A Flow Starvation Model for TPJBs and Evaluation of FRFs: A Contribution Towards Understanding the Onset of Low Frequency Shaft Motions

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Funded by Hitachi, Ltd. R&D Group

Accepted for journal publication
Tilting Pad Journal Bearings (TPJBs) support rotors with minimal destabilizing forces.
→ Power loss reduces up to 35%.
→ Bearing max. temperature reduces up to 25%.

Haragonzo (1990), Brockwell and DeCamillo (1994): Direct lubricated bearings without end seals (evacuated) operate with less drag and have lower film & pad temperatures than flooded bearings.

Dmochowski and Blair (2006): Bearing power loss and temperature reduce as a result of the TPJB flow evacuation. Reduced oil flow has no effect on min. film thickness but reduces bearing stiffness and damping.

Direct lubrication & bearing without end seals
→ Lower supply flow and faster oil evacuation
→ Churning loss ↓ → Power loss and temperature↓

A too low oil flowrate (starvation) causes rotor sub synchronous vibrations (SSV)
SSV Issues due to lubricant starvation

DeCamillo et al. (2008) experimentally find.

- For low flow, evacuated TPJB shows less SSV than sealed ends bearing.
- At intermediate oil flow, SSV is pervasive in evacuated - direct lube TPJBs.
- LOP configuration produces larger amplitude SSV than LBP.
- Shaft SSV correlates with vibrations of one of the pads. Unloaded pads have the largest SSV but rarely correlate with shaft SSV.
- Adding side grooves suppresses SSV.
A model for flow starved bearings
Reynolds and thermal energy transport

On k\(^{th}\) pad,

**Reynolds equation:**

\[
\nabla \cdot \left\{ \frac{(h_k^p)^3}{12\mu} \nabla P^k \right\} = \frac{\partial h_k^p}{\partial t} + \frac{\Omega}{2} \frac{\partial h_k^p}{\partial \theta} + \frac{\rho (h_k^p)^2}{12\mu} \frac{\partial^2 h_k^p}{\partial t^2}
\]

**Thermal energy transport eqn.:**

\[
C_v \nabla \cdot (\rho h_k^p \nabla T_k^p) + \dot{Q}_k = \frac{12\mu}{h_k^p} \left( (V_k^p)^2 + \frac{(\Omega R_j)^2}{12} + \left[ U_k^p - \frac{\Omega R_j}{2} \right]^2 \right)
\]

**Film thickness**

\[
h_k^p = C_p + e_x \cos \theta + e_y \sin \theta + \left( \xi_{p}^k - \bar{r}_p \right) \cos(\theta - \theta_p^k) + \left( \eta_{p}^k - R_j \delta^k \right) \sin(\theta - \theta_p^k)
\]

+ pivot and pad surface (mechanical and thermal) deformation equations.
In a lubricant starved pad, fluid filling the gap between a pad and the journal starts at angle $\theta_f$. Pad has a lesser effective arc length (and pivot arc).

*Concept introduced by He et al. (2005)*
Flow distribution into each bearing pad

$e_s = 0$ (no load) supply flow distributes evenly as

$$Q_s^i = \frac{Q_{s}^{total}}{N_{pad}}$$

Operation with load ($W$) $\rightarrow$ journal eccentricity:
- Unloaded pads (low pressure) $\rightarrow$ demand more lubricant

With a reduced supply flow, flow into each pad decreases while the flow fraction $\sigma_i = Q_i/Q_{total}$ is constant; as per Dmochowski & Blair (2006) when noting that a reduction in flow has no effect on minimum film thickness.
Numerical model for flow starved bearings

Solve flow equations numerically using FE methods. Iterative search to satisfy inlet flow and update effective arc length.

