Tilting Pad Journal Bearings: On Bridging the \textit{HOT Gap} between Experimental Results and Model Predictions

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Literature review – recent

- Influence of pivot flexibility on TPJBs force coefficients.
- Controversy on the frequency dependency of force coefficients $K&C$.
- Temporal fluid inertia effects neglected in conventional models.
- Knowledge of empirical parameters (hot bearing clearances, pivot stiffness, pad temperatures etc.)
- Accuracy of $[K-C-M]$ curve-fitting procedure at frequency range with pivot flexibility

### Experimental Work at TAMU

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### TPJB recent model validations

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A justification

- Influence of pivot flexibility on TPJBs force coefficients.
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Needs

**Effects** to consider for accurate prediction of tilting pad journal bearing performance

- Pivot flexibility
- Hot bearing clearance
- Pad flexibility
- Flow turbulence
- Fluid inertia
- Feeding type (direct, LEG)
- Flooded vs. evacuated

**Pivot flexibility** reduces the force coefficients in heavily loaded TPJBs.

For LOP/LBP, preload (0~0.5)
Film thickness in a pad

Film thickness:

\[ h = C_p + e_x \cos \theta + e_y \sin \theta + \]
\[ \left( \xi_{piv} - r_p \right) \cos(\theta - \theta_p) + \left( \eta_{piv} - R_d \delta_p \right) \sin(\theta - \theta_p) \]
Reynolds equation for thin film bearing

- Laminar flow
- Includes temporal fluid inertia effects
- Average viscosity across film

On k\textsuperscript{th} pad

\[
\frac{1}{R_J^2} \frac{\partial}{\partial \theta} \left\{ \frac{h^3}{12\mu} \frac{\partial P}{\partial \theta} \right\} + \frac{\partial}{\partial z} \left\{ \frac{h^3}{12\mu} \frac{\partial P}{\partial z} \right\} = \frac{\partial h}{\partial t} + \frac{\Omega}{2} \frac{\partial h}{\partial \theta} + \frac{\rho h^2}{12\mu} \frac{\partial^2 h}{\partial t^2}
\]

- \( h \): fluid film thickness
- \( \mu \): lubricant viscosity
- \( R_J \): journal radius
- \( P \): hydrodynamic pressure
- \( \Omega \): journal speed
- \( W \): static load
- \( \Omega \): journal speed
- \( h \): fluid film thickness
- \( P \): hydrodynamic pressure
- \( \mu \): lubricant viscosity
- \( R_J \): journal radius
Thermal energy transport in thin film

\( T \): film temperature

\( h \): film thickness

\( U, W \): circ. & axial flow velocities

\( \mu, \rho, C_v \): viscosity & density, specific heat

\( h_B, h_J \): heat convection coefficients

\( T_B, T_J \): bearing and journal temperatures

\( \Omega \): journal speed

\[\begin{align*}
\rho C_v & \left[ \frac{\partial}{R \partial \Theta} \left( U h T \right) + \frac{\partial}{\partial z} \left( W h T \right) \right] + h_B \left( T - T_B \right) + h_J \left( T - T_J \right) \\
& = + \frac{12 \mu}{h} \left( W^2 + \frac{\Omega^2 R^2}{12} + \left[ U - \frac{\Omega R}{2} \right]^2 \right)
\end{align*}\]

CONVECTION + DIFFUSION = DISSIPATION

(Energy Disposed) = (Energy Generated)

Neglects temperature variations across-film. Use bulk-flow velocities and temperature.
Journal & pads kinetics

Forces on journal and forces on a pad:

\[- \begin{bmatrix} W_{X_0} \\ W_{Y_0} \end{bmatrix} = \begin{bmatrix} F_{X_0} \\ F_{Y_0} \end{bmatrix} = \sum_{k=1}^{N_{pad}} \begin{bmatrix} F_{X_0}^k \\ F_{Y_0}^k \end{bmatrix} \]

\[\begin{bmatrix} F_{X_0}^k \\ F_{Y_0}^k \end{bmatrix} = \int_{-L/2}^{L/2} \int_{\theta_p^k}^{\theta_p^k + \pi} P_0^k \begin{bmatrix} \cos(\theta_p^k) \\ \sin(\theta_p^k) \end{bmatrix} R_f \, d\theta \, dz\]

