



ASME

FEDSM 2025

FLUIDS ENGINEERING DIVISION
SUMMER MEETING

DOUBLETREE BY HILTON PHILADELPHIA CENTER CITY
PHILADELPHIA, PA, USA

Wet Annular Seals in Multiphase Pumps: Leakage and Rotordynamic Force Coefficients and a Method to Promote Seal Direct Stiffness



Dr. Luis San Andrés is Professor Emeritus at Texas A&M University. He was the Mast-Childs Chair and Professor of Mechanical Engineering (2014-2023). Since 1990-2025, students and Luis conducted research in lubrication of bearings and seals, vibrations and rotordynamics.

Dr. San Andrés received the **ASME-IGTI 2022 Aircraft Engine Technology Award** for creative contributions to aircraft engine technology, the **2023 Mayo D. Hersey Award** for enduring contributions to foil bearing technology, and the **2025 ASME Henry R. Worthington Medal** for groundbreaking work on pump seals for cryogenic turbopumps and multiple-phase pump seals.





TURBOMACHINERY LABORATORY
TEXAS A&M ENGINEERING EXPERIMENT STATION

Making a Difference in Industrial & Aerospace Turbomachinery

<http://turbolab.tamu.edu>

Dr. Eric Petersen, Director
June 2025

ASME
FEDSM 2025
FLUIDS ENGINEERING DIVISION
SUMMER MEETING



Facilities Turbomachinery Laboratory

37000 ft² total area with 12 test-cell suites, each 520 ft² sound-and seismically-isolated,
+
600VAC x 400 A bus = 240 kW=
(330 HP motor) in each cell.

+ Air Compressors

1.1 m³/s @ 8.6 bar
(2350 scfm @ 125 psi),
0.40 m³/s @ 20 bar,
0.54 m³/s @ 24 bar,
75 bar (1,100 psi)



New facilities in 2025 – Combustion & Propulsion – liquid and solid.



The Turbo Lab

Research Goals:

Conduct fundamental and applied research of the highest quality, meeting the needs of the turbomachinery industry for the 21st century (**performance and reliability**).

Thrust Areas:

- (1) Turbine Combustion & Cooling Technologies.
- (2) **Aerodynamic Performance and Efficiency of Turbomachinery (Pumps).**
- (3) Rotordynamics of Turbomachinery including Fluid-Structure Interaction Forces (**Compressors & Pumps**)
- (4) **Oil-Free Micro-Turbomachinery (< 400 kW)**



The Turbo Lab : Summary Outreach



Turbomachinery & Pump Symposia

- Annual event
- Houston, TX
- Approx. 4,500 attendees
- 350+ exhibitors
- Began:
Turbo 1971
Pump 1984

Asia Turbomachinery & Pump Symposium

- Biennial event since 2016
- Kuala Lumpur
- ~ 900+ attendees
- 50+ exhibitors

Continuing Education Stand-alone Short Courses

- 4-5 courses annually
- Houston, TX

Faculty & Graduate Student Involvement

- 6-8 ME faculty
 - 2 Aero faculty
 - 90+ graduate students
- TYP
\$700k/faculty per year

Turbomachinery Research Consortium (TRC)

- **30+ companies (O&G OEMs and end users)**
- \$35k annual fee
- Approx. 13-17 research projects funded annually.

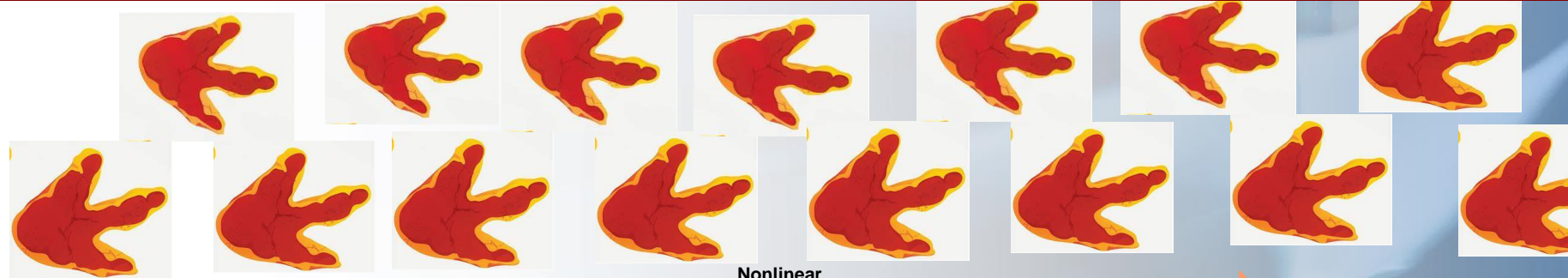
Global footprint!

Middle East Turbomachinery Symposium (METS)

Past (2011, 2013, 2015): 1000 attendees + 60+ exhibitors

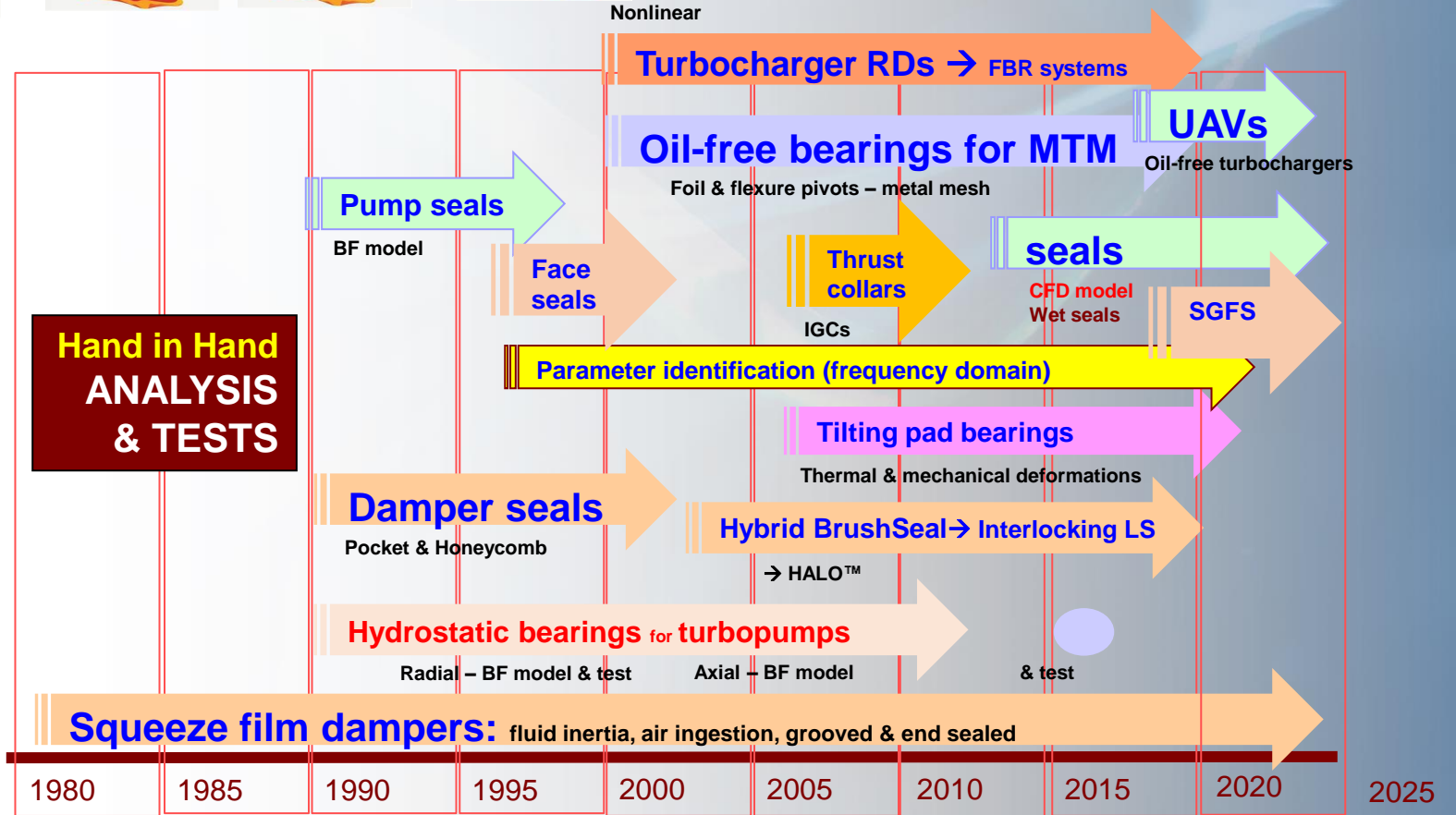


A dinosaur walk since last millennium



Funding Sources

John Crane, Baker-Hughes, Trane, Elliott Co.
Blue Origin,
 Army Research Lab (CUP),
NSF, NASA GRC,
Pratt & Whitney,
Northrop Grumman,
Rocketdyne,
 Honeywell TT,
 Danfoss TurboCor
 Borg-Warner TC,
Torishima Pumps,
 MHI, Hitachi RL,
Samsung, Key Yang,
Hyundai HI, Capstone MT
Siemens, TRC



