

## Notes 2. Appendix

# One-Dimensional Fluid Film Bearings

Analysis for load capacity and drag in

**A. Plane Slider Bearing**

read about the Kingsbury bearing at the end of this document (Aug 2012)

**B. Rayleigh Step Bearing**

**C. Elementary Squeeze Film Flow**

**Objective: To understand the static load performance of simple 1D bearings. Actual configurations in practice follow similar design guidelines to ensure optimum performance.**

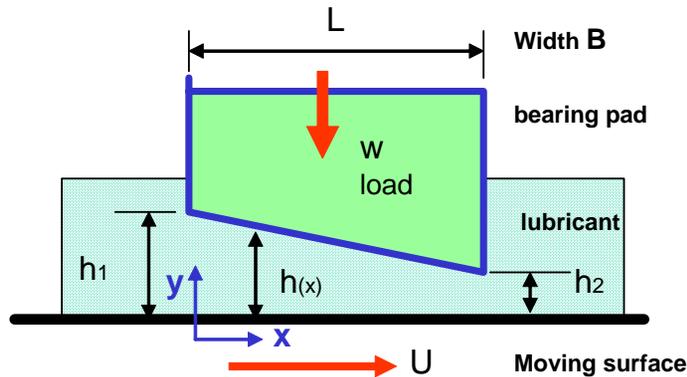
Modern Lubrication. Luis San Andrés © 2009

# Analysis of 1D slider bearing:

Luis San Andres (c) 2009 (NUS)

Figure 1 shows the geometry and coordinate system of a one-dimensional slider (thrust) bearing. The film thickness ( $h$ ) has a linear taper along the direction of the surface velocity  $U$ . The film wedge generates a hydrodynamic pressure field that supports an applied load  $w$ .

Note that the (exit) minimum film thickness  $h_2$  is unknown and must be determined as part of the bearing design. The bearing taper ( $h_1-h_2$ ) is a design parameter (determined from analysis).



film thickness,  $h(x)$

$h_1$  at inlet (leading edge)

$h_2$  at exit (trailing edge)

Fig. 1 Geometry of taper slider bearing

## Nomenclature:

- $U$  surface speed
- $L$  bearing length
- $B$  bearing width,  $B \gg L$
- $\mu$  lubricant viscosity
- $w$  Load

## Assumptions:

- incompressible lubricant, isoviscous,
- steady state operation,  $dh/dt=0$
- width  $B \gg L$
- no fluid inertia effects

$$h(x) = h_1 + (h_2 - h_1) \cdot \frac{x}{L} \quad (1)$$

$$h_1 > h_2$$

**Reynolds eqn.** for generation of hydrodynamic pressure (p) reduces to:

$$\frac{d}{dx} \left( \frac{h^3}{12 \cdot \mu} \cdot \frac{d}{dx} p - \frac{U \cdot h}{2} \right) = 0 \quad \text{with } p=0 \text{ (ambient) at } x=0 \text{ and } L \text{ (inlet and exit of bearing)} \quad (2)$$

Define the following dimensionless variables

$$X = \frac{x}{L} \quad P = \frac{p}{\frac{6 \cdot \mu \cdot U \cdot L}{h_2^2}} \quad H = \frac{h}{h_2} \quad H(X, \alpha) := \alpha + (1 - \alpha) \cdot X \quad (3)$$

where  $\alpha = \frac{h_1}{h_2}$  is a **film thickness ratio or taper ratio**

hence, Eq. (2) becomes

$$\frac{d}{dX} \left( H^3 \cdot \frac{d}{dX} P - H \right) = 0$$

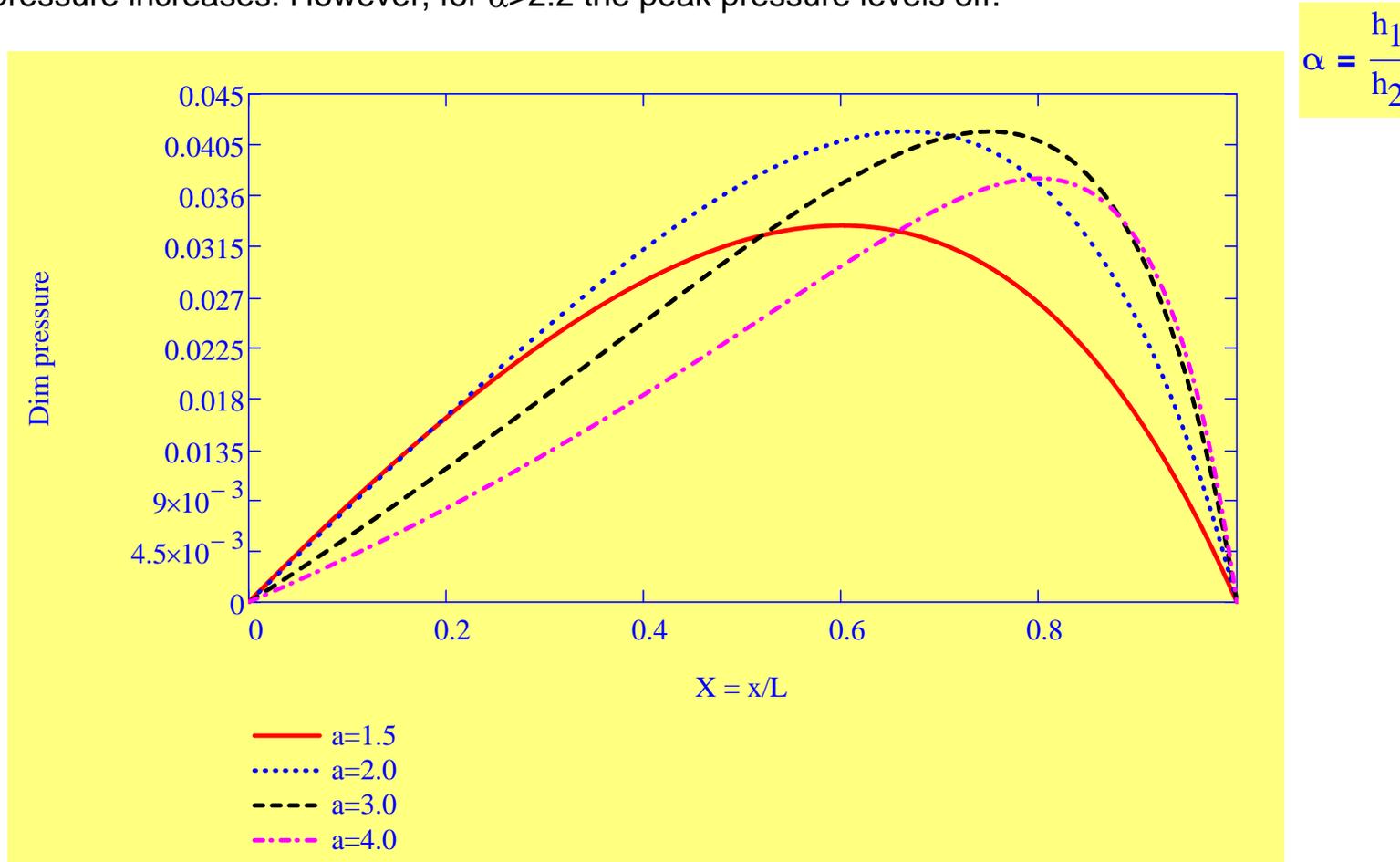
A first integral of this equation renders a constant, proportional to the flow rate  $Q_x$ , i.e.

$$\left( H^3 \cdot \frac{d}{dX} P - H \right) = -Q_X \quad (4)$$

Integration of Eq. (4) is rather simple for the film **tapered** profile. After some algebraic manipulation and application of the pressure boundary conditions at the inlet and exit planes of the bearing, the end result is

$$P(X, \alpha) := \frac{\alpha}{1 - \alpha^2} \cdot \left( \frac{1}{H(X, \alpha)^2} - \frac{1}{\alpha^2} \right) - \frac{1}{1 - \alpha} \cdot \left( \frac{1}{H(X, \alpha)} - \frac{1}{\alpha} \right) \quad (5)$$

Figure 2 depicts the pressure profile for four film taper ratios,  $\alpha=1.5, 2, 3$  and  $4$ . Note that as  $\alpha$  increases the peak pressure increases. However, for  $\alpha>2.2$  the peak pressure levels off.



**Fig 2. Dimensionless pressure for 1D-slider bearing and increasing film thickness (inlet/exit) ratios  $\alpha$**

The analysis determines

$$P_{\max}(\alpha) := \frac{(\alpha - 1)}{4 \cdot \alpha \cdot (1 + \alpha)}$$

at location

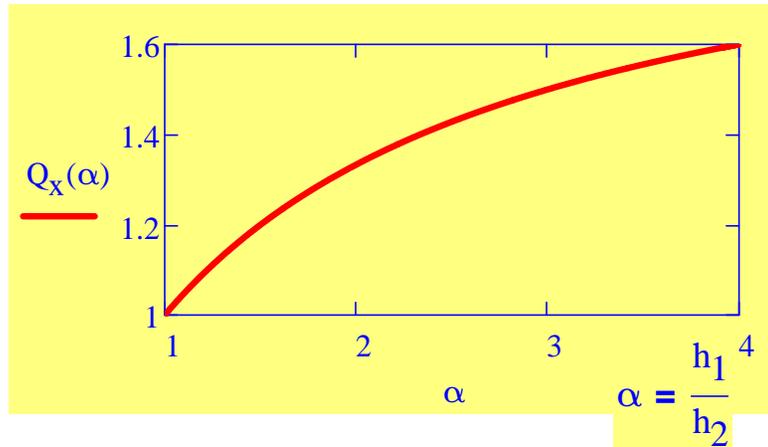
$$x_{P_{\max}}(\alpha) := \frac{\alpha}{\alpha + 1}$$

and volumetric flow rate

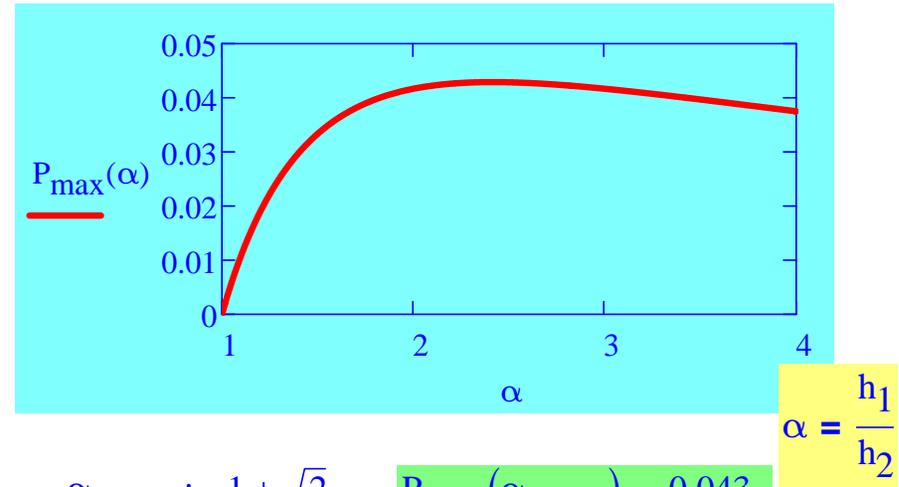
$$q_x(\alpha) = \left( h_2 \cdot \frac{U}{2} \cdot B \right) \cdot Q_x \quad (7)$$

where

$$Q_x(\alpha) := \frac{2 \cdot \alpha}{1 + \alpha}$$



(6)



$$\alpha_{P_{\max}} := 1 + \sqrt{2}$$

$$P_{\max}(\alpha_{P_{\max}}) = 0.043$$

$$\alpha_{P_{\max}} = 2.414$$

Integration of the pressure field over the pad surface renders the bearing reaction force opposing the applied

load  $w$ , i.e

$$w = \int_0^L P(x) dx \cdot B = \frac{6 \cdot \mu \cdot U \cdot L^2 \cdot B}{h_2^2} \cdot W(\alpha) \quad W = \int_0^1 P(X) dX \quad (8a)$$

