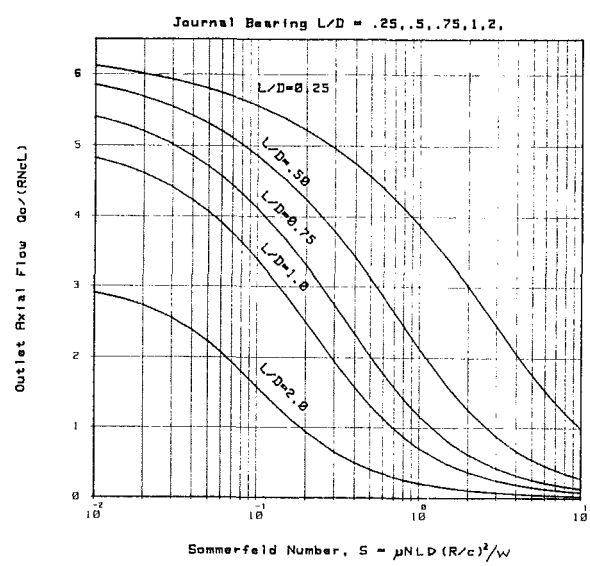

Fig. 4 Approximate lateral flow coefficient (Cqo) versus Sommerfeld number. *L/D* ratio varies.

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(1958), Hashish et al. (1980) and B. Reason (1981), is reported to be excellent for all coefficients and is ommitted here due to the nature of this technical brief. The author (San Andres 1984, 1985, 1987a, 1987b) has established that the present solution improves considerably Warner's approximation, specially for large eccentricities and low L/D ratios.

Figures 3 and 4 show the coefficient of friction ($C\mu$), and the dimensionless side flow coefficient (Cqo), as functions of the Sommerfeld number for increasing L/D ratios. These figures are intended to help in the rapid global design of journal bearings which by nature is iterative and involves the particular viscosity-temperature relationship of the lubricant. For particular cases, given the values of the Sommerfeld number and L/D ratio, the formulae given in equations (3) to (11) can be easily implemented in a programmable calculator and the bearing coefficients determined with the aid of a simple root solver algorithm.

Conclusions

An analysis for the approximate calculation of design coefficients for statically loaded cylindrical journal bearings is presented. The approximate solution is exact in the limits, as $L/D \rightarrow \infty$ for long bearing model, and as $L/D \rightarrow 0$ for the short bearing model, and comparisons with numerical results for finite length bearings are highly satisfactory. The simplicity of the solution enables it to be used in manual design computations with great accuracy.

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