Today's Outcome

1. Why wet (bubbly) seals? Where are they found?
2. How does the gas content affect seal leakage and drag?
3. How does gas content affect the stiffness and damping coefficients of a *wet* seal?
4. Why a wavy surface seal is a better option than a grooved seal in a two-phase viscous flow pump?
5. Low frequency self-excited vibrations & their source.
6. Why does gas injection increase the centering stiffness of seals in pumps & turbines?



A need: subsea pumping & compression

Subsea Engineering or SURF

Subsea

Umbilicals

Risers

Flowlines

Wet compression
systems a must!

High pressure & extreme temperature

Bloomberg : Since 2018 offshore oil production tops shale oil on generation of jobs.

Extreme engineering enables five year or longer reliability for subsea production facilities (North Sea & Brazil → Gulf of Mexico → Arctic).

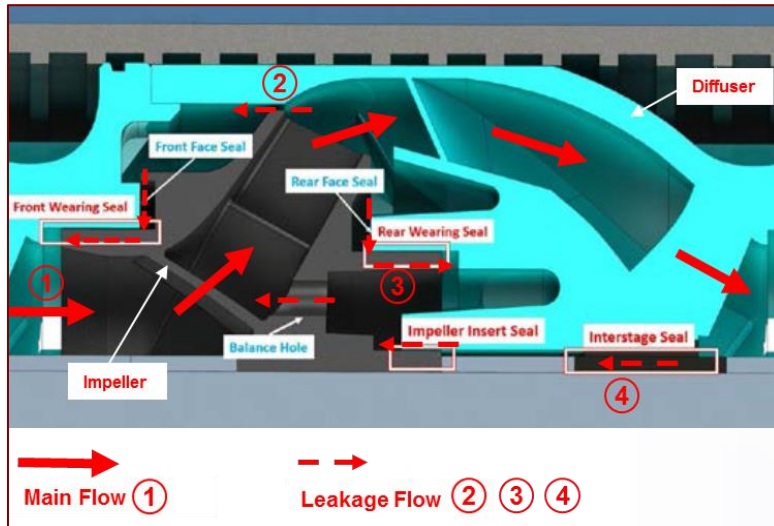


Subsea O&G Production Facilities

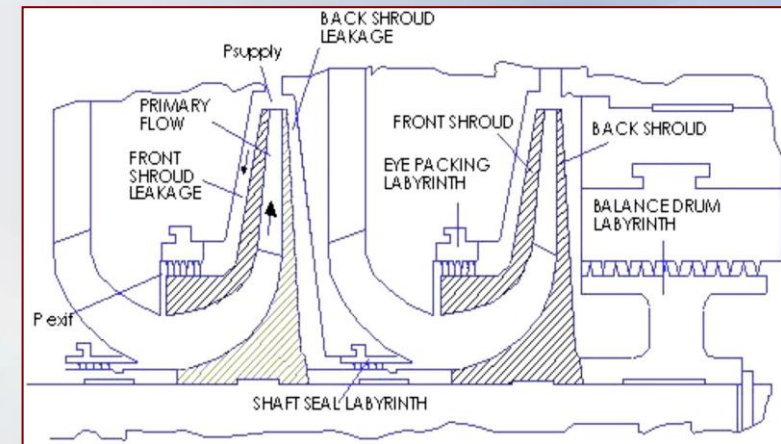


GVF: Gas volume fraction (pump)

LVF=1-GVF: Liquid volume fraction (compressor)



Electrical submersible pump stage (GVF: 0 to 100%)



Compressor stage (LVF: 0 to 5%)

Bubbly/Wet seals affect pump/compressor performance (leakage and torque) and reduce system rotordynamic stability.

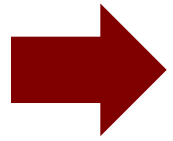
Subsea O&G production facilities with improved reliability and reduced operational cost demand a concerted effort to quantify effect of two-phase flow in sealing components.



Prior art: a shallow review



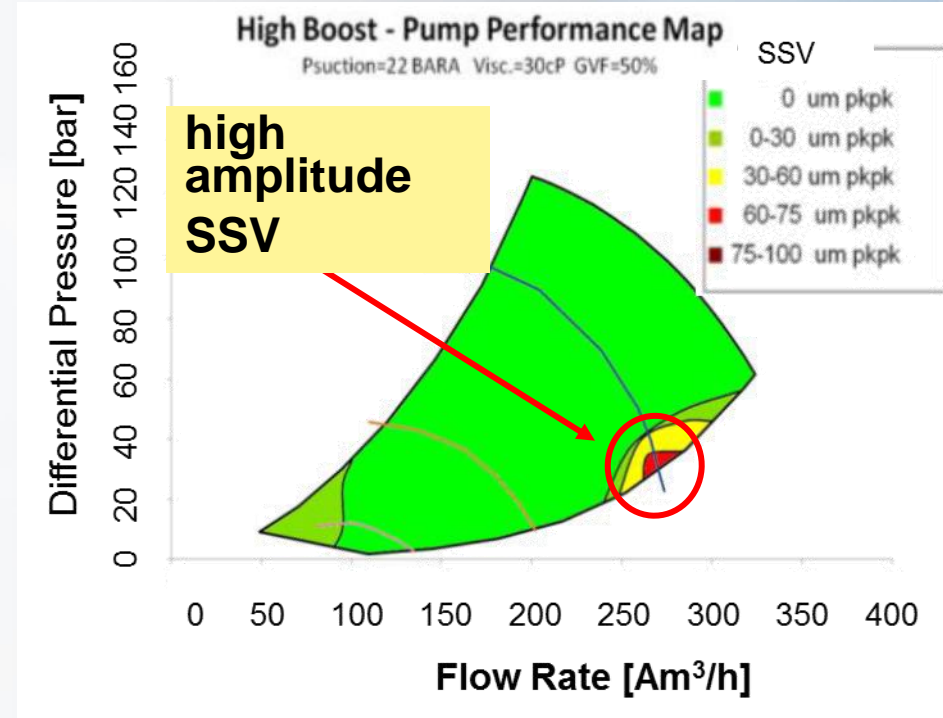
Balance drum seal in multiphase pump (2013)



Helico-axial pump (1.5 to 4.6 krpm)

Pump operates stable with liquid. (ΔP 150 bar, $\mu_l = 30$ cP)

Rotor asynchronous vibrations (SSV) appear under some two-phase flow conditions: **low differential pressure with a high-viscosity mixture.**

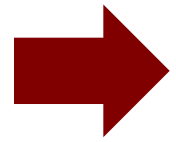


Bibet et al. (2013)

Bibet, P. J., et al., 2013, "Design and Verification Testing of a New Balance Piston for High Boost Multiphase Pumps," Proc. 29th International Pump User Symposium, Houston, TX.



Balance drum seal in multiphase pump (2018)



Helico-axial pump (1.5 to 4.6 krpm)

ΔP 110 bar, $\mu_f=800$ cP, GVF= 0 \rightarrow 1.