Substitution of the pressure field above gives, after considerable algebraic manipulation:

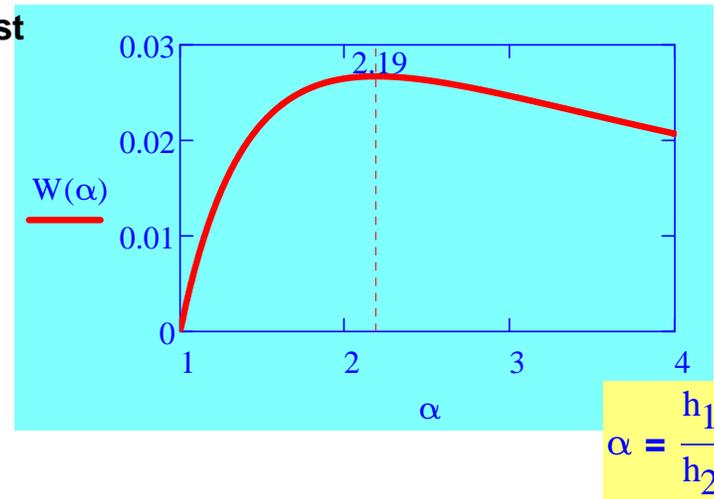
$$W(\alpha) := \frac{1}{(1-\alpha)^2} \left[ \ln(\alpha) + 2 \cdot \frac{(1-\alpha)}{1+\alpha} \right] \quad (8b)$$

There is an optimum film ratio  $\alpha$  that determines the **largest load capacity**. This optimum ratio is determined from

$$\frac{d}{d\alpha} W(\alpha) = 0$$

$$\alpha_{opt} := 2.1889$$

$$W(\alpha_{opt}) = 0.0267$$



Note that too large taper ratios,  $\alpha > \alpha_{opt}$  act to reduce the load capacity. The formula above shows that the machined taper  $(h_1-h_2) \sim 2.18 h_1$  for maximum load carrying.

It is also important to determine the **shear force** due to the fluid being dragged into the thin film region. This

force equals

$$f = \int_0^L \tau_w(x) dx \cdot B$$

(9a)

where  $\tau_w$  is the shear stress at the moving wall

$$\tau_w(x) = \mu \cdot \left( \frac{d}{dy} V_x \right) \quad \text{at } y=0 \quad (10a)$$

The fluid velocity field  $V_x$  adds the Poiseuille and Couette contributions, i.e. due to pressure and shear, respectively; i.e.

$$V_x = \frac{1}{2 \cdot \mu} \cdot \left( \frac{d}{dx} p \right) \cdot (y^2 - h \cdot y) + U \cdot \left( 1 - \frac{y}{h} \right)$$

Then

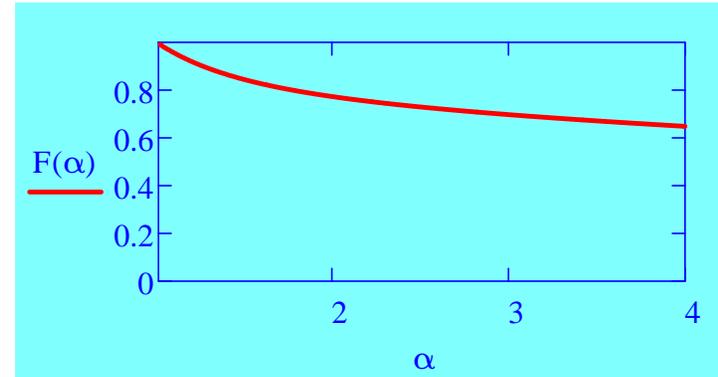
$$\tau_w = \mu \cdot \frac{dV_x}{dy} = \frac{-h}{2} \cdot \frac{d}{dx} p - \mu \cdot \frac{U}{h} \quad \text{at } y=h \text{ (moving wall)}$$

Substitution of the pressure profile above and integration over the pad surface gives a shear force

$$f = \frac{\mu \cdot U \cdot L \cdot B}{h_2} \cdot F(\alpha) \quad \underline{F(\alpha) := -\left(\frac{6}{1+\alpha} + \frac{4 \cdot \ln(\alpha)}{1-\alpha}\right)} \quad (10b)$$

The **power lost** dragging the fluid is

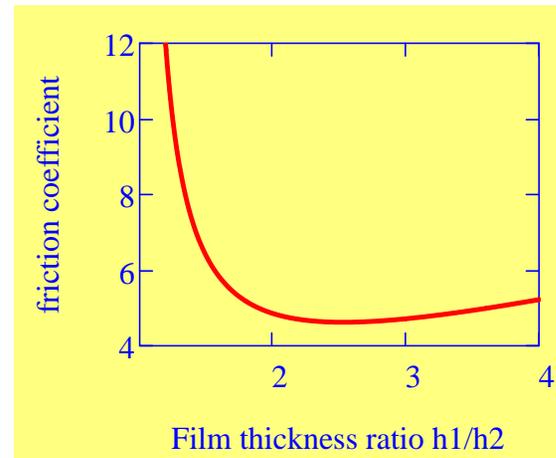
$$P_w = f \cdot U \quad (11)$$



It is customary to define a **coefficient of friction**  $\mu_f$  relating the shear force to the applied load, i.e

$$\mu_f = \frac{f}{w} = \frac{h_2}{L} \cdot \mu \mu(\alpha) \quad \mu \mu(\alpha) := \frac{F(\alpha)}{W(\alpha) \cdot 6}$$


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Since  $h_2/L \ll 1$ , the friction coefficient  $\mu_f$  is actually much smaller than the dimensionless coefficient  $\mu$  displayed in the graph above.

Figure 3 below depicts the dimensionless peak pressure, load capacity, shear force and flow rate for the 1D-slider bearing as a function of the film thicknesses ratio  $\alpha$

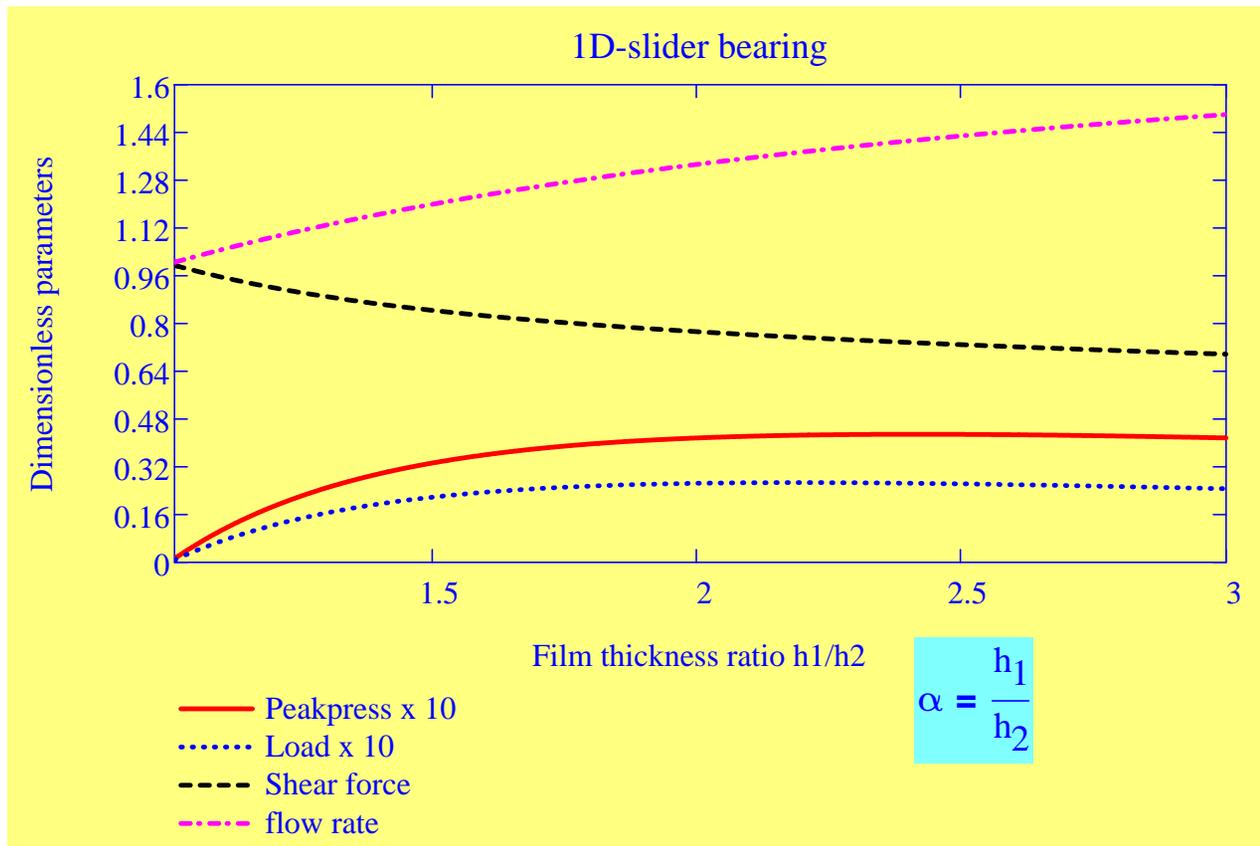


Fig 3. Performance parameters for tapered 1D-slider bearing. Increasing film ratios  $\alpha$

## Thermal effects

Thus far the analysis considers the lubricant viscosity to remain invariant. However, most lubricants (mineral oils) have a viscosity strongly dependent on its temperature. In actuality, as the lubricant flows through the film thickness it becomes hotter because it must carry away the mechanical power dissipated within the film.

Analysis of fluid film bearings with thermal effects is complicated since the film temperature changes even across the film thickness. Such analysis is presently out of scope, i.e. within the framework of the notes hereby presented.

Nonetheless, a simple method follows to estimate in a global form -as in a **lumped system**- the overall temperature raise of the lubricant and its effective lubricant viscosity to use in the analysis and design of a bearing.

The mechanical power is not only carried away by the lubricant flow but also conducted to and through the bearing bounding solid surfaces - bearing and moving collar.

Recall that the mechanical power dissipated equals  $P_w = f U$  and converted into heat that is carried away by the lubricant. A balance of mechanical power and heat flow gives

$$\kappa \cdot P_w = \rho \cdot C_p \cdot q_x \cdot \Delta T \quad \Delta T = T_{\text{exit}} - T_{\text{inlet}} \quad (12)$$

where  $\rho$  and  $C_p$  are the lubricant density and heat capacity, respectively;  $q_x$  is the flow rate, and  $\Delta T$  is the temperature raise. Above  $\kappa$  is an (empirical) coefficient denoting the fraction of mechanical power converted into heat. Typically  $\kappa=0.8$ .