Read more:
San Andres & Tao [2013], San Andres & Li [2015]  
ASME J. Eng. Gas Turbines Power
Equations of motion for rotor supported on TPJBs
DOFs and equations of motion

Degrees of freedom

\[ z = \{ x, y \}^T \]  lateral rotor displacements

Rotor-TPJB, including pad tilt angles

\[ z = \{ x, y, \delta_1, \ldots, \delta_{N_{pad}} \}^T \]

EOM for rotor-bearing system

\[ M \ddot{z} + C \dot{z} + Kz = F_0 e^{i\omega t} \]

In the frequency domain:

\[ \left( K - \omega^2 M + i\omega C \right) z_0 = G(\omega) z_0 = F_0 \]

Let \[ z = z_0 e^{i\omega t} \]

\( G(\omega) \) is a system complex stiffness matrix
G matrix for whole system

\[
G(\omega) = \begin{bmatrix}
\sum_{i} g_{xx}^i - M_J \omega^2 & \sum_{i} g_{xy}^i & g_{x\delta}^1 & g_{x\delta}^2 & g_{x\delta}^3 & g_{x\delta}^4 \\
\sum_{i} g_{yx}^i & \sum_{i} g_{yy}^i - M_J \omega^2 & g_{y\delta}^1 & g_{y\delta}^2 & g_{y\delta}^3 & g_{y\delta}^4 \\
g_{\delta x}^1 & g_{\delta y}^1 & g_{\delta \delta}^1 - \omega^2 I_{pad}^1 & 0 & 0 & 0 \\
g_{\delta x}^2 & g_{\delta y}^2 & 0 & g_{\delta \delta}^2 - \omega^2 I_{pad}^2 & 0 & 0 \\
g_{\delta x}^3 & g_{\delta y}^3 & 0 & 0 & g_{\delta \delta}^3 - \omega^2 I_{pad}^3 & 0 \\
g_{\delta x}^4 & g_{\delta y}^4 & 0 & 0 & 0 & g_{\delta \delta}^4 - \omega^2 I_{pad}^4 \\
\end{bmatrix}
\]

\[g_{xx}^i = K_{xx}^i + i \omega C_{xx}^i - \omega^2 M_{xx}^i; \quad g_{x\delta}^\beta = K_{x\delta}^\beta + i \omega C_{x\delta}^\beta - \omega^2 M_{x\delta}^\beta\]

\[g_{\delta x}^\beta = K_{\delta x}^\beta + i \omega C_{\delta x}^\beta - \omega^2 M_{\delta x}^\beta; \quad g_{\delta \delta}^\beta = K_{\delta \delta}^\beta + i \omega C_{\delta \delta}^\beta - \omega^2 M_{\delta \delta}^\beta\]

\((K^i, C^i, M^i)\) are pad stiffness, damping and added mass coefficients [3 DOF \((x, y, \delta^i)\) each]. \(G\) size = \((2+N_{pad})^2\)

Note: computational model includes also pivot deformation and pad transverse displacement (+ 2 DOF x \(N_{pad}\))
2-DOF rotor-bearing system

with frequency reduced force coefficients (as per Lund):

\[
\tilde{G}(\omega) = \left[ \begin{array}{cc}
\tilde{K}_{XX(\omega)} + i\omega\tilde{C}_{XX(\omega)} & \tilde{K}_{XY(\omega)} + i\omega\tilde{C}_{XY(\omega)} \\
\tilde{K}_{YX(\omega)} + i\omega\tilde{C}_{YX(\omega)} & \tilde{K}_{YY(\omega)} + i\omega\tilde{C}_{YY(\omega)} 
\end{array} \right] - \omega^2 \left[ \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right] M_J
\]

\(M_J = \) rotor mass and \((K,C)\) are 2×2 bearing stiffness & damping coefficients.