For pump $\Delta P \sim 40$ bar, inlet GVF ~ 0.6

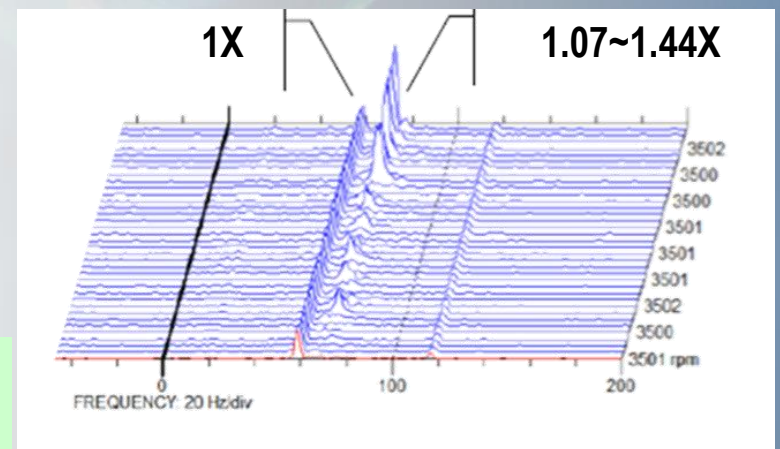
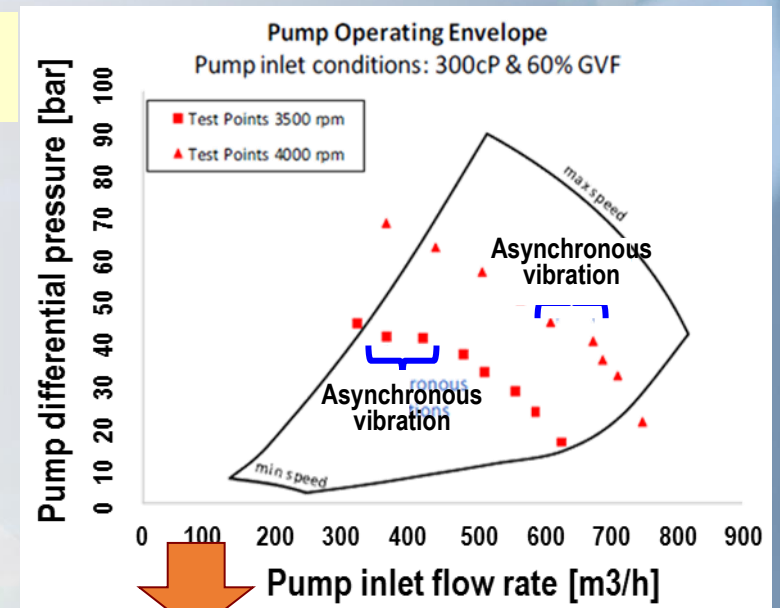
Rotor super-synchronous vibration (1.07 ~1.44 X).

Seal mass flow 'jumps' when ΔP is held constant.

Negative seal direct stiffness predicted.

REMEDY: Changed seal clearance profile affects axial flow velocity development and removes super synchronous vibration.

Ekeberg, I., Bibet, P., Knudsen, H., Torbergsen, E., Kjellnes, H.F., Angeltveit, R., and Klepsvik, K., 2018, "Design and Verification Testing of Balance Piston for High-Viscosity Multiphase Pumps," *Proc. 47th Turbomachinery & 34th International Pump Symposium*, Houston, TX.



Research on Wet Gas / Bubbly Seals at the Turbo Lab

Childs and students (2012-2018)

J. Eng. Gas Turb. Power, 2017

J. Tribol., 2018

Measured leakage and rotordynamic force coefficients of wet seals with an air in silicon oil mixture. **GVF: 0 → 10%, LVF: 0 → 8%.**

Max pressure: 70 bar, shaft speed 20 krpm ($R\Omega=96$ m/s)

San Andrés and students (2014-2022)

Tribol. Trans., 2016

J. Eng. Gas Turb. Power: 2017, 2018, 2019, 2022

2018 ATPS/TPS

AJKFluids 2019

WTC 2022

Quantify leakage and dynamic force coefficients of wet seals [five types] with air in ISO VG10 oil mixture. **GVF: 0 → 1,**

Max supply pressure: 5 bar, shaft speed: 5.2 krpm ($R\Omega=35$ m/s).

Application: subsea multiphase pumps and wet gas compressors





In sum ...

and **this lecture**

In the subsea oil and gas industry, multiphase pumps and wet gas compressors enable long distance tie back system and eliminate oil and gas separation stations.

Seals must be able to operate without affecting the system efficiency and dynamic stability.

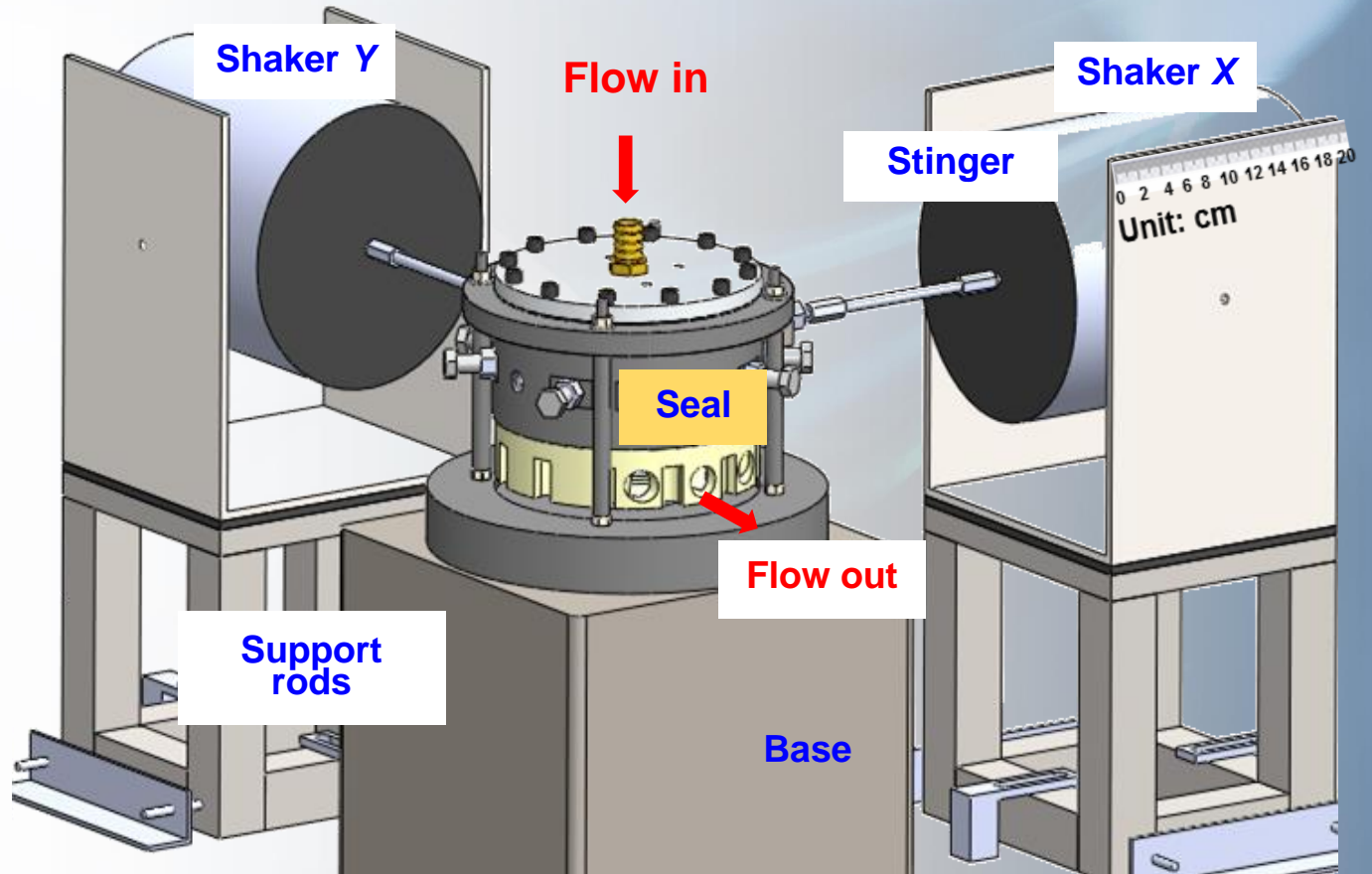
The lecture presents measurements of leakage and force coefficients for two annular seals operating with an air in oil mixture ranging from pure liquid to mostly air. **Some interesting features reported!**