Substitution of the shear force ( $f$ ) and flow rate ( $q_x$ ) into the equation above gives

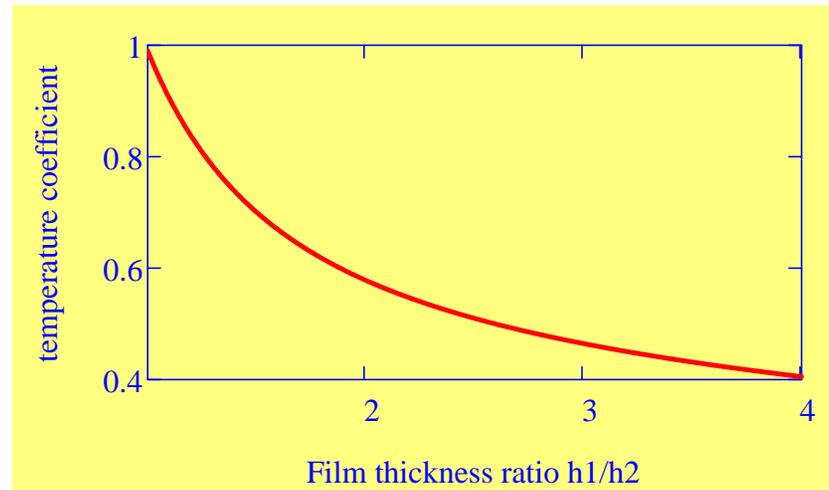
$$\Delta T = \kappa \cdot \left( \frac{\mu \cdot U \cdot L}{\rho \cdot C_p \cdot h^2} \right) \cdot \delta T(\alpha) \quad \delta T(\alpha) := \frac{F(\alpha)}{Q_x(\alpha)} \quad (13)$$

In the expression above, the viscosity is evaluated at an effective temperature,  $T_{eff}$ , which is taken as a weighted average between the inlet or supply temperature and the calculated exit temperature. Typically,

$$T_{eff} = T_{inlet} + \frac{\Delta T}{2} \quad (14)$$

In general, for applications not generating very high hydrodynamic pressures (GPa), the lubricant viscosity is an exponential decaying function of temperature.

$$\mu_{lub}(T) = \mu_{ref} \cdot e^{-\alpha_v \cdot (T - T_{ref})} \quad (15)$$



where  $T_{ref}$  and  $\mu_{ref}$  are reference lubricant temperature and viscosity, respectively.  $\alpha_v$  is a viscosity temperature coefficient

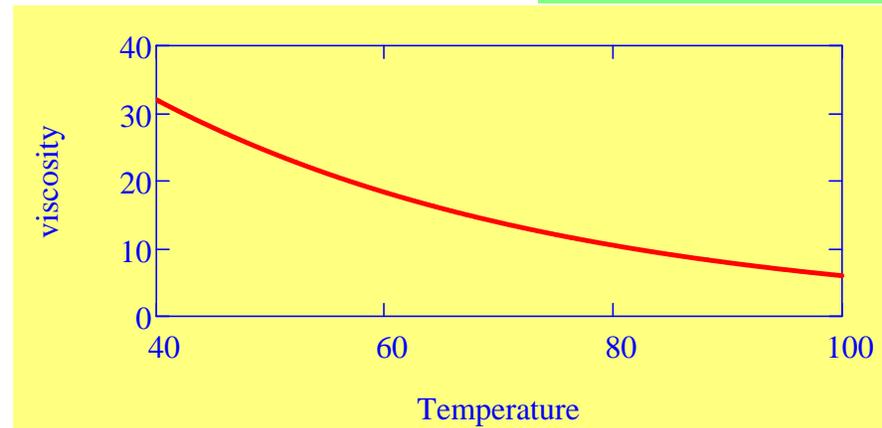
Lubricant technical specification charts provide the lubricant viscosity at two temperatures, 40C and 100C. Thus, the viscosity-temperature coefficient follows from, for example:

$$T_1 := 40 \quad T_2 := 100$$

$$\mu_1 := 32 \quad \mu_2 := 6$$

$$\alpha_v := \frac{\ln\left(\frac{\mu_1}{\mu_2}\right)}{(T_2 - T_1)}$$

$$\mu_{\text{lub}}(T) := \mu_1 \cdot e^{-\alpha_v \cdot (T - T_1)}$$



The bearing engineering design procedure follows an iterative procedure. Given the taper for the bearing ( $h_2-h_1$ ), surface velocity  $U$  and applied load  $w$ :

- a.** assume exit film thickness  $h_2$  and effective temperature, set effective viscosity and
  - b.** calculate bearing reaction load, flow rate, shear force and temperature raise
    - if bearing load > applied load, film thickness  $h_2$  is too small, increase  $h_2$
    - if bearing load < applied load, film thickness  $h_2$  is too large, reduce  $h_2$
  - c.** once **b.** is satisfied, check the effective temperature, if same as prior calculated then process has converged. Otherwise, reset effective temperature, calculate new viscosity and return to **b**
- A MATHCAD worksheet is provided for you to perform the analysis.

# Analysis of 1D Rayleigh step bearing:

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Figure 1 shows the geometry and coordinate system of a one-dimensional Rayleigh step bearing. The film thickness  $h$  is a constant over each flow region, namely the ridge or step and the film land. The bottom surface moves with velocity  $U$ . The sudden change in film thickness generates a hydrodynamic pressure field that supports an applied load  $w$ .

Note that the minimum film thickness  $h_2$  is unknown and must be determined as part of the bearing design. The bearing step height ( $h_1 - h_2$ ) is a design parameter determined from the analysis.

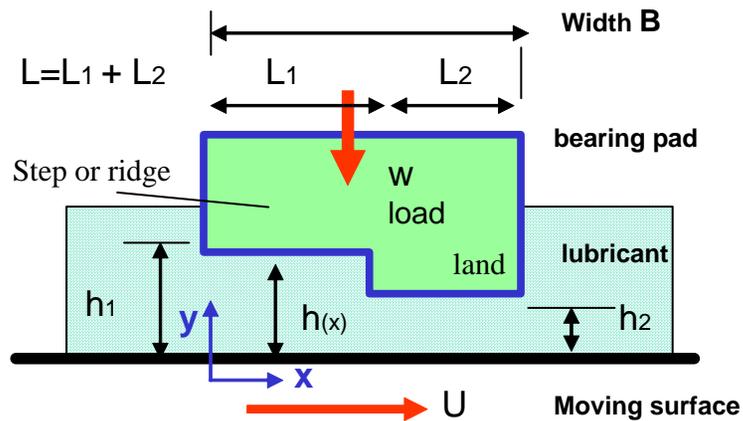


Fig. 1 Geometry of Rayleigh step bearing

$h(x)$  film thickness,

$h_1$  at inlet (leading edge)

$h_2$  at exit (trailing edge)

Two coordinate systems  $x_1$  and  $x_2$  aid to formulate the analysis

$$\begin{aligned}
 0 \leq x_1 \leq L_1 & \quad h(x_1) = h_1 & \quad h_1 > h_2 \\
 0 \leq x_2 \leq L_1 & \quad h(x_2) = h_2 & \quad (1)
 \end{aligned}$$

## Nomenclature:

$U$  surface speed       $w$  Load  
 $L$  bearing length  
 $B$  bearing width,  $B \gg L$   
 $\mu$  lubricant viscosity

## Assumptions:

- incompressible lubricant, isoviscous,
- steady state operation,  $dh/dt=0$
- **width  $B \gg L$**
- no fluid inertia effects
- rigid surfaces

Over each region, Reynolds eqn. for generation of hydrodynamic pressure (p) reduces to:

$$\frac{d}{dx} \left( \frac{h^3}{12 \cdot \mu} \cdot \frac{d}{dx} p - \frac{U \cdot h}{2} \right) = 0 \quad (2)$$

Integration of Reynolds Eqn over each flow region (step and land) is straightforward since the film thickness is constant

$$\frac{h_1^3}{12 \cdot \mu} \cdot \frac{d}{dx_1} p_1 - \frac{U \cdot h_1}{2} = -q_x \quad \frac{h_2^3}{12 \cdot \mu} \cdot \frac{d}{dx_2} p_2 - \frac{U \cdot h_2}{2} = -q_x \quad (3)$$

where  $q_x$  is the volumetric flow rate per unit width B. This flow rate is constant and equal in the two zones. Note that Eq. (3) shows the pressure gradient to be constant over each region, step and land; thus, the pressure varies linearly within each region, as shown in Figure 2.

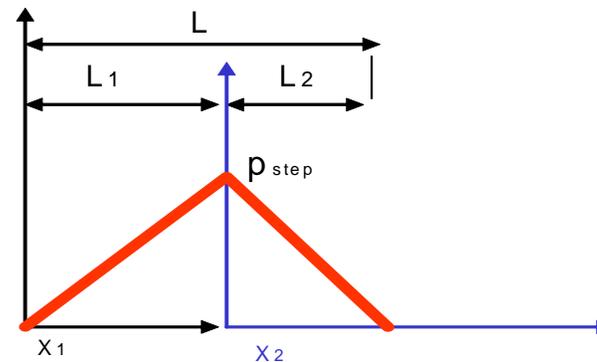
for the step region, the boundary conditions are

$$\begin{aligned} \text{at } x_1 := 0 & \quad p_1(0) = 0 \\ \text{at } x_1 = L_1 & \quad p_1(0) = p_{\text{step}} \end{aligned} \quad (4a)$$

while for the land region, the boundary conditions are

$$\begin{aligned} \text{at } x_2 := 0 & \quad p_2(0) = p_{\text{step}} \\ \text{at } x_2 = L_2 & \quad p_2(0) = 0 \end{aligned} \quad (4b)$$

where  $p_{\text{step}}$  is the pressure at the step-land interface. Note that this pressure is also the highest within the film flow region. The step pressure is determined by equating the flow rates in Eqs. (3)



**Fig. 2 pressure profile for Rayleigh step bearing**

Define the following dimensionless parameters

$$\alpha = \frac{h_1}{h_2}$$

film thickness step to land ratio

$$\beta = \frac{L_2}{L_1}$$

step to land length ratio (5)

The analysis determines the step pressure to equal

$$P_{\text{step}} = \frac{6 \cdot \mu \cdot U \cdot L_2}{h_2^2} \cdot P_{\text{step}}(\alpha, \beta)$$

$$P_{\text{step}}(\alpha, \beta) := \frac{\alpha - 1}{1 + \alpha^3 \cdot \beta}$$