$k=1, \ldots, N_{pad}$

Fluid film moment on pad

\[M_\delta = -R_d \left( -F_X \sin \theta_p + F_Y \cos \theta_p \right) = -R_d F_\eta \]

Pad equations of motion about pivot $P$

\[
\begin{bmatrix}
\ddot{\delta}_p \\
\ddot{\xi}_{piv} \\
\ddot{\eta}_{piv}
\end{bmatrix} = \begin{bmatrix} M_\delta \\ F_\xi_{piv} \\ F_\eta_{piv} \end{bmatrix} + \begin{bmatrix} M_{\delta_{piv}} \\ F_{\xi_{piv}} \\ F_{\eta_{piv}} \end{bmatrix}
\]

Inertia matrix

\[
M_{pad} = \begin{bmatrix}
l_p & m \, l_\eta & -m \, l_\xi \\
m \, l_\eta & m & 0 \\
-m \, l_\xi & 0 & m
\end{bmatrix}
\]

$l_\xi, l_\eta$, radial and transverse distances from pad mass center to pivot point
Nonlinear pivot deflection & stiffness

Pivot deformation is typically nonlinear depending on the load \( F_{piv} \), area of contact, hardness of the materials, surface conditions.

Pivot and housing:

- \( E_p, E_h \) Elastic modulus
- \( \nu_p, \nu_h \) Poisson’s ratios
- \( D_p, D_h \) Diameter of the curvature
- \( L \) Contact length

- Sphere on a sphere
- Sphere on a cylinder
- Cylinder on a cylinder
- Load-deflection function (empirical)

\[ F_{piv} = a_0 + a_1 \xi_{piv} + a_2 (\xi_{piv})^2 + a_3 (\xi_{piv})^3 + a_4 (\xi_{piv})^4 \]

\[ K_{piv} = \frac{\partial F_{piv}}{\partial \xi_{piv}} \]

Test rig

Test TPJB

102mm (4 in.)
For small amplitude motions about the equilibrium position, bearing dynamic forces are modeled in linear form as

\[ \mathbf{F} = -K \mathbf{z} - C \ddot{\mathbf{z}} - M \dddot{\mathbf{z}} \]

\( K, C, M \) coefficients extracted from dynamic load tests as curve fits to measured impedances \( Z \)

\[ \text{Re}(Z) \rightarrow (K - \omega^2 M), \text{Im}(Z) \rightarrow \omega C \]

\( \mathbf{F} = \{F_x, F_y\}^T \): lateral reaction force

\( \mathbf{z} = \{x(t), y(t)\}^T \): journal center displacement

\( K \): bearing stiffness matrix

\( C \): bearing damping matrix

\( M \): bearing mass matrix

\( Z \): impedance

## Predictions for a five-pad TPJB

*(Kulhanek and Childs*) Five pad, tilting pad bearing (LBP)