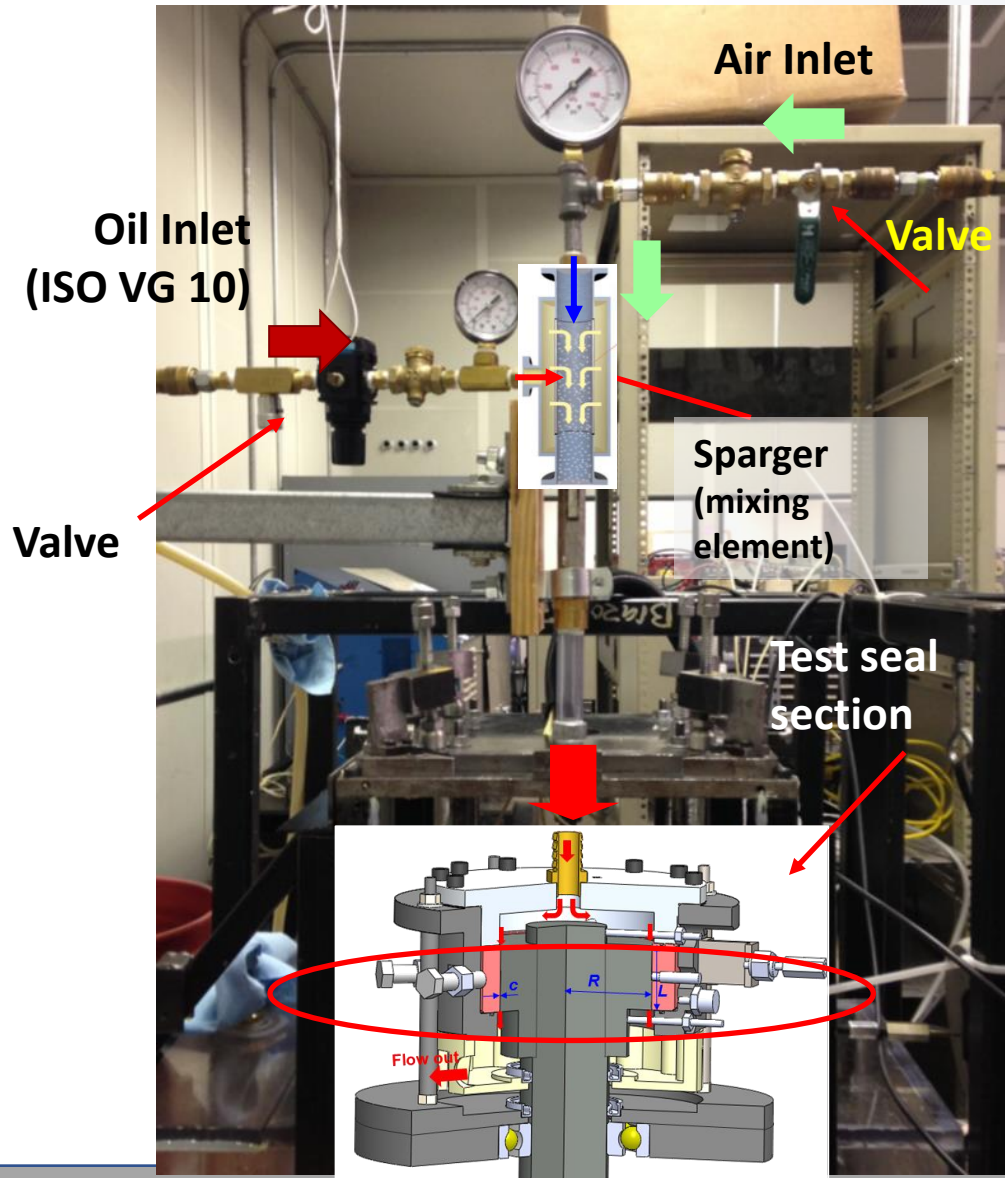


Wet Gas Test Rig

- Controlled motion test rig with "floating" seal housing and spinning rigid shaft.
- Shakers exert frequency-dependent loads to excite system toward obtaining seal force coefficients.



Wet Gas / Bubbly Liquid Seal Rig



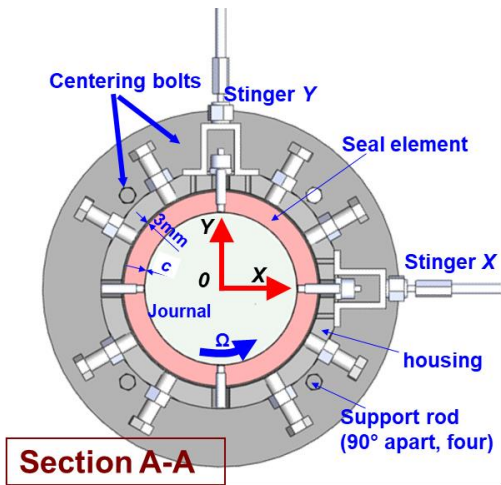
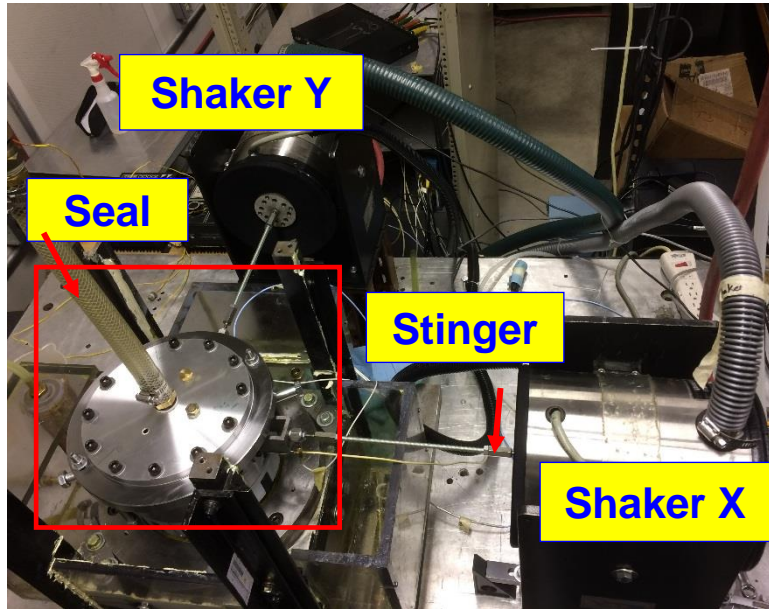
Rotor Diameter (D)	127 mm
Rotor Length (L)	46 mm
Supply pressure (P_s)	1.0~4.5 bar (abs)
Oil ISO VG 10 density(ρ_l)	830 kg/m³
viscosity (μ_l)	10.6 cP at 34 °C
Air density (ρ_{ga})	1.2 kg/m³ at 1bar
viscosity(μ_{ga})	0.02 cP at 20 °C
Sparger pore size	2 μ m
Air bubble size	Up to 4 mm