(6) Note the peak pressure is largest at  $\alpha \sim 2$

and flow rate,

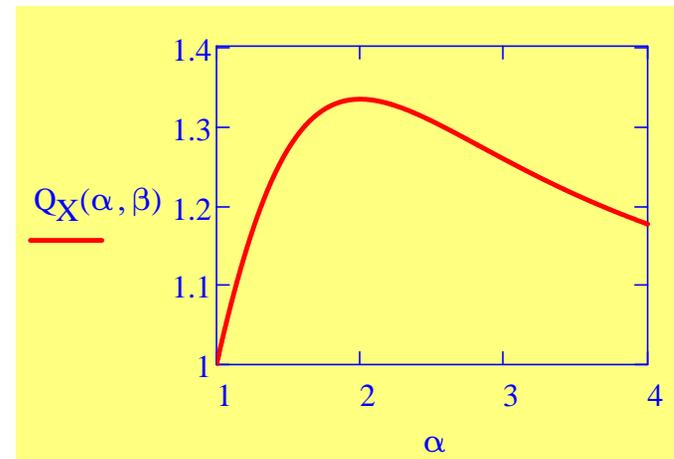
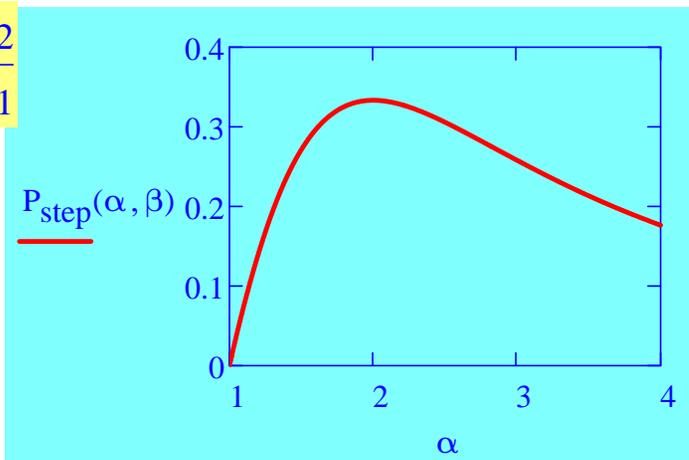
$$q_x = \frac{U}{2} \cdot h_2 \cdot Q_X(\alpha, \beta)$$

$$Q_X(\alpha, \beta) := \alpha \cdot \frac{(\alpha^2 \cdot \beta + 1)}{1 + \alpha^3 \cdot \beta}$$

(7)

$$\beta := 0.25$$

$$\beta = \frac{L_2}{L_1}$$



Since the pressure is linear over each region, integration of the pressure field over the bearing surface is straightforward and renders the bearing reaction force opposing the applied load  $w$ , i.e

$$w = \int_0^L P(x) dx \cdot B = \frac{L \cdot B}{2} \cdot p_{\text{step}}$$

$$W = \int_0^1 P(X) dX \quad (8a)$$



It is more meaningful to define the load in terms of the following **land to total length** ratio:

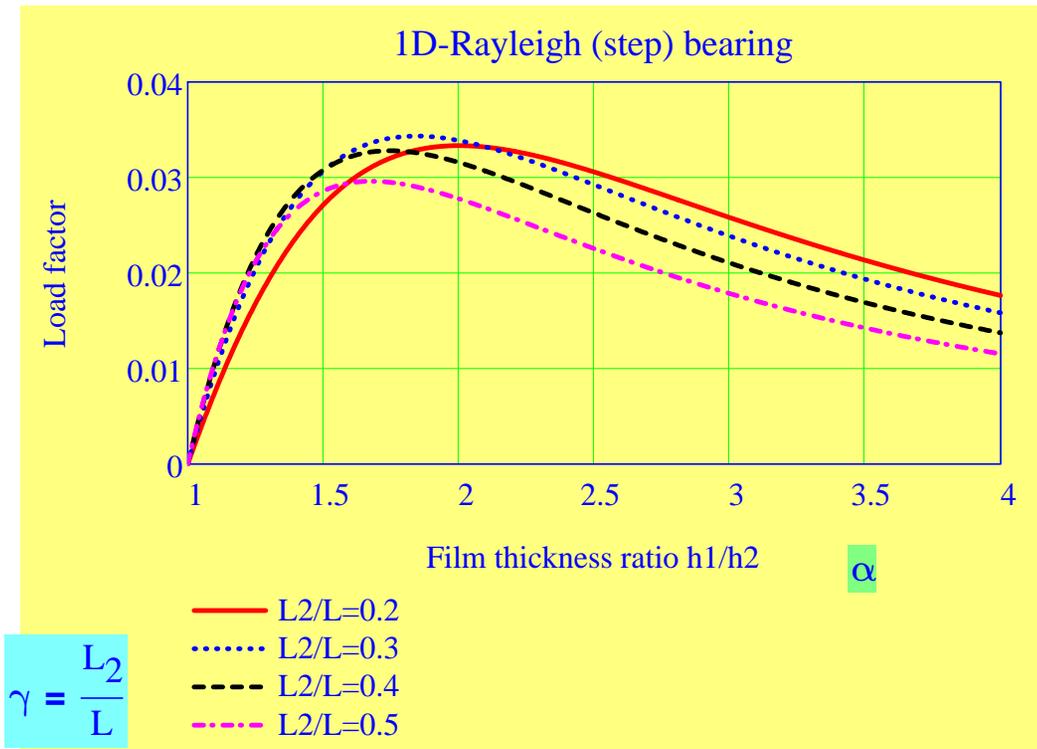
Hence,  $\beta(\gamma) := \frac{\gamma}{1-\gamma}$  and

$$\gamma = \frac{L_2}{L}$$

$$W = 6 \cdot \frac{\mu \cdot U \cdot B}{h_2^2} \cdot L^2 \cdot W(\gamma, \alpha)$$

$$W(\gamma, \alpha) := \frac{\gamma}{2} \cdot \frac{(\alpha - 1)}{1 + \alpha^3 \cdot \frac{\gamma}{1 - \gamma}}$$

Figure 3 depicts the dimensionless load factor versus step to land ratio for four land to bearing length ratios  $\gamma$ . Clearly the maximum load occurs over a narrow range of film thickness ratios (1.5 to 2.25) and for land lengths around 30% of the total bearing length, i.e. steps extending to 70% of the bearing length.



**Fig. 3 Load factor for Rayleigh step bearing for three land lengths**

Using MATHCAD one can determine easily the optimum film thickness ratio for a range of land lengths

Let: 
$$W(\gamma, \alpha) := \frac{\gamma}{2} \cdot \frac{(\alpha - 1)}{1 + \alpha^3 \cdot \frac{\gamma}{1 - \gamma}}$$

Define 
$$g = \frac{d}{d\alpha} W$$

$$\frac{d}{d\alpha} \left[ \gamma \cdot \frac{(\alpha - 1)}{1 + \alpha^3 \cdot \frac{\gamma}{1 - \gamma}} \right]$$

$$g(\alpha, \gamma) := \frac{\gamma}{\left[1 + \alpha^3 \cdot \frac{\gamma}{(1-\gamma)}\right]} - 3 \cdot \gamma^2 \cdot \frac{(\alpha-1)}{\left[1 + \alpha^3 \cdot \frac{\gamma}{(1-\gamma)}\right]^2} \cdot \frac{\alpha^2}{(1-\gamma)}$$



Set  $\gamma$ :

$$\frac{L2}{L}$$

$$\gamma := .30$$

change as needed

guess  $\alpha$ :

$$\alpha_g := 1.7 < 3$$

Solve for  $g = 0$

$$\alpha_{opt} := \text{root}(g(\alpha_g, \gamma), \alpha_g, 1.2, 3)$$

$$\alpha_{opt} = 1.843$$

$$W(\gamma, \alpha_{opt}) = 0.0343$$

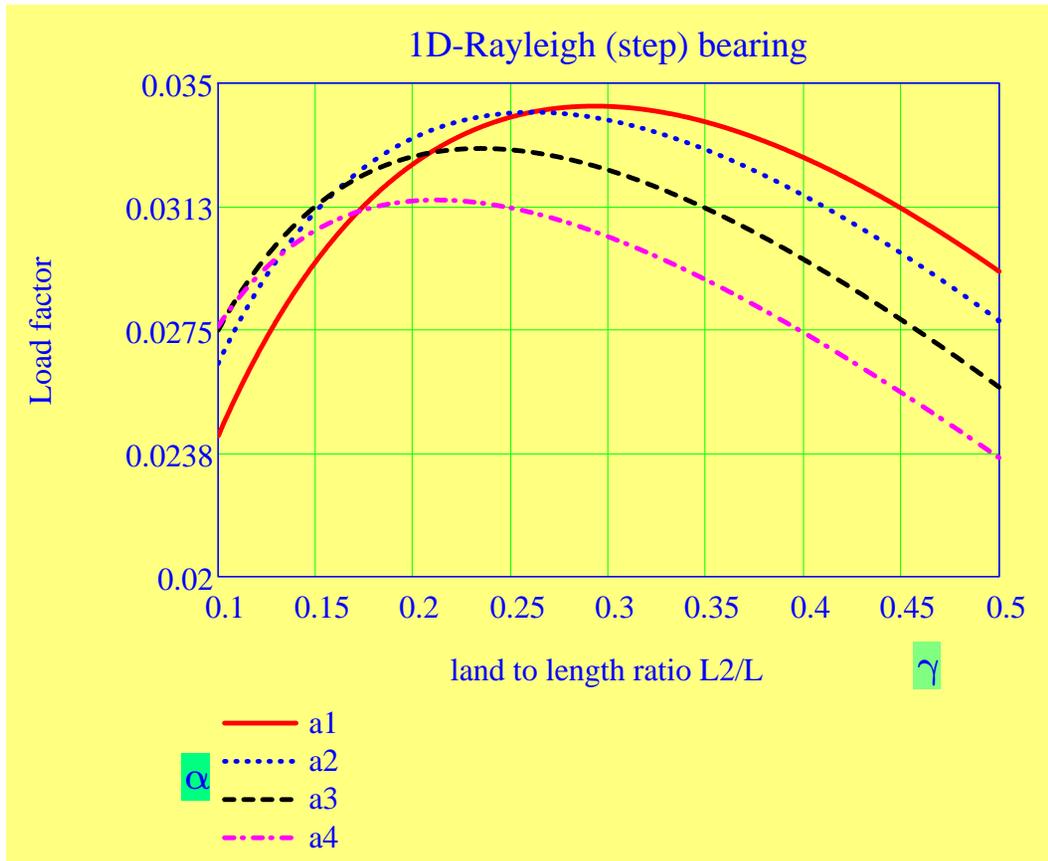
**Compare with slider-bearing:**

$$\alpha_{opt} := 2.1889$$

$$W(\alpha_{opt}) = 0.0267$$

## Examples

$$\alpha_1 := 1.8 \quad \alpha_2 := \alpha_1 + .2 \quad \alpha_3 := \alpha_2 + .2 \quad \alpha_4 := \alpha_3 + .2$$



The maximum load occurs for land to length ratios  $0.22 < \gamma < 0.30$

$$\alpha_1 = 1.8$$

$$\alpha_2 = 2$$

$$\alpha_3 = 2.2$$

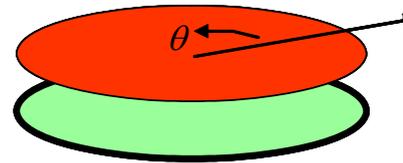
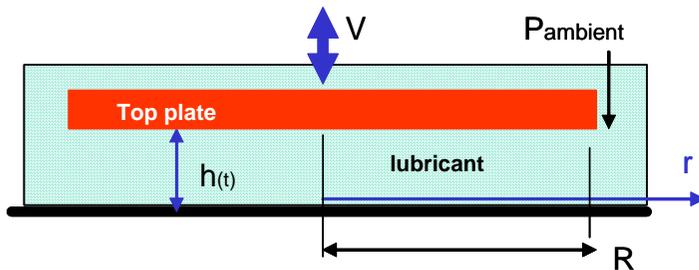
$$\alpha_4 = 2.4$$

# Analysis of simple squeeze film flow

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Figure 1 shows the simplest squeeze film flow. Consider two circular (rigid) plates **fully immersed in a lubricant pool**. The plates are perfectly smooth and aligned with each other at all times. The film thickness separating the plates is a function of time only.