### Specific load, \( W/LD \)
345 ~3,101 kPa (50 ~ 450 psi)

### Journal speed, \( \Omega \)
7 krpm-16 krpm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Number of pads, ( N_{pad} )</td>
<td>5</td>
</tr>
<tr>
<td>Configuration</td>
<td>LBP</td>
</tr>
<tr>
<td>Rotor diameter, ( D )</td>
<td>101.6 mm (4 inch)</td>
</tr>
<tr>
<td>Pad axial length, ( L )</td>
<td>60.3 mm (2.3 inch)</td>
</tr>
<tr>
<td>Pad arc angle, ( \Theta_P )</td>
<td>58°</td>
</tr>
<tr>
<td>Pivot offset</td>
<td>50%</td>
</tr>
<tr>
<td>Pad preload, ( r_p )</td>
<td>0.27</td>
</tr>
<tr>
<td>Nominal bearing clearance, ( C_B )</td>
<td>81 ( \mu )m (3.18 mil)</td>
</tr>
<tr>
<td>Nominal pad clearance, ( C_P )</td>
<td>112 ( \mu )m (4.41 mil)</td>
</tr>
<tr>
<td>Pad inertia, ( I_P )</td>
<td>2.49 kg.cm(^2) (0.85 lb.in(^2))</td>
</tr>
<tr>
<td>Oil inlet temperature</td>
<td>~44 °C (111 °F)</td>
</tr>
<tr>
<td>Lubricant type</td>
<td>ISO VG32, DTE 797</td>
</tr>
<tr>
<td>Oil supply viscosity, ( \mu_0 )</td>
<td>0.0228 Pa.s</td>
</tr>
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</table>

**Unknown pivot stiffness and hot clearance**

Leading edge groove (LEG)  
**Pivot type:** Rocker-back

\[
\frac{1}{K_{piv}} = \frac{1}{K_m} - \frac{1}{K_f}
\]

- \( K_{piv} \), pivot stiffness
- \( K_m \), experimental static stiffness
- \( K_f \), predicted film stiffness (rigid pivot)

**Estimated pivot stiffness:**  
641 MN/m

**Estimated hot bearing clearance=**  
\[ C_B - \alpha_{cb} \Delta T_{avg} \]

- \( C_B \), cold bearing clearance
- \( \alpha_{cb} \), thermal expansion coefficient
- \( \Delta T_{avg} \), average temperature change

**Bearing hot clearance reduces by up to 21% to 64 \( \mu \)m. Preload increases to 0.32**

**Murphy (2014) reports:** ~700 MN/m  

**Bearing hot clearance change may vary 10% ~ 30%**
Journal eccentricity vs. static load

Predictions and test data agree at low loads. At high loads, pivot stiffness is too low.

Temperature vs. static load

Rotor speed $\Omega = 7$ krpm, 16 krpm
Unit load $W/LD = 3,101$ kPa (450 psi)

Predicted film temperatures show reasonable agreement with recorded pad temperatures.

**Impedances**

Specific load = 1,723 kPa (250 psi)

**Real part** \( \text{Re}(Z) \leftarrow K \cdot M \omega^2 \)

**Symbols:** test data  
**Lines:** prediction

Predictions for \( \text{Re}(Z) \) correlate well with test data, in particular at low frequencies (\( \omega < \Omega = 1X \)).

At 7 krpm, predicted $\text{Im}(Z_{yy})$ agrees with test data & $\text{Im}(Z_{xx})$ is over predicted. At 16 krpm, correlation is best.

Symbols: test data
Lines: prediction

Stiffness coefficients

Symbols: test data
Lines: prediction

Stiffnesses increase with load, shaft speed has little effect.

Very good agreement between predictions and tests. Best at 16 krpm.

Damping coefficients

Symbols: test data
Lines: prediction

\[ C_{YY} \text{ grows with load at lowest shaft speed. Predicted } C_{YY} \text{ is lower than test data. Largest disagreement at peak load (3,103 kPa=450 psi).} \]

Symbols:
- \( C_{YY} \): Damping coefficient
- \( C_{XX} \): Damping coefficient

Lines:
- Prediction

Rotor speed \( \Omega = 7 \text{ krpm} \)

Rotor speed \( \Omega = 16 \text{ krpm} \)

At 16 krpm, virtual masses are small & negative. At 7 krpm, predicted larger (negative) magnitude shows a dynamic stiffness hardening effect. Prediction depends on frequency range for [K-C-M] model.

Quantify the effect of pivot stiffness on the performance of the TPJB tested by Kulhanek and Childs (2012).