**Frequency reduced**

\[
\text{Re} \left( \tilde{K}_{\alpha\beta(\omega)} - i\omega\tilde{C}_{\alpha\beta(\omega)} \right) \rightarrow \tilde{K}_{\alpha\beta} - \omega^2 \tilde{M}_{\alpha\beta}
\]

\[
\text{Im} \left( \tilde{K}_{\alpha\beta(\omega)} - i\omega\tilde{C}_{\alpha\beta(\omega)} \right) \rightarrow i\omega\tilde{C}_{\alpha\beta}
\]

**KCM model (Childs)**

\[
\bar{G}(\omega) = \left[ \begin{array}{cc}
\bar{K}_{XX} + i\omega\bar{C}_{XX} & \bar{K}_{XY} + i\omega\bar{C}_{XY} \\
\bar{K}_{YX} + i\omega\bar{C}_{YX} & \bar{K}_{YY} + i\omega\bar{C}_{YY} 
\end{array} \right] - \omega^2 \left[ \begin{array}{ccc} \bar{M}_{XX} + M_J & \bar{M}_{XY} \\ \bar{M}_{YX} & \bar{M}_{YY} + M_J \end{array} \right]
\]

with KCM coefficients
Frequency response functions (FRF) for rotor-bearing system

FRFs show natural frequencies and damping ratios for various modes of operation → evidence sensitivity of a journal or pads to external (periodic) forces
Simple rotor-TPJB system

\[ \left( K - \omega^2 M + i\omega C \right) z_0 = G(\omega) z_0 = F_0 \quad \text{EOM} \]

Full coefficient
\[ z_0 = G^{-1}(\omega) F_0 \]

Freq. reduced
\[ z_0 = \tilde{G}^{-1}(\omega) F_0 \]

KCM
\[ z = \{x, y\}^T \]
Effect of lubricant starvation on bearing performance

(1) Brockwell et al. (1994)

5-pads LBP TPJB TEST
Flow reduced to 40% nominal.
Measured pad temperatures.
Five-pad TPJB (LBP): Brockwell et al. (1994)

*Nominal flow rate=28 LPM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter, ( D )</td>
<td>98 mm</td>
</tr>
<tr>
<td>Diametrical clearance</td>
<td>304 µm</td>
</tr>
<tr>
<td>Length, ( L )</td>
<td>38 mm</td>
</tr>
<tr>
<td>Pad arc length, ( \Theta_p )</td>
<td>56°</td>
</tr>
<tr>
<td>Pad preload</td>
<td>0.0</td>
</tr>
<tr>
<td>Pivot offset</td>
<td>0.6</td>
</tr>
<tr>
<td>Lubricant viscosity, ( \mu )</td>
<td>21 mPa-s</td>
</tr>
<tr>
<td>density, ( \rho )</td>
<td>856 kg/m³</td>
</tr>
<tr>
<td>supply temperature</td>
<td>49 °C</td>
</tr>
<tr>
<td>Rotor speed, ( \Omega )</td>
<td>16,500 rpm</td>
</tr>
<tr>
<td>Applied load, ( W )</td>
<td>5,338 N</td>
</tr>
<tr>
<td>Specific load, ( W/(LD) )</td>
<td>1.4 MPa</td>
</tr>
</tbody>
</table>
Flow starvation: temperature & pressure

1X = 16.5 krpm, W/LD = 1.4 MPa

As flow rate decreases, so does the effective pad length and film peak pressure.

→ Pad temperature increases.

Measured & predicted temperatures agree.
Flow starvation: temperature & power loss

1X = 16.5 krpm, \( W/(LD) = 1.4 \text{ MPa} \)

As flow rate decreases,
→ Pad temperature increases
→ Power loss decreases
Effect of lubricant starvation on bearing performance

(2) Dimond et al. (2010)