The top circular plate (of radius  $R$ ) moves towards or away from the bottom plate with velocity  $V = dh/dt$  (rate of change of film thickness). None of the plates rotates. There is no mechanical deformation of the plates



## Assumptions:

- incompressible lubricant, isoviscous,
- unsteady operation,  $dh/dt \neq 0$
- no fluid inertia effects
- no air entrainment
- rigid plates
- plates are fully submerged in a lubricant bath (to avoid air entrainment)

**Fig. 1 Geometry of two circular plates for squeeze film flow. Plates submerged in a lubricant pool.**

## Nomenclature:

$h(t)$  film thickness, only a function of time

$V = \frac{d}{dt}h$  top surface speed

$R$  plate outer radius

$\mu$  lubricant viscosity

$F$  squeeze film force

The film thickness is NOT a function of the radial (r) or angular ( $\theta$ ) coordinates. (**Axisymmetric flow**). Hence, Reynolds equation in polar coordinate reduces to

$$\frac{1}{r} \cdot \frac{d}{dr} (r \cdot q_r) + \frac{d}{dt} h = 0 \quad \text{or} \quad \frac{1}{r} \cdot \frac{d}{dr} \left( r \cdot \frac{h^3}{12 \cdot \mu} \cdot \frac{d}{dr} P \right) = V \quad (1) \quad \text{or} \quad \frac{1}{r} \cdot \frac{d}{dr} (-r \cdot q_r) = V$$

where  $q_r$  is the radial flow rate  $q_r = \frac{-h^3}{12 \cdot \mu} \cdot \frac{d}{dr} P$  (2)

A first integral of Reynolds Eqn. is straightforward,  $q_r = -V \cdot \frac{r}{2}$  (3)

Note that  $q_r = 0$  at  $r=0$  because there cannot be any flow in or out of the center of the plates (a uniqueness condition). In addition, the radial flow increases linearly with the radial coordinate, being a maximum at  $r=R$ .

$q_r > 0$ , flow leaves the plates, if  $V = dh/dt < 0$ , that is when the film thickness is decreasing; while there is lubricant inflow.  $q_r < 0$  if  $V > 0$ , when the film thickness is increasing. This last condition occurs if and only if the plates are submerged in a pool of lubricant. Otherwise, air entrains into the film; thus invalidating the major assumption for the analysis.

Substitution of (2) into (3) and integration leads to the pressure field

$$P(r) = P_a - 3 \cdot \mu \cdot \frac{V}{h^3} \cdot (R^2 - r^2) \quad (4)$$

where  $P_a$  is the ambient pressure at the plate boundary,  $r=R$ . Eq. (4) shows that the pressure has a parabolic shape with a peak (max or min) value at the plate center,  $r=0$ .

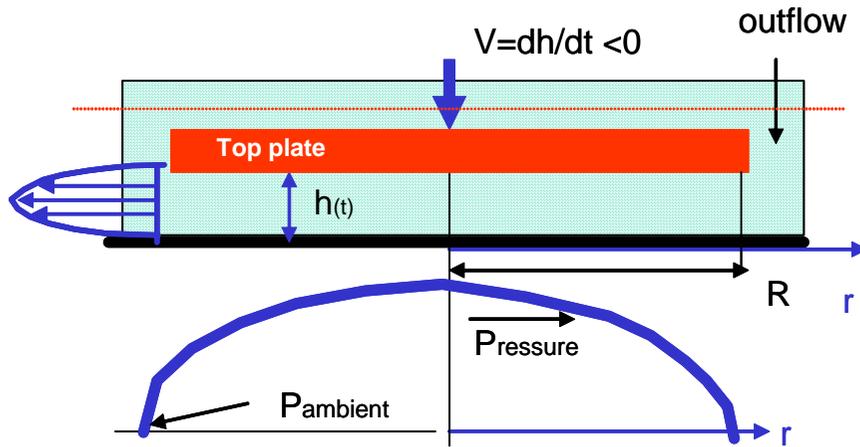
The peak pressure, above the ambient value is

$$P_{\text{peak}} = -3 \cdot \mu \cdot \frac{V}{h^3} \cdot R^2$$

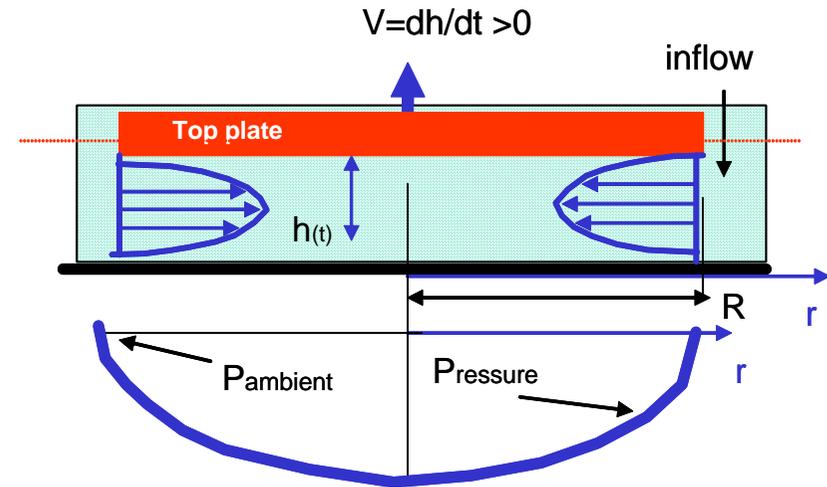
(5)

Note that the peak pressure  $>0$  if  $V < 0$ , when the film thickness is decreasing

Figures 2 and 3 show details of the pressure profile and exit flow out or into the gap between the plates for the conditions of positive squeeze ( $V < 0$ ) and negative squeeze ( $V > 0$ ), respectively.



**Fig. 2 Positive squeeze film flow,  $dh/dt < 0$ ,  $P > P_{\text{ambient}}$ , flow leaving gap**



**Fig. 3 Negative squeeze film flow,  $dh/dt > 0$ ,  $P < P_{\text{ambient}}$ , inflow into gap**

The pressure acting on the plates generates a dynamic force,  $F$ , given by

$$F = \int_0^R (P - P_a) \cdot r \, dr \cdot (2 \cdot \pi) \quad (6a)$$

Substitution of (4) into (eq. (6a) renders

$$F = \frac{3}{2} \cdot \pi \cdot \mu \cdot \frac{-V}{h^3} R^4 \quad (6b) \quad \text{where} \quad V = \frac{d}{dt}h \quad \text{and} \quad h(t) \quad \text{are the instantaneous velocity and film thickness}$$

- a)** If  $\underline{V} := 0$  then  $\underline{F} := 0$  a squeeze film cannot generate a force unless  $V \neq 0$
- b)** If  $V < 0$  then  $F > 0$  a support load, opposite to the velocity of approach of both plates  
**(positive squeeze action)**
- c)** If  $V > 0$  then  $F < 0$  a load opposing the velocity of separation of both plates  
**(negative squeeze action)**

Clearly (c) occurs provided there is no lubricant cavitation, since  $P_{\text{peak}} < 0$ . This condition will only happen for sufficiently large ambient pressures. In practice, however, the needed static pressure is too large, and thus impractical to implement.

Consider (top) plate periodic motions with frequency  $\omega$   $h(t) = h_0 + \Delta h \cdot \sin(\omega \cdot t)$

$$\Delta h < h_0$$

in this case,

$$V(t) = \Delta h \cdot \omega \cdot \cos(\omega \cdot t)$$

and the squeeze film reaction force equals

$$F = \frac{3}{2} \cdot \pi \cdot \mu \cdot \omega \cdot \frac{-(\Delta h \cdot \cos(\omega \cdot t))}{(h_0 + \Delta h \cdot \sin(\omega \cdot t))^3} R^4 \quad (7)$$

Define the following dimensionless variables

$$\tau = \omega \cdot t$$
$$\Delta H = \frac{\Delta h}{h_0} \quad F_0 = \frac{3}{2} \cdot \pi \cdot \mu \cdot \omega \cdot \frac{R^4}{h_0^2}$$

then

$$F(\tau) = F_0 \cdot \left[ -\Delta H \cdot \frac{\cos(\tau)}{(1 + \Delta H \cdot \sin(\tau))^3} \right] = F_0 \cdot g(\Delta H, \tau)$$

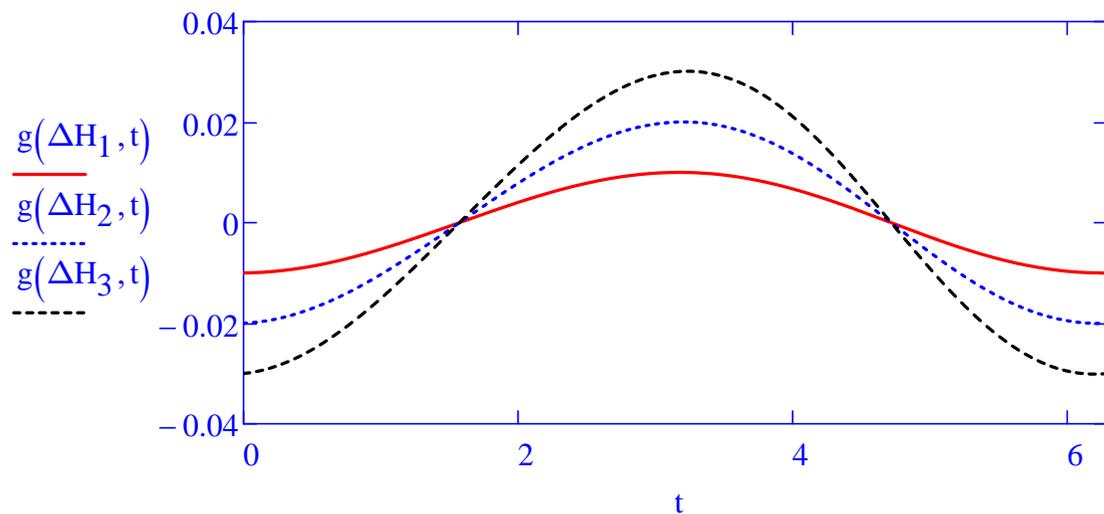
define

$$g(\Delta H, \tau) := -\Delta H \cdot \frac{\cos(\tau)}{(1 + \Delta H \cdot \sin(\tau))^3}$$

and graph the **dynamic pressure field** for one period of dynamic motion ( $2\pi/\omega$ ). Note how quickly the squeeze film pressure increases as the amplitude  $\Delta H$  grows and approaches  $H=1$ . Furthermore, the film pressure, albeit periodic, has multiple super frequency components as  $\Delta H \gg 0$ .