$L/D=0.36$ short length

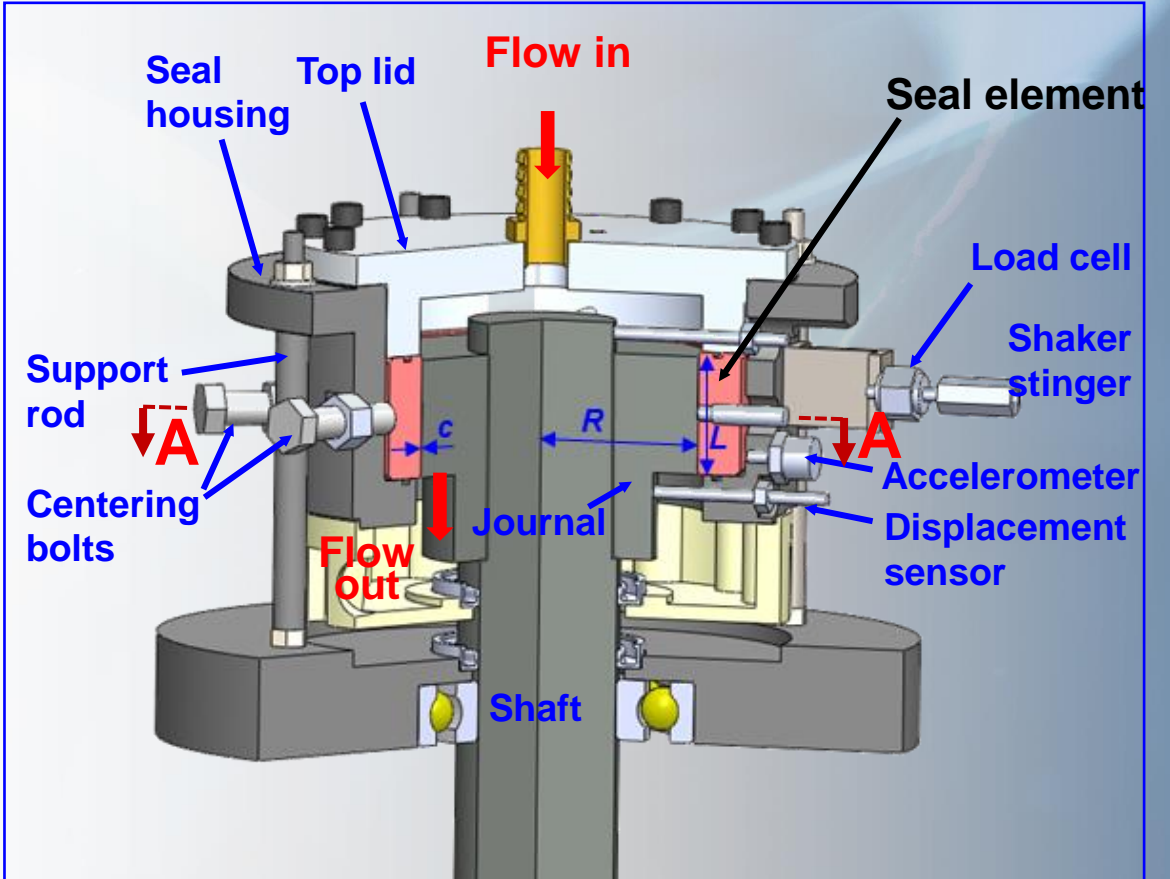
APPLICABLE to impeller interstage seals and/or neck ring seals.



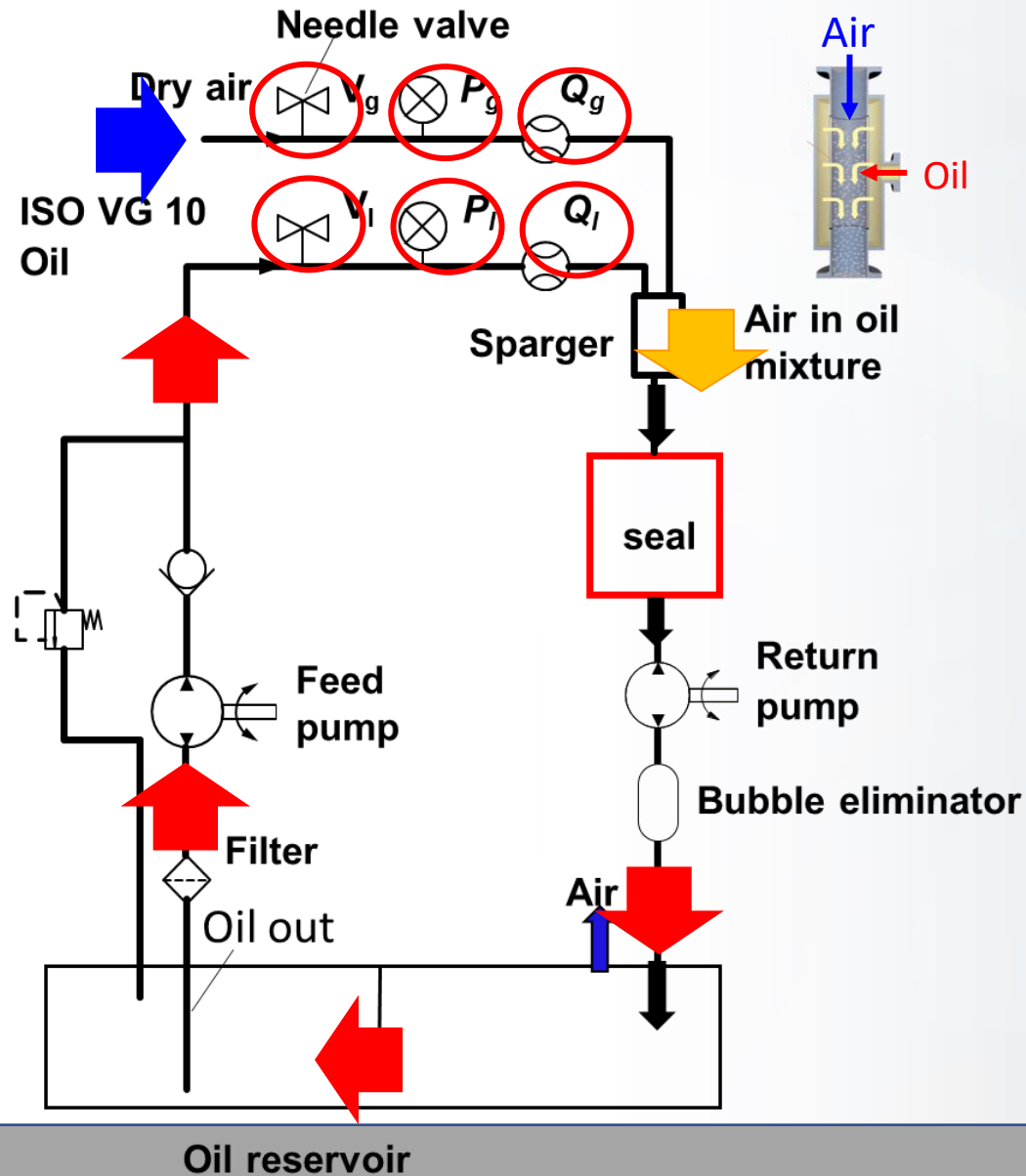
Cartridge Housing Seal



Rotor speed, Ω_{\max}	3.5 krpm
surface speed, $\frac{1}{2}D\Omega_{\max}$	23.3 m/s



Air & Oil Systems and their Mixing



α : Gas volume fraction (GVF)

P_s : pressure at seal inlet plane

P_a : ambient pressure= 1 bar(a)