4-pads LBP TPJB
Small journal size
Preloaded pads (0.3)
Center pivot
Four-pad TPJB (LBP): Dimond et al. (2010)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter, $D$</td>
<td>127 mm</td>
</tr>
<tr>
<td>Diametral clearance, $C_D=2C_B$</td>
<td>305 µm</td>
</tr>
<tr>
<td>Length, $L$</td>
<td>95.3 mm</td>
</tr>
<tr>
<td>Pad arc length, $\Theta_p$</td>
<td>75°</td>
</tr>
<tr>
<td>Pad preload</td>
<td>0.3</td>
</tr>
<tr>
<td>Pivot offset</td>
<td>0.5</td>
</tr>
<tr>
<td>Lubricant viscosity, $\mu$</td>
<td>33 mPa-s</td>
</tr>
<tr>
<td>density, $\rho$</td>
<td>850 kg/m³</td>
</tr>
<tr>
<td>supply temperature</td>
<td>32 °C</td>
</tr>
<tr>
<td>Pad inertia, $I_{pad}$</td>
<td>716 kg-mm²</td>
</tr>
<tr>
<td>Rotor speed, $\Omega$</td>
<td>5,000 rpm</td>
</tr>
<tr>
<td>(83.3 Hz)</td>
<td></td>
</tr>
<tr>
<td>Applied load, $W$</td>
<td>8.3 kN</td>
</tr>
<tr>
<td>Specific load, $W/(LD)$</td>
<td>689 kPa</td>
</tr>
</tbody>
</table>

* Nominal flow rate=45LPM

* Bearing geometry adapted from Dimond(2009) and set $M_J=\frac{1}{2} W/g$. 

$\frac{1}{2}$ Rotor Mass $M_J = 850$ kg
Flow starvation: **Pressure**

- **Flowrate ↓ → Unloaded pads (Pads #1&2) do not generate pressure → Peak pressure: 71% flow < 41% flow**

**1X= 5 kpm (83.3 Hz)**
How does the flow reduction affect the system natural frequency and damping ratio?

**Flow starvation: force coefficients**

1X= 5 kpm (83.3 Hz)

<table>
<thead>
<tr>
<th>Flow (LPM)</th>
<th>$K_{xx} = K_{yy}$ (MN/m)</th>
<th>$C_{xx} = C_{yy}$ (kN.s/m)</th>
<th>$M_{xx} = M_{yy}$ (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45 (100%)</td>
<td>328</td>
<td>861</td>
<td>299</td>
</tr>
<tr>
<td>32 (71%)</td>
<td>243</td>
<td>356</td>
<td>358</td>
</tr>
<tr>
<td>18 (41%)</td>
<td>415</td>
<td>282</td>
<td>401</td>
</tr>
</tbody>
</table>

Bearing stiffness increases for lowest flow rate (41%) while damping decreases quickly with more flow starvation.

How does the flow reduction affect the system natural frequency and damping ratio?
### Flow starvation: Natural frequency and damping ratio

1X = 5,000 rpm (83.3 Hz)

<table>
<thead>
<tr>
<th>Flow</th>
<th>Modes</th>
<th>KCM</th>
<th>Frequency Reduced</th>
</tr>
</thead>
<tbody>
<tr>
<td>100%</td>
<td>1,2</td>
<td>66</td>
<td>85</td>
</tr>
<tr>
<td>71%</td>
<td>3,4</td>
<td>65</td>
<td>71</td>
</tr>
<tr>
<td>41%</td>
<td>KCM</td>
<td>86</td>
<td>92</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Flow</th>
<th>Modes</th>
<th>KCM</th>
<th>Frequency Reduced</th>
</tr>
</thead>
<tbody>
<tr>
<td>100%</td>
<td>1,2</td>
<td>0.48</td>
<td>0.70</td>
</tr>
<tr>
<td>71%</td>
<td>3,4</td>
<td>0.19</td>
<td>0.33</td>
</tr>
<tr>
<td>41%</td>
<td>KCM</td>
<td>0.15</td>
<td>0.20</td>
</tr>
</tbody>
</table>

With flow starvation → system natural frequency increases and damping ratio quickly decreases.