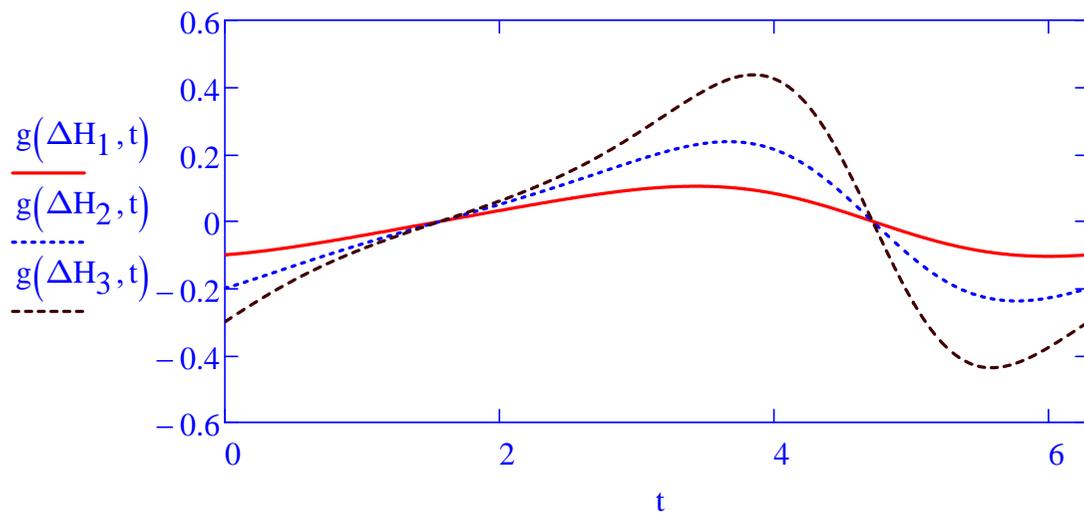
In the graphs below, note the change in scale (vertical)

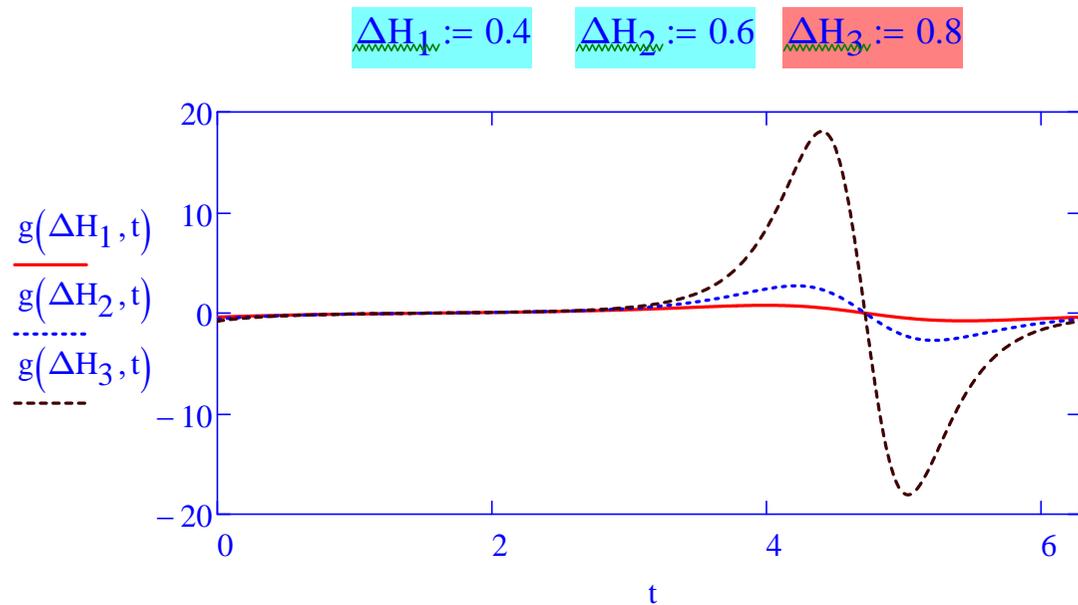
$\Delta H_1 := 0.01$   $\Delta H_2 := 0.02$   $\Delta H_3 := 0.03$



ONE period of motion

$\Delta H_1 := 0.1$   $\Delta H_2 := 0.2$   $\Delta H_3 := 0.3$





**Closure:**

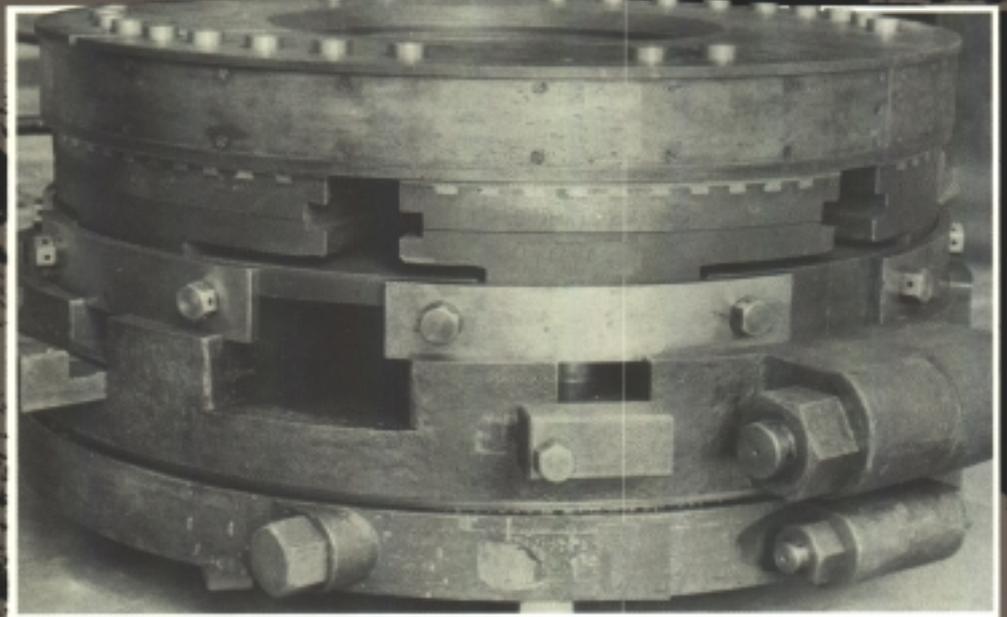
The analysis above neglects fluid inertia (**a severe omission**) and assumes there is NO air entrainment or fluid cavitation (gaseous or vapor) [**a more severe omission**]



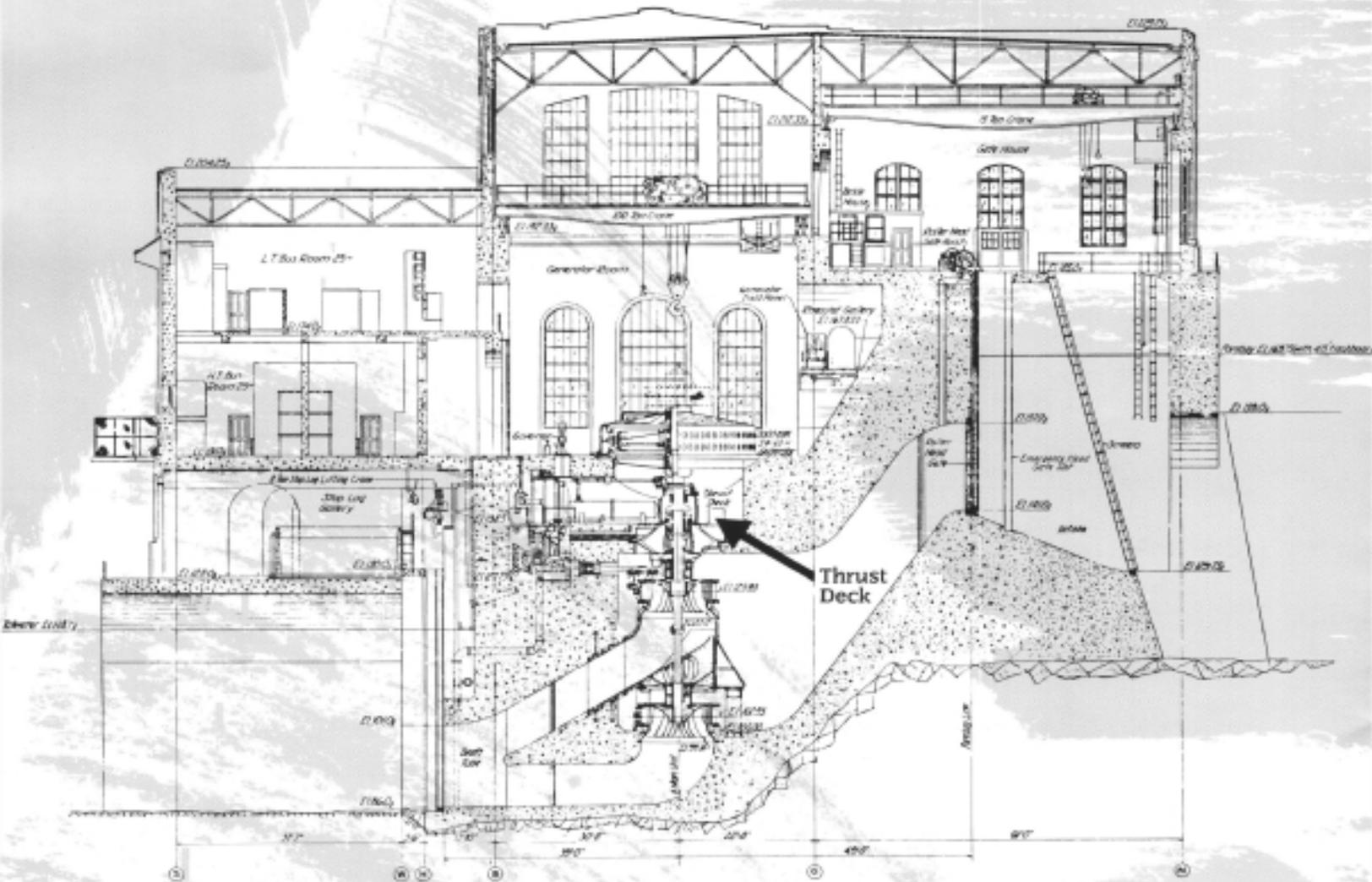
The American Society of  
Mechanical Engineers

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The American Society of Mechanical Engineers designates the first Kingsbury thrust bearing at Holtwood Hydroelectric Station as an International Historic Mechanical Engineering Landmark on June 27, 1987.



The Kingsbury bearing at Holtwood



There's an invention that's been working for 75 years along the Susquehanna River in Lancaster County, Pa., with negligible wear, while withstanding a force of more than 220 tons.

The Susquehanna Section of the American Society of Mechanical Engineers dedicated that invention — a 48-inch-diameter thrust bearing — on Saturday, June 27, 1987, as its 23rd International Historic Mechanical Engineering Landmark.

The bearing is the brainchild of the late professor Albert Kingsbury, an engineering genius who personally supervised its installation in the 10,000-horsepower Unit 5 of the 10-unit Holtwood Hydroelectric Station, June 22 through 27, 1912. Holtwood today is owned and operated by Pennsylvania Power & Light Co. It was built and operated until 1955 by Pennsylvania Water & Power Co.

All bearings in rotating machinery need to overcome the effects of friction between revolving parts and stationary parts. A thrust bearing specifically overcomes the friction created when a shaft exerts a force in a direction parallel to its axis of rotation.

Helicopter rotors and airplane or boat propellers, for instance, need thrust bearings on their shafts. So do hydroelectric turbine-generator units.

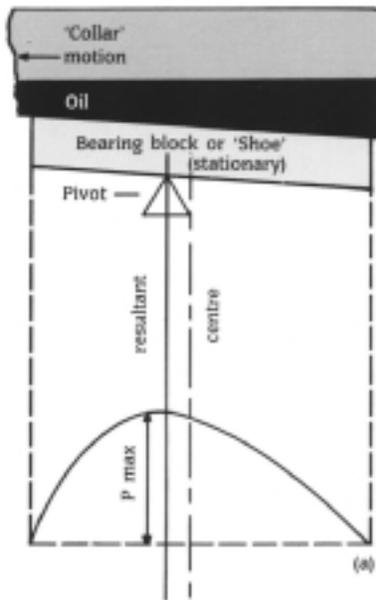
Until Kingsbury came onto the scene, units like Holtwood's represented the upper size limit of hydroelectric design. The rotating elements at Holtwood Unit 5 have a combined weight of more than 180 tons, and the downward force of water passing through the turbine adds another 45 tons.