Q_g : gas flow rate at P_s

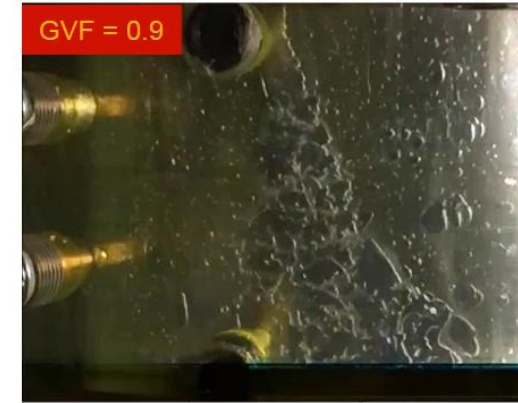
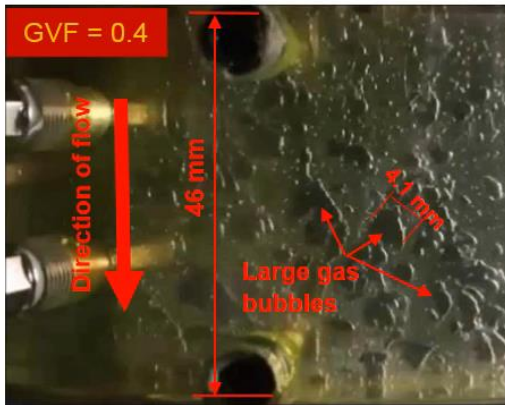
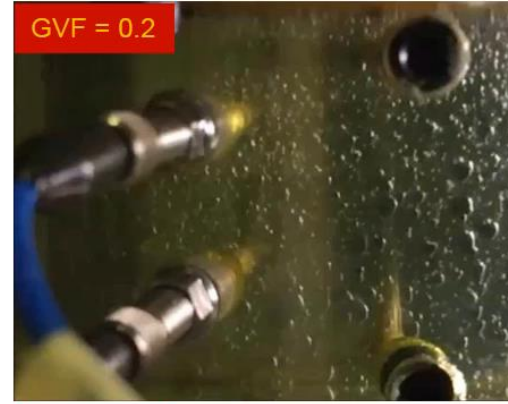
Q_l : liquid flow rate

GVF at inlet:

$$\alpha_{in} = \frac{Q_g \left(\frac{P_a}{P_s} \right)}{Q_l + Q_g \left(\frac{P_a}{P_s} \right)}$$



Visualization → $GVF = 0-0.9$. $P_s/P_a=2.5$. Speed 0 rpm (pain seal)



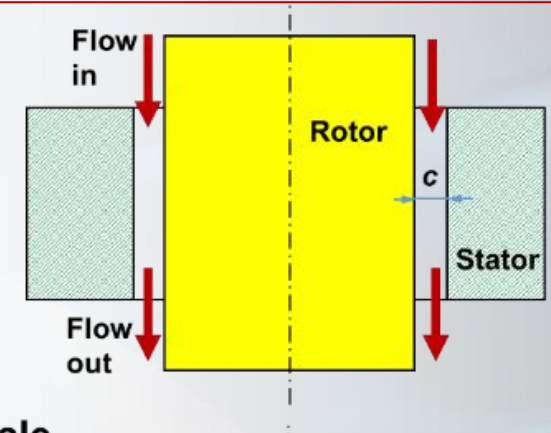
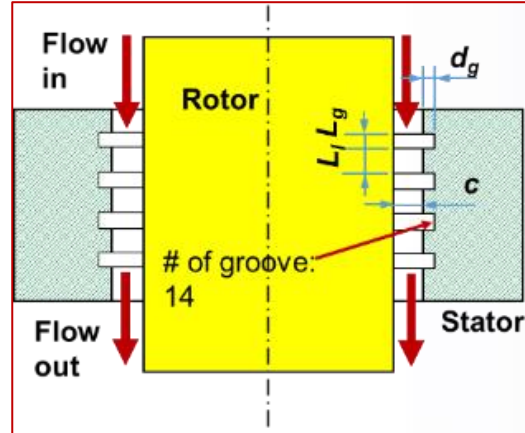
Gas content increases

ASME 2019

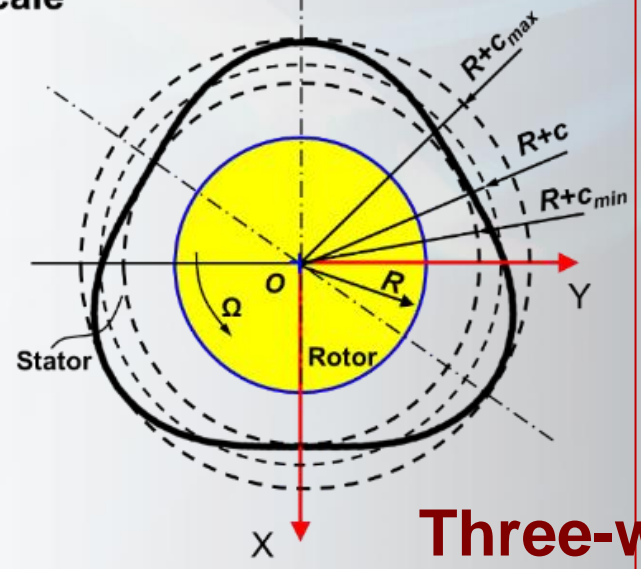
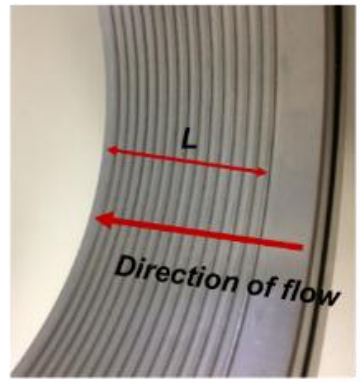




Two test seals



Not to scale



Grooved seal

$c_r=0.211$ mm, $d_g=0.543$ mm, $l_g=1.5$ mm, $l_r=0.904$ mm, $N_g=14$

Three-wave seal

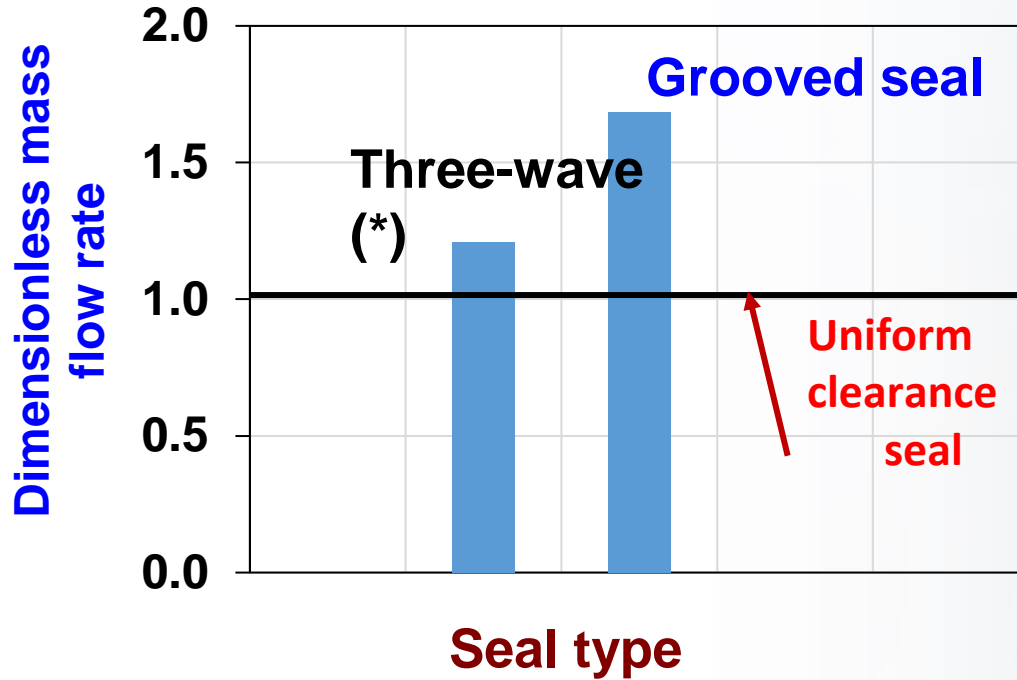
$c_{max}=0.274$ mm, $c_{min}=0.108$ mm, $c_{av}=0.191$ mm