KCM model overpredicts damping and nat frequency.
As flow decreases, damping ratio decreases and natural frequency increases (crosses 1X). KCM model predicts the most damping.
Effect of lubricant starvation on bearing performance

(3) Commercial turbo machine

SSV observed under flow starvation. 5-pads LOP TPJB Center pivot (0.5) No preload pads Heavy rotor (12t)
**Turbo machine: Rotor-TPJB (LOP)**

![Diagram of turbo machine with rotor and pads]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter, $D$</td>
<td>432 mm</td>
</tr>
<tr>
<td>Length, $L$</td>
<td>254 mm</td>
</tr>
<tr>
<td>Diametral clearance</td>
<td>874 $\mu$m</td>
</tr>
<tr>
<td>Pad arc length, $\Theta_p$</td>
<td>56°</td>
</tr>
<tr>
<td>Preload, $r_p$</td>
<td>0.0</td>
</tr>
<tr>
<td>Pivot offset</td>
<td>0.5</td>
</tr>
<tr>
<td>Lubricant temp. at supply</td>
<td>45 °C</td>
</tr>
<tr>
<td>Viscosity, $\mu$</td>
<td>18 cPoise</td>
</tr>
<tr>
<td>Density, $\rho$</td>
<td>860 kg/m$^3$</td>
</tr>
<tr>
<td><strong>Rotor speed, $\Omega$</strong></td>
<td>3,600 rpm</td>
</tr>
<tr>
<td>Pad mass, $M_{pad}$</td>
<td>40 kg</td>
</tr>
<tr>
<td>Inertia, $I_{pad}$</td>
<td>0.395 kg-m$^2$</td>
</tr>
<tr>
<td>External radial load, $W$</td>
<td>117 kN</td>
</tr>
<tr>
<td><strong>Specific load, $W/(LD)$</strong></td>
<td>1.070 MPa</td>
</tr>
</tbody>
</table>

$\frac{1}{2}$ Rotor Mass $M_J = 11,980$ kg
Known operation (pads w/o preload)

Top pads are unloaded. If fully lubricated there is an increase in drag power loss. The large amount of flow pads 4&5 require ($Q_s$) is a waste.
Known operation of bearing

In actual operation:
flow supplied is lesser than one predicted (~ 50%)
→ The top (unloaded) pads take less flow, i.e. not fully flooded.
→ Delivered flow is enough to fill the three loaded pads: (3-pads model require $0.5Q_s$)
Predictions: 5-pad vs 3-pad (flooded and starved)

1X = 3.6 krpm, W/(LD) = 1.1 MPa

Film thickness

- Starved (3-pads, 88% flow)
- Flooded (3-pads)
- Flooded (5-pads)

Pressure

- Starved (3-pads, 88% flow)
- Flooded (3-pads)
- Flooded (5-pads)

Temperature

- Starved (3-pads, 88% flow)
- Flooded (3-pads)
- Flooded (5-pads)

5-pad and 3-pad models show similar results.

**Starved 3-pad** model predicts ~0 pressure for pads #1 & #3.

→ **Pad #2** is one generating pressure, identical for three cases.
$K, C$ coefficients: 5-pad vs 3-pad (flooded and starved)

Pad #2 has a full film albeit the other pads starve $\rightarrow$ Constant $K_{YY}$ & $C_{YY}$

$K_{YY} >> K_{XX}$, $C_{YY} >> C_{XX}$. For 88% flow, $P_{\text{Pads}#1,3,4,5}$~0 bar $\rightarrow (K_{XX}, C_{XX})$~0
Predictions: 1 pad (starves) vs 3 & 5 pads

For fully flooded, 5-pad: $Q_s$, 3-pad: $\frac{1}{2} Q_s$, 1-pad: $\frac{1}{4} Q_s$

**Flowrate ↓**

→ reduction in $Q_s$ increases $e$ and $K_{YY}$ but decreases $C_{YY}$.