Roller thrust bearings once used in such installations rarely lasted more than two months before needing repair or replacement.

Then Kingsbury came along with the deceptively simple idea that instead of roller or ball bearings, a thin film of oil could support the massive weight — and practically eliminate mechanical wear in the bargain.

The principle of the discovery — in Kingsbury's own words — was this:

**In reading (a) paper dealing with flat surfaces, it occurred to me that here was a possible solution to the troublesome problem of thrust bearings ...if an extensive flat surface rubbed over a flat surface slightly inclined thereto, oil being present, there would be a pressure distributed about as sketched...**



*The diagram at left is taken from a sketch by Kingsbury himself, showing his insight into the pressure distribution in an offset bearing shoe. Below, a model of the Kingsbury thrust bearing is affixed to Holtwood's Unit 5 generator, so that visitors can see the ingenious way its parts interact.*

The maximum pressure would occur somewhat beyond the center of the bearing block in the direction of motion and the resultant would be between that maximum and the center line of the block. It occurred to me that if the block were supported from below on a pivot, at about the theoretical center of pressure, the oil pressures would automatically take the theoretical form, with a resulting small bearing friction and absence of wear of the metal parts, and that in this way a thrust bearing could be made, with several such blocks set around in a circle and with proper arrangements for lubrication.



*Former Holtwood superintendent W. Roger Small Jr. checks the glass inspection port on the Unit 5 thrust deck that enables operators to view the oil-bathed bearing during operation.*

Kingsbury's design could support 100 times the load of the previously used roller bearings.

The top half of his bearing design is a flat, cast-iron ring, called a "runner." The runner rests on six flat shoes, shaped like wedges of pie to match the shape of the ring.

The entire bearing is bathed in 570 gallons of oil. Each of the shoes is pivot-mounted so that it can rock a bit. How this happens is shown in the diagram, copied from Kingsbury's original sketch in which he showed the pressure distribution in the offset shoes.

The rotating motion of the cast-iron runner squeezes oil between it and the shoes, and the oil actually supports the weight, with no physical contact between the runner and the shoes.

As a kind of engineering "bonus," the design is such that the faster a unit runs, the more weight the bearing is capable of carrying.

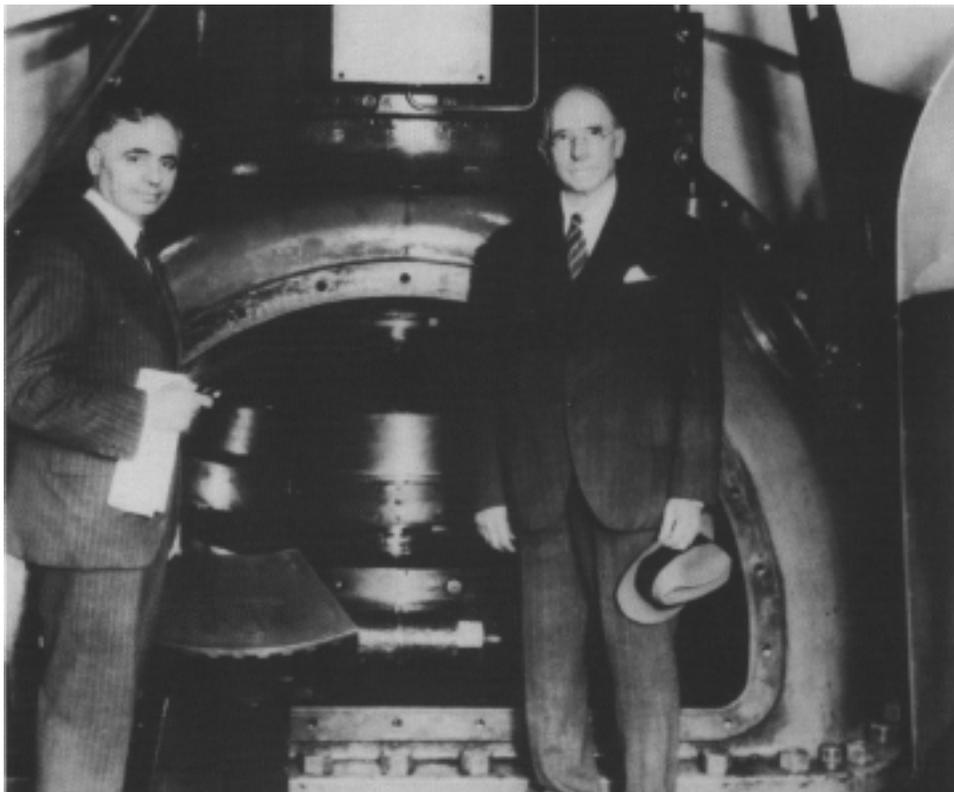
Kingsbury returned to Holtwood once — to mark the 25th anniversary of the installation of bearing No. 1 in the plant's Unit 5. Amid all the pomp and ceremony of the occasion, he took time out to smear his initials in the oil film of a Kingsbury bearing shoe that the owners had on display.

Incidentally, the contract between PW&P and Kingsbury, which agreement the professor described as "a stiff one," was for \$2,650 for the construction and installation of that first bearing.

At Holtwood today, a model of the bearing is attached to Unit 5, along with a plaque reading:

**"The first Kingsbury thrust bearing ever installed on a hydro-electric generation unit was put into service in this machine on June 22, 1912. It carries a weight of 220 tons.**

*On the 25th anniversary of its installation, Kingsbury (right) returned to Holtwood for a look at the Unit 5 thrust bearing. With him is Frederick A. Allner, who eventually became a Pennsylvania Water & Power Co vice president. The arc-shaped piece in front of Allner is one of the bearing shoes that had been removed for the occasion.*



**"When the generator was rebuilt for 60-cycle service in 1950, the original Kingsbury bearing was retained, as it was in perfect condition.**

**"Not a single part has ever been replaced."**

## The Kingsbury company

Albert Kingsbury was born in Morris, Ill., in 1863, the son of a Quaker mother and Presbyterian father. His lineage went back to English immigrants who landed in Massachusetts in the 17th century.

A well-rounded individual with a sense of humor, Kingsbury was equally at ease working with machinery, singing, playing the flute or reading in Spanish, Italian, Greek, French, German or Danish.

Throughout his early life, there remained a thread of interest in coefficients of friction that appeared to have begun when he undertook the testing of bearing metals while studying mechanical engineering at Cornell University.

Kingsbury took over the testing of half-journal bearings at Cornell in research underwritten by the Pennsylvania Railroad Co. After carefully scraping and refitting all the test bearings there, he discovered that they exhibited identical characteristics and showed no detectable wear.

He was to remain intrigued by the mysteries of friction and the properties of lubricants for the rest of his life, whether teaching at New Hampshire College (Durham) after his graduation, working in private industry or teaching again at Worcester Polytechnic Institute.

The inspiration for Kingsbury's tilting-pad bearing came when he read an 1886 paper by Osborne Reynolds on properties of fluid-film-lubricated bearings. Kingsbury built a successful thrust bearing in 1898, while at New Hampshire College.

Eventually lured away from the academic life by his desire to work more closely with lubrication problems, Kingsbury nonetheless was awarded two honorary doctorates in recognition of his contributions to the knowledge of tribology — the study of friction and ways to overcome its effects.

He applied for a U.S. patent in 1907, and eventually was awarded Patent No. 947,242 in 1910.

It was when Kingsbury was working at the East Pittsburgh works of the Westinghouse Electric and Manufacturing Co. that this daringly innovative engineer chanced upon a daringly innovative company — Pennsylvania Water & Power Co. — that was in need of a bearing of the sort Kingsbury wanted to demonstrate on a commercial scale.

PW&P was a struggling company between 1910 — when Holtwood went into operation — and 1914, when the utility was able to turn around its financial fortunes.

Kingsbury, for his part, took the money from a matured insurance policy and used it to pay Westinghouse for building the first thrust bearing that was installed at Holtwood.

Both he and PW&P were betting their futures on the success of his bearing in replacing the roller bearings that used to wear out in a matter of months at Holtwood and similar hydroelectric installations.

The first time it was installed, it looked like an overheated failure. But Kingsbury took it back to East Pittsburgh and applied that same careful scraping technique whose results puzzled him at Cornell. Within five days, the bearing was installed and running without problems.

After three months, the bearing was taken apart and found to be in perfect condition. PW&P bought more, and eventually put them on all 10 Holtwood hydroelectric units. Calculations after that first inspection showed that the bearing should last 330 years before the shoes' bearing surface would be half worn away.

After four years and another inspection, recalculation indicated a more than 1,320-year life span.

When the unit was again inspected in 1969, the bearing was still in nearly new condition.

Kingsbury's bearing made possible the design of much larger hydroelectric units, such as those of the Tennessee Valley Authority and Bonneville Power Authority and at the Hoover and Grand Coulee dams.

In addition, Kingsbury bearings have been used extensively in marine propulsion — for the propeller shafts of large ships and even nuclear-powered submarines. The first such application was on U.S. Navy ships in 1917.

## History of Holtwood Hydro

The Holtwood Hydroelectric Station was built between 1905 and 1910 by the Pennsylvania Water & Power Co. PW&P merged with Pennsylvania Power & Light Co., the plant's present owner and operator, in 1955.

The Kingsbury thrust bearing is far from the only technology pioneered at Holtwood.

A hydraulic laboratory existed there for many years, and in it were tested not only the runner (turbine blade) design for the Safe Harbor Hydroelectric Development, eight miles upriver from Holtwood, but also runners for many large hydroelectric developments throughout the country, including Bonneville and Santee-Cooper.

Another interesting facet of "river technology" was the dredging and burning of anthracite culm, or "fines" (waste coal that washed into the river from the anthracite belt as far north as Luzerne and Lackawanna counties).

For years, commercial dredging was done in the Holtwood impoundment (Lake Aldred), and later in the Safe Harbor impoundment (Lake Clarke). Indeed, Holtwood Steam Electric Station was built to burn river coal, and did so until cleaner coal-mining and processing methods shut off the flow of available "fines" and environmental regulations in 1972 made it impractical to continue dredging.

Other technological pioneering at Holtwood included PW&P's testing of lightning-protection systems and water-deluge fire-

fighting systems to protect large transformers. The systems protecting transformers at all PP&L plants today are direct descendants of the prototypes developed at Holtwood.