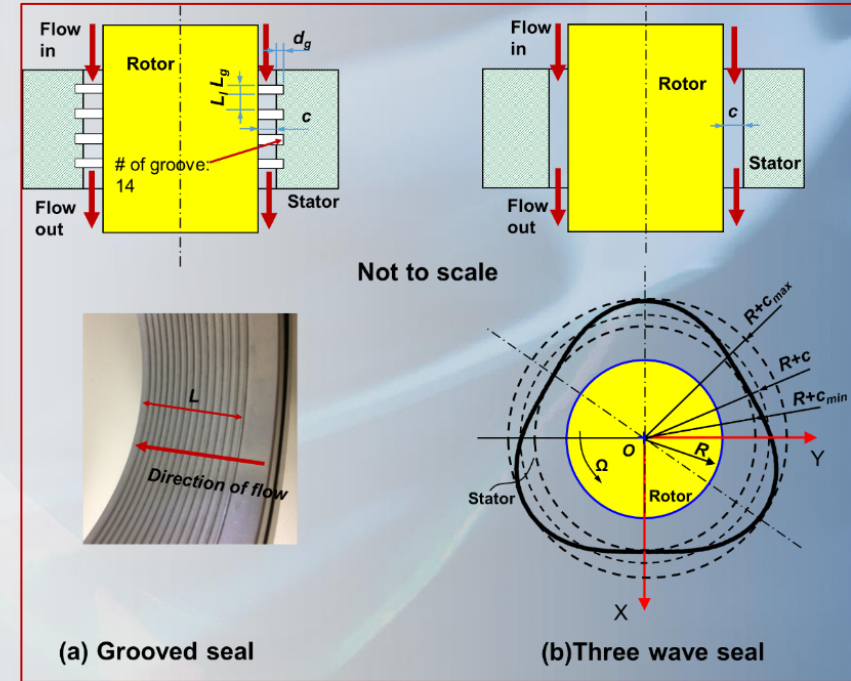
Measured Leakage: Liquid Only **LVF=1**

Normalized to

$$\dot{m}_{pl} = \frac{1}{12} \frac{\rho_l}{\mu_l} \pi D c^3 \frac{\Delta P}{L}$$



(*) Lu, X., and San Andrés, L., 2019, "ASME J. Eng. Gas Turb. Power, 141.



Grooved seal not as effective as three wave seal (*) since flow is laminar.



Leakage vs. Gas Content

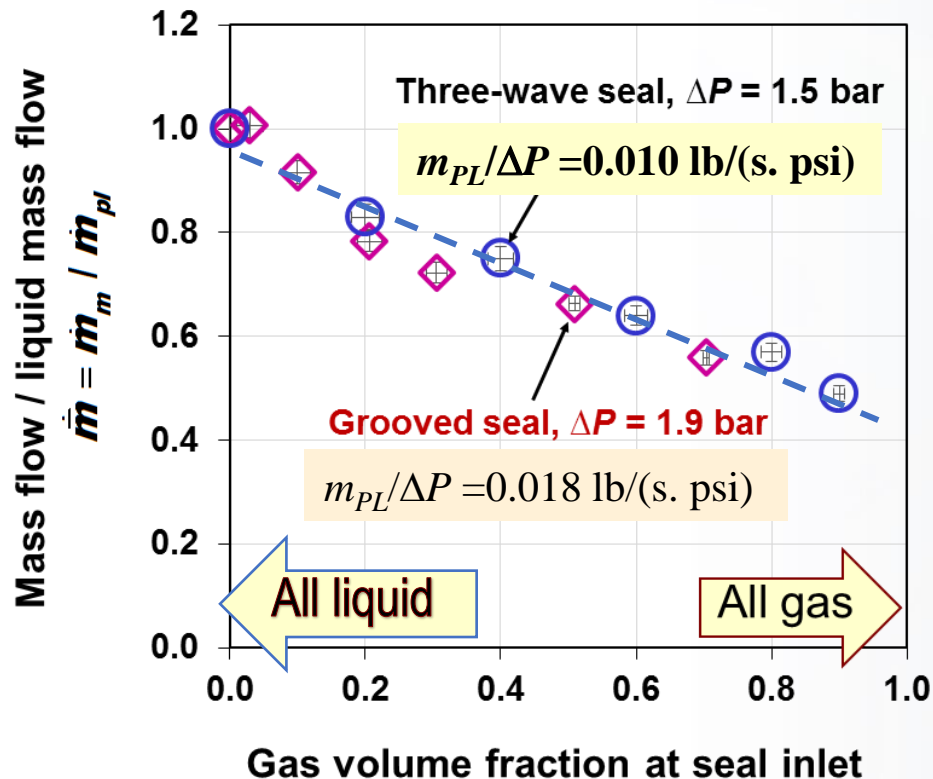
Grooved seal vs
Three-wave seal

$$\dot{\bar{m}} = \frac{\dot{m}_{mixture}}{\dot{m}_{liquid}}$$

Normalized with respect to liquid (GFV=0)

Rotor speed, Ω_{max} 3.5 krpm

Surface speed 23.3 m/s



Leakage for two seals drops as GVF increases.

For LVF=1 (GVF=0), grooved seal leaks (117 g/s) more than three wave seal (65 g/s).

Inlet GVF	0	0.10	0.5	0.7
GMF	0	0.05%	0.43%	1%



Grooved Seal Drag Torque vs. GVF

normalized to all liquid condition

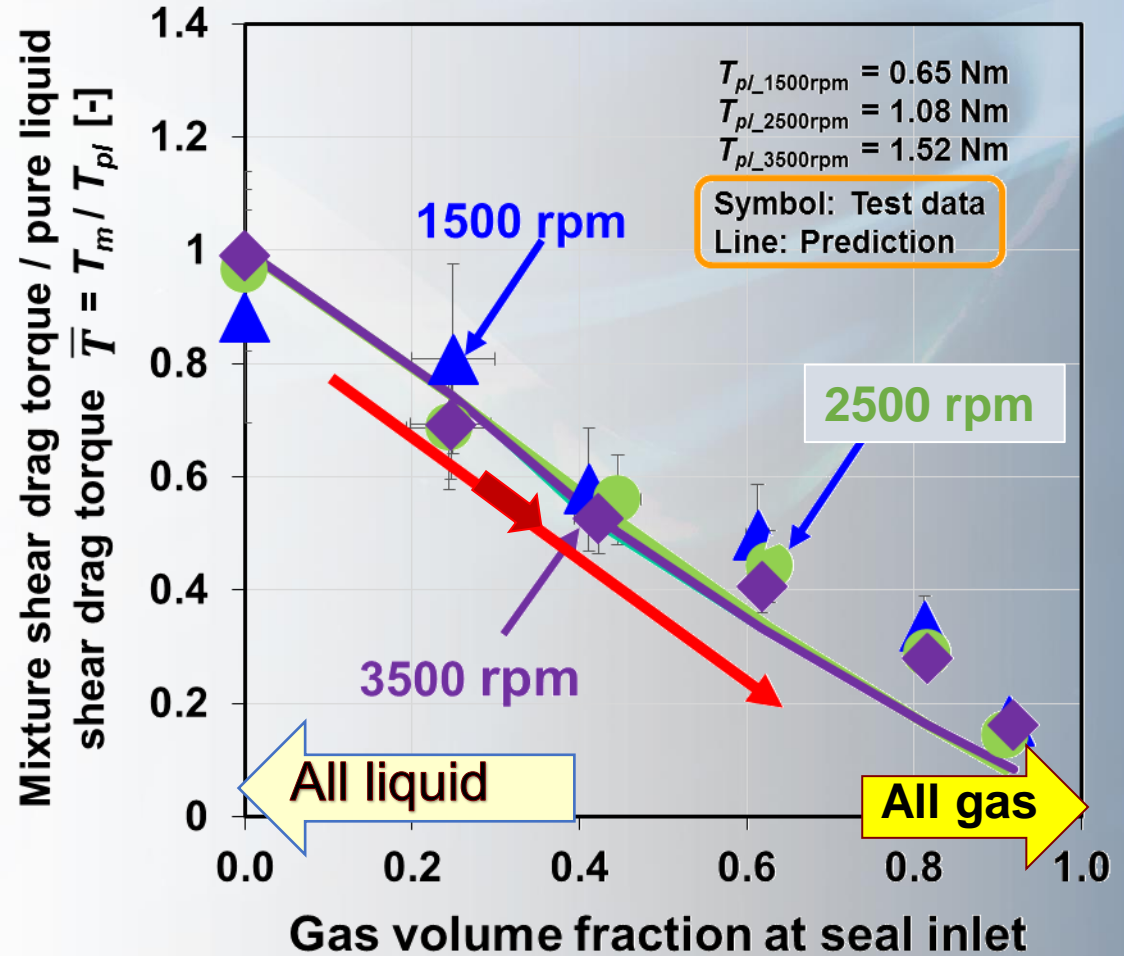
Torque linearly decreases with GVF.