Select three pivots (flexible to rigid):

\[
k_{piv} = \frac{K_{piv} C_p}{W_{max}}
\]

- \(k_{piv} = 2.4\)
- \(k_{piv} = 5.9\)
- \(k_{piv} = \infty\)

\(K_{piv}\), dimensional pivot stiffness
\(C_p\), cold pad clearance
\(W_{max}\), maximum static load


\(L/D \approx 0.6\)
Journal center displacement exceeds bearing clearance for large loads and a bearing with flexible pivots.

\[ S = \frac{\mu NLD}{W} \left( \frac{R}{C_P} \right)^2 \]
Effect of Pivot Stiffness on Friction Coefficient

\[ f = \frac{\text{Torque}}{(WR)} \]

\[ \bar{r}_p = 0.5 \]

\[ \bar{r}_p = 0 \]

Preload \( r_p \) decreases

\[ S = \frac{\mu NLD}{W} \left( \frac{R}{C_p} \right)^2 \]

friction coefficient not affected by pivot.

\[ f_{\text{rigid pivot}} \approx f(k_{piv} = 5.9) \approx f(k_{piv} = 2.4) \]
Effect of Pivot Stiffness on TPJB

Stiffness coefficients of bearing with large preload ($r_p=0.5$) are more sensitive to pivot flexibility.

\[ k = \frac{K C_P}{W} \]

\[ S = \frac{\mu NLD (R/C_P)^2}{W} \]

- $K_{YY}$ Solid
- $K_{XX}$ Dash

Load increases
Effect of Pivot Stiffness on TPJB

Damping Coefficients

Pivot flexibility has a more pronounced effect in reducing damping than the bearing stiffnesses.
Virtual masses become **negative** as pivot stiffness decreases. Pivot flexibility leads to a hardening of the bearing dynamic stiffness.
Quantify the effect of pad inertia and fluid inertia on the impedances of test TPJB - Kulhanek and Childs (2012).

- Temporal Fluid Inertia
- Pad Mass Moment of Inertia
- Pad Mass

OBJECTIVE: Update predictions of impedances vs. frequency to assess importance of pivot flexibility.

\[ k_{piv} = \infty \rightarrow \text{Rigid pivot} \]

\[ k_{piv} = 3.8 \rightarrow K_{piv} = 641 \text{ MN/m} \]
Impedances

**Real part** \( \text{Re}(Z) \leftarrow K \cdot M \omega^2 \)

\[ k_{piv} = \infty \]

**Specific load** = 3,103 kPa (450 psi)

<table>
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<th>Test data</th>
<th>Neglect all three effects</th>
<th>Include all three effects</th>
<th>Include fluid inertia</th>
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Poor predictions when neglecting pivot flexibility.

**Rotor speed** \( \Omega = 7 \text{ krpm} \)

**Rotor speed** \( \Omega = 16 \text{ krpm} \)
Impedances

Specific load = 3,103 kPa (450 psi)

Imaginary part \( \text{Im}(Z) \leftarrow C \omega \)

\( k_{piv} = \infty \)

Rotor speed \( \Omega = 7 \text{ krpm} \)

Rotor speed \( \Omega = 16 \text{ krpm} \)

Poor predictions neglecting pivot flexibility.
At 7 krpm, fluid inertia reduces $K$ at high frequency.

Specific load = 3,103 kPa (450 psi)

**Impedances**

**Real part** $\text{Re}(Z) \leftrightarrow K - M \dot{\omega}^2$

$k_{piv} = 3.8$

**Rotor speed $\Omega = 7$ krpm**

**Rotor speed $\Omega = 16$ krpm**

- Test data
- Neglect all three effects
- Include all three effects
- Include fluid inertia
No major impact of pad mass and its moment of inertia on damping.
Conclusions

• Good correlation between test data in: and predictions when applying a-priori knowledge of pivot stiffness and bearing & pad hot clearances.

• Selection of an adequate frequency range is crucial when determining the bearing force coefficients extracted from a $[K-C-M]$ curve-fitting procedure.

• Stiffness and damping coefficients of LBP TPJBs ($r_p=0.5$) with a heavy load (small Sommerfeld #) are more sensitive to pivot flexibility than a lightly loaded bearing with zero preload.
  