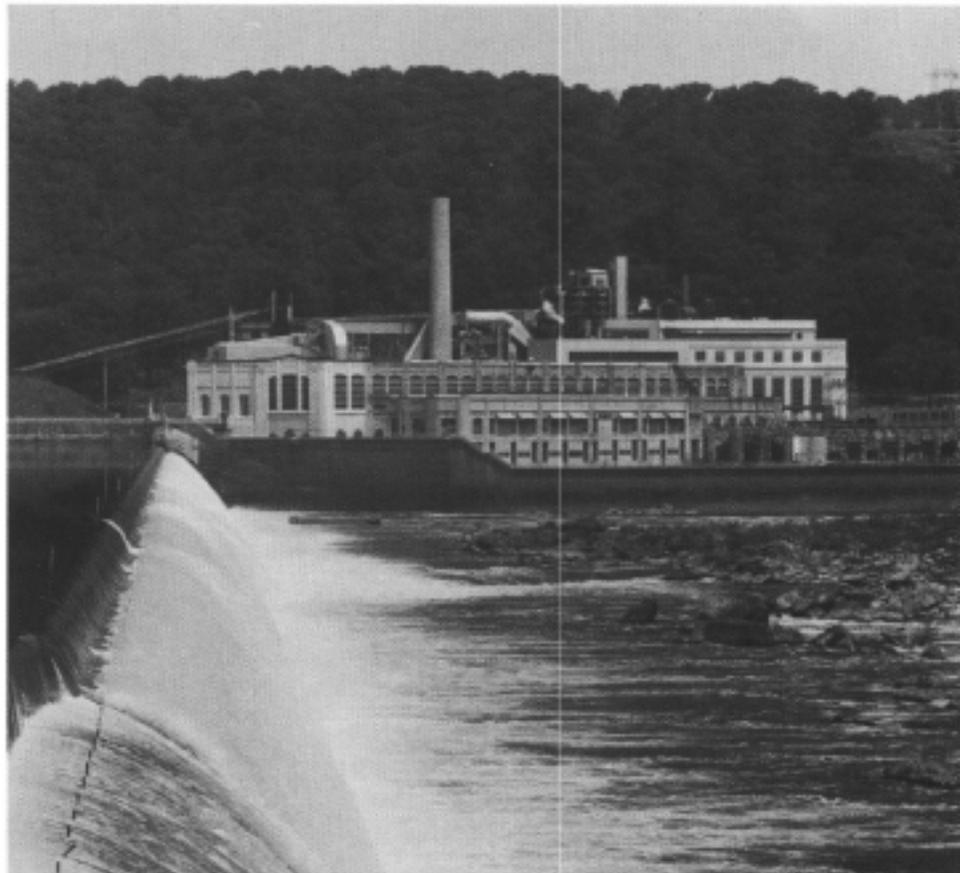
Notwithstanding the embryonic state of large-project engineering and construction techniques when Holtwood was built, its massive concrete dam has withstood all major floods on the Susquehanna, including the devastating flood of 1936, the assault of Tropical Storm Agnes in 1972 and a massive ice jam in 1978.

Thanks to PP&L's ongoing "Extended Life" program for its generating stations, Holtwood is expected to have a useful life well into the next century, and far longer than might have been projected for it in 1910.

At the time of the Kingsbury thrust bearing's dedication as an International Historic Mechanical Engineering Landmark, a complete rebuild of Holtwood's hydroelectric Unit 8 was in progress. Other units will be rebuilt on a continuing schedule.

With that kind of maintenance and with original equipment of the quality of the Kingsbury bearing, it's possible that Holtwood Hydroelectric Station may never "wear out."

*Seen here from the York County side of the river, Holtwood Hydroelectric Station has been a familiar landmark along the Susquehanna for more than 76 years.*



## The ASME's History and Heritage Program

The ASME History and Heritage program began in September 1971. To implement and achieve its goals, ASME formed a History and Heritage Committee, composed of mechanical engineers, historians of technology, and the curator of Mechanical and Civil Engineering at the Smithsonian Institution. The committee provides a public service by examining, noting, recording and acknowledging mechanical engineering achievements of particular significance. For further information, please contact the Public Information Department, American Society of Mechanical Engineers, 354 East 47th Street, New York, N.Y. 10017, (212) 705-7740.

### About the Landmarks

The Kingsbury Bearing is the 23rd International Historic Mechanical Engineering Landmark to be designated. Additionally, since the ASME National Historic Mechanical Engineering Program began in 1971, 87 National Historic Mechanical Engineering Landmarks, one International Mechanical Engineering Heritage Site, one International Mechanical Engineering Heritage Collection, and two National Mechanical Engineering Heritage Sites have been recognized. Each reflects its influence on society in its immediate locale, nationwide or throughout the world.

According to David P. Kitlan, the ASME Susquehanna Section's History & Heritage Committee chairman, this region of Pennsylvania is particularly rich in examples of engineering innovation and progress, and the section has sponsored more National Historic Mechanical Engineering Landmarks than any other in the country.

The Kingsbury bearing, however, is the section's first International ASME Historic Mechanical Engineering Landmark, so designated because of the global consequences of Kingsbury's invention and its applications.

Other engineering accomplishments already recognized as "landmarks" in Pennsylvania include: The Pennsylvania Railroad "GG-1" electric locomotive No. 4800 at Strasburg; the Worthington cross-compound steam pumping engine at York; the Kaplan hydroelectric turbine at York Haven Hydroelectric Plant, York Haven; the Cornwall Iron Furnace in Lebanon County; the Fairmount Water Works in Philadelphia; the steam engines of the USS Olympia, berthed in Philadelphia; the Monongahela and Duquesne inclines at Pittsburgh; and the Drake oil well at Titusville.

An ASME landmark represents a progressive step in the evolution of mechanical engineering. Site designations note an event or development of clear historical importance to mechanical engineers. Collections mark the contributions of a number of objects with special significance to the historical development of mechanical engineering.

The ASME Historical Mechanical Engineering Program illuminates our technological heritage and serves to encourage the preservation of the physical remains of historically important works. It provides an annotated roster for engineers, students, educators, historians and travelers. It helps establish persistent reminders of where we have been and where we are going along the divergent paths of discovery.

### Text of the Kingsbury Plaque at Holtwood

*(Editor's note: The ASME plans to erect a plaque for the Michell bearing at another location. That plaque will repeat the last paragraph from the Kingsbury plaque, but with Kingsbury's name substituted for Michell's.)*

**INTERNATIONAL HISTORIC MECHANICAL ENGINEERING LANDMARK  
KINGSBURY THRUST BEARING  
HOLTWOOD UNIT #5  
HOLTWOOD, PA.  
1912**

THE LOAD IN A KINGSBURY BEARING IS CARRIED BY A WEDGE-SHAPED OIL FILM FORMED BETWEEN THE SHAFT THRUST-COLLAR AND A SERIES OF STATIONARY PIVOTED PADS OR SEGMENTS. THIS RESULTS IN AN EXTREMELY LOW COEFFICIENT OF FRICTION AND NEGLIGIBLE BEARING WEAR.

ALBERT KINGSBURY (1863-1944) DEVELOPED THE PRINCIPLE IN THE COURSE OF BEARING AND LUBRICATION INVESTIGATIONS COMMENCING IN 1888 WHILE A STUDENT. HIS FIRST EXPERIMENTAL BEARING WAS TESTED IN 1904, AND HE FILED FOR A PATENT IN 1907 — GRANTED IN 1910.

THE FIRST KINGSBURY BEARING IN HYDROELECTRIC SERVICE — ONE OF ITS MAJOR APPLICATIONS — WAS INSTALLED HERE IN 1912. IT REMAINS IN FULL USE TODAY. KINGSBURY THRUST AND JOURNAL BEARINGS HAVE BEEN APPLIED TO LARGE MACHINERY OF ALL TYPES THROUGHOUT THE WORLD.

IN ONE OF THOSE COINCIDENCES WITH WHICH THE HISTORY OF TECHNOLOGY IS STREWN, AUSTRALIAN A. G. M. MICHELL SIMULTANEOUSLY AND INDEPENDENTLY INVENTED A BEARING ON THE SAME PRINCIPLE, THE TYPE BEING KNOWN IN MANY PARTS OF THE WORLD BY HIS NAME.

## History and description of PP&L

Pennsylvania Power & Light Co., incorporated June 4, 1920, now serves 2.5 million people in all or parts of 29 Central Eastern Pennsylvania counties.

Thomas A. Edison himself established several of the companies that were precursors of PP&L, and he built the world's first three-wire electric supply system in Sunbury. The first electrically lighted hotel and church both were located within what is now PP&L's service area.

Ideally located near a majority of the mid-Atlantic states' population, PP&L contributes to the economic health and growth of the area not only by providing dependable and economical electricity, but by an aggressive economic development program to nurture existing companies and encourage others to locate in its territory.

Holtwood Hydroelectric Station, whose first unit was completed in 1910, before PP&L was formed, is the oldest of PP&L's generating facilities. The Holtwood dam was, for a short time, the longest in the nation, and the generating plant was the largest hydro station as well.

Also on the site is the Holtwood Steam Electric Station, location of the nation's largest anthracite-burning boiler.

Other PP&L hydro facilities are the wholly owned Wallenpaupack Hydroelectric Station, completed in 1926, and the Safe Harbor Hydroelectric Development (one-third interest), which dates back to 1931.

The Sunbury Steam Electric Station, which went on-line in 1949, is PP&L's other anthracite-burning facility. Its four anthracite-fired boilers make it the largest anthracite-burning generating plant in the nation.

The company has four plants where bituminous coal is burned: Sunbury; Martins Creek Steam Electric Station — on line in 1954; Brunner Island Steam Electric Station — 1961; and Montour Steam Electric Station — 1972.

The Martins Creek station also is the location of two heavy-oil-burning units, dating to 1975.

PP&L's newest large plant is the nuclear-powered Susquehanna Steam Electric Station, with an in-service date of 1983.

In addition, PP&L owns part interests in the Keystone and Conemaugh plants in western Pennsylvania.

More than 25 light-oil-fueled combustion turbines are located throughout the PP&L system to provide additional peak-load power when needed.

All PP&L's plants combined have a capacity of more than 8.8 million kilowatts.

## The ASME

Formed in 1880, the American Society of Mechanical Engineers is a professional society dedicated to the maintenance of high engineering standards and to education of the public in matters relating to engineering.

## Acknowledgments

### The American Society of Mechanical Engineers

Richard Rosenberg, President  
Richard A. Hirsch, Vice President, Region III  
Michael R. C. Grandia, History and Heritage Chairman, Region III  
Dr. David L. Belden, Executive Director

The ASME Susquehanna Section

Theodore Taormina, Chairman  
William J. Stewart, Secretary-Treasurer  
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Carron Garvin-Donohue, Staff liaison

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John T. Kauffman, Executive Vice President-Operations  
Thomas M. Crimmins Jr., Vice President-Power Production  
Alden F. Wagner Jr., Superintendent of Plant-Holtwood Operations  
N. Christian Porse, Supervisor-Hydro Generating Plant  
James K. Witman, Power Production Engineer

Kingsbury, Inc.

Margaretta Clulow, Chairman  
George Olsen, President  
Richard S. Gregory, Vice President and General Manager  
Andrew M. Mikula, Director of Marketing

## References for Further Reading:

Mechanical Engineering magazine, December 1950, p. 957  
PP&L Insights newsletter, June 24, 1983, p. 2  
PP&L REPORTER magazine, October 1985, p. 10



### Specifications for the first Kingsbury bearing at Holtwood:

- Designed for a 12,000-kilowatt water-wheel-driven generator
- Capable of supporting a 400,000-lb. load in continuous operation
- To operate between 94 and 116 RPM
- Lubrication to be a high grade oil known as "Renown Engine Oil"
- Intake temperature of oil to be not more than 40 degrees C
- Must be capable of 10 RPM for 15 minutes and also 20 RPM for one hour without undue heating of any part, providing oil is supplied at 17.5 gallons per minute
- Must be capable of operating at a runaway speed of 170 RPM for one hour, providing oil is supplied at 17.5 gallons per minute
- Must be capable of operating during one-half hour of interruption of oil circulation, providing no oil is lost from the casing – or for 20 minutes at an overspeed not to exceed 40 percent above 116 RPM
- Diameter: 48 inches
- Height: 24 inches
- Approximate weight: 2.5 tons

*The ASME was assisted in preparation of this publication by Pennsylvania Power & Light Co. and Kingsbury, Inc*