→ For 1-pad, 5 & 3-pad models: $e$, $K_{YY}$, and $C_{YY}$ remain constant as Pad #2 is flooded although other pads are flow starved.

$K_{YY} \gg K_{XX}, C_{YY} \gg C_{XX}$
Natural frequencies and damping ratios for 5-pad, 3-pad and 1-pad models – fully flooded condition (pads effective arc=actual arc)

<table>
<thead>
<tr>
<th>Modes</th>
<th>Natural frequency (Hz)</th>
<th>5,6</th>
<th>7,8</th>
<th>9,10</th>
<th>11,12</th>
<th>13,14</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 Pads</td>
<td>9</td>
<td>17</td>
<td>60</td>
<td>114</td>
<td>117</td>
<td>180</td>
</tr>
<tr>
<td>3 Pads</td>
<td></td>
<td>17</td>
<td>61</td>
<td>111</td>
<td>116</td>
<td>181</td>
</tr>
<tr>
<td>1 Pad</td>
<td></td>
<td>60</td>
<td>179</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Modes</th>
<th>Damping ratio</th>
<th>1,2</th>
<th>3,4</th>
<th>5,6</th>
<th>7,8</th>
<th>9,10</th>
<th>11,12</th>
<th>13,14</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 Pads</td>
<td>0.03</td>
<td>0.11</td>
<td>0.31</td>
<td>0.88</td>
<td>0.87</td>
<td>0.99</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Pads</td>
<td></td>
<td>0.11</td>
<td>0.31</td>
<td>0.86</td>
<td>0.85</td>
<td>0.99</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Pad</td>
<td></td>
<td>0.32</td>
<td></td>
<td>0.99</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Only 5-pads model predicts top pads’ tilting mode.

→ Low damping ratio 1-pad model predicts two pairs of N.F.s for \{y, \delta_2\}.

For same mode, predicted N.F.s are ~same, regardless of model.
5-pad model gives lowest natural frequency (9 Hz) with small damping ratio (0.03) → tilting of unloaded pads (4&5).

5-pad and 3-pad models → identical amplitude for 2\textsuperscript{nd} mode at 17 Hz → tilting of loaded pads 1-3.
Pad resonance frequency (~9 Hz) does not excite rotor.

→ DeCamillo (2008) noted pad flutter does not excite test rotor.

Predicted nat. frequency at 17 Hz is slightly lower than measured SSV frequency at ~20 Hz.
At 88% of nominal flow, the peak amplitudes are unbounded as the damping ratio is negative ($\zeta<0$). The natural quickly drops.
Reducing flow to 90% decreases nat. frequency and damping ratio (17 → 11 Hz, $\zeta = 0.11 \rightarrow 0.01$).

As flow decreases (to 88%), upstream and downstream pads (1&3) starve → natural freq. drops to 5 Hz, → Damping turns negative → SSV Hash
Conclusion

A Flow Starvation Model for TPJBs and Evaluation of FRFs: A Contribution towards Understanding the Onset of Low Frequency Shaft Motions
A model predicts the performance of TPJBs operating with an oil supply flowrate well below its design condition.

A dynamic force coefficients model constructs FRF for a rotor supported on TPJBs

Example 1 (Brockwell) Predicted pads’ temperature agrees well with measurements in a flow starved bearing.
Conclusion

As supplied flowrate decreases:

(a) Pad temperature increases and power loss decreases.

(b) The force coefficients rapidly change, and the system N.F. and damping ratios drastically decreases.

(c) Flutter of unloaded pads does not excite a rotor.

(d) LOP configuration is more sensitive to the decrease in the flow rate than LBP TPJBs.

(e) Frequency reduced coefficients cannot predict pads’ modes of vibration or onset of SSV from flow starvation.
Acknowledgment

Thanks to Hitachi, Ltd. R&D Group

Questions (?)

Learn more: http://rotorlab.tamu.edu