  Other predictions for LOP not shown for brevity.

• Pad mass and pad mass moment of inertia have no major impact on TPJB impedances. Fluid inertia reduces the dynamic stiffness at a low shaft speed (7 krpm).
Acknowledgments

Thanks to the Turbomachinery Research Consortium and students of Prof. Childs.

Questions (?)

Learn more at http://rotorlab.tamu.edu
Small amplitudes perturbation analysis

- Consider small journal motion displacements with frequency ($\omega$) about the equilibrium position:

$$
\begin{bmatrix}
    e_X(t) \\
    e_Y(t)
\end{bmatrix}
= 
\begin{bmatrix}
    e_{X0} \\
    e_{Y0}
\end{bmatrix}
+ 
\begin{bmatrix}
    \Delta e_X \\
    \Delta e_Y
\end{bmatrix} e^{i\omega t}
$$

- The journal motions induce changes in the $k^{th}$ pad rotation ($\delta$) and pivot displacements ($\xi, \eta$) with frequency ($\omega$):

$$
\begin{bmatrix}
    \delta_p(t) \\
    \xi_{piv}(t) \\
    \eta_{piv}(t)
\end{bmatrix}
= 
\begin{bmatrix}
    \delta_{p0} \\
    \xi_{piv0} \\
    \eta_{piv0}
\end{bmatrix}
+ 
\begin{bmatrix}
    \Delta \delta_p \\
    \Delta \xi_{piv} \\
    \Delta \eta_{piv}
\end{bmatrix} e^{i\omega t}
$$

- Journal and pad motions induce changes in the film thickness and pressure fields.
Reduced force coefficients

25 force impedances for $k^{th}$ pad

\[ K + i\omega C = Z_{\alpha\beta} = \int_{-L/2}^{L/2} \int_{\theta_1}^{\theta_1 + \Theta_p} P_\beta h_\alpha R d\theta dz \]

\( \alpha, \beta = X, Y, \xi, \eta, \delta \)

Use same whirl frequency to reduce to 4 impedances \((Z_{R\alpha\beta}, \alpha, \beta = X, Y)\)

\[ Z_{R} = A^T \tilde{Z}_R A \]

\[ \tilde{Z}_R = \tilde{Z} - \tilde{Z}_{JP} \left[ \tilde{Z}_s - \omega^2 \tilde{M}_{pad} + \tilde{Z}_P \right]^{-1} \tilde{Z}_{PJ} \]

transformation matrix

\[
A = \begin{bmatrix}
\cos \theta_p & \sin \theta_p \\
-\sin \theta_p & \cos \theta_p
\end{bmatrix}
\]
Pad inlet thermal mixing coefficient

Thermal mixing coefficient $\lambda$ ($0 < \lambda < 1$) is empirical.

$$F_{in} = F_s + \lambda F_h$$
$$F_{in} T_{in} = F_s T_s + \lambda F_h T_h$$

$F_s$, $F_h$, $F_{in}$, $T_s$, $T_h$, $T_{in}$: Volumetric flow rates and fluid flow temperatures

$\lambda \approx 0.6$-0.95 for conventional lubricant feed arrangements with deep grooves and wide holes.

$\lambda$ small ($\approx 0.5$) for TPJBs with LEG feed arrangements and spray bars (and scrappers).
Effect of Pivot Stiffness on Journal eccentricity

Load configuration has negligible effect on journal eccentricity.
Friction coefficient does not vary with load configurations and pivot flexibility.

\[ f(\text{rigid pivot}) \approx f(k_{\text{piv}}=5.9) \approx f(k_{\text{piv}}=2.4) \]
A bearing with rigid pivot has more orthotropic stiffness at low Sommerfeld number.
Effect of Pivot Stiffness on TPJB Damping coefficients

Load configuration has no major impact on dampings. Pivot flexibility reduces damping at small Sommerfeld number.
Virtual masses between the LOP and LBP bearings are most notable for low $S$ (<0.5).