GVF = 0 to 0.9 → 85% reduction in drag torque.

Torque change → Sudden overspeed will affect the pump and motor reliability!

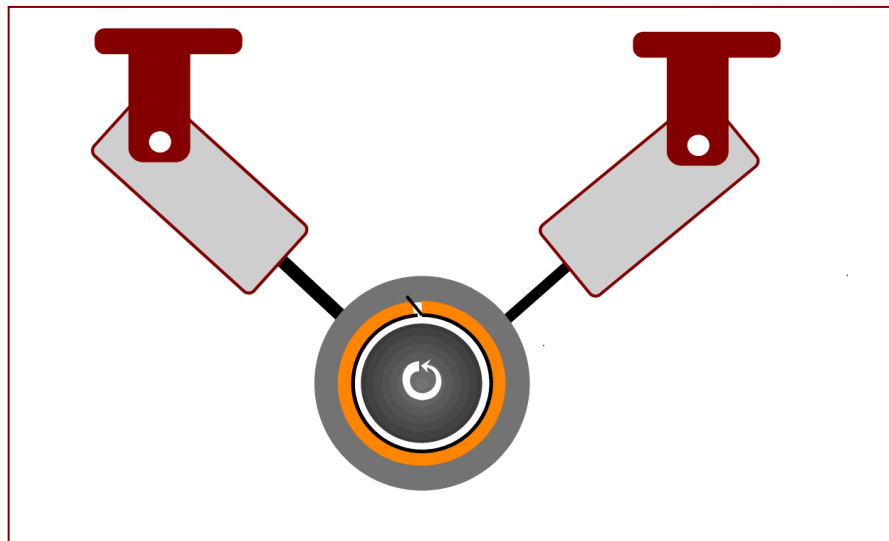
$$T_{seal} \sim \frac{2\pi \mu_{(GVF)} \Omega R^3 L}{c}$$

prediction

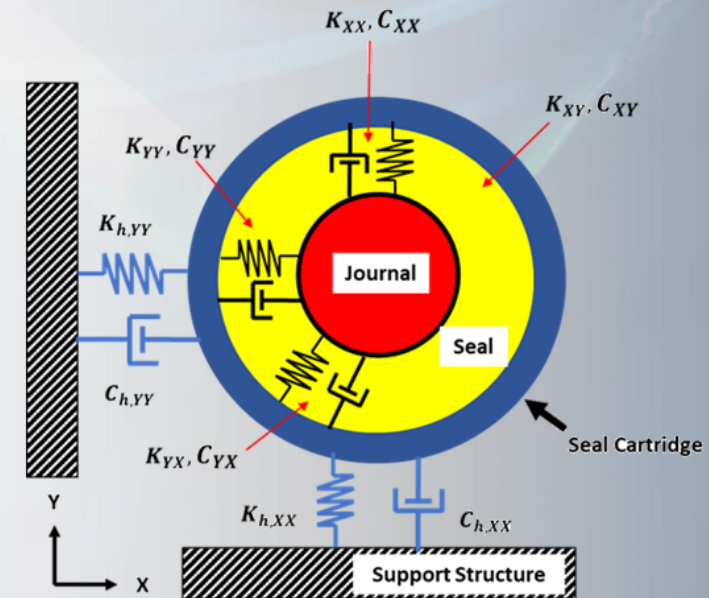
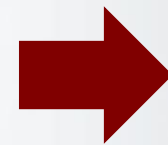




Experimental Identification of Seal Rotordynamic Force Coefficients

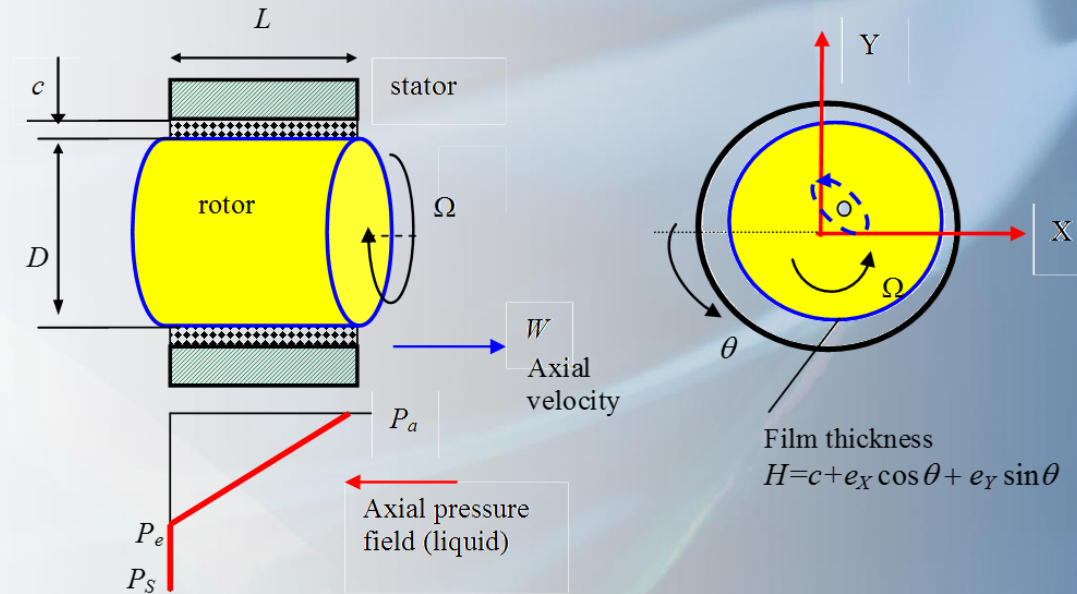


2-DOF system for seal and support structure



Dynamic Force Coefficients

For small amplitudes of rotor motion, a seal force is represented with **stiffness (K)**, **damping (C)** and **added mass (M)** force coefficients. However, wet seals have frequency dependent K & C :



$$\begin{Bmatrix} F_X \\ F_Y \end{Bmatrix} = - \begin{bmatrix} K_{(\omega)} & k_{(\omega)} \\ -k_{(\omega)} & K_{(\omega)} \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} - \begin{bmatrix} C_{(\omega)} & c_{(\omega)} \\ -c_{(\omega)} & C_{(\omega)} \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix}$$

For two-phase flow or a gas



Identification of Force Coefficients

- 1) Apply Load $\mathbf{F} = \mathbf{F}_0 \sin(\omega t) \rightarrow$ Measure vectors of displacements $\mathbf{z} = \{x, y\}^T$, & accelerations $\mathbf{a} = \{a_x, a_y\}^T$
- 2) $\bar{\mathbf{F}}, \bar{\mathbf{A}}, \bar{\mathbf{Z}}$ = Discrete Fourier Transform of $\mathbf{F}, \mathbf{a}, \mathbf{z}$
- 3) $\bar{\mathbf{F}} - \mathbf{M}_h \bar{\mathbf{A}} - [\mathbf{K}_h + i\omega \mathbf{C}_h] \bar{\mathbf{Z}} \rightarrow \mathbf{H}_{(\omega)} \bar{\mathbf{Z}}$ $[\mathbf{M}, \mathbf{K}, \mathbf{C}]_h$ = mass, stiffness, damping of support structure

Components of seal complex stiffness \mathbf{H}

$$\text{Re}(\mathbf{H}_{(\omega)}) \rightarrow \mathbf{K}_{(\omega)}$$

Dynamic Stiffness

$$\text{Im}(\mathbf{H}_{(\omega)}) \rightarrow \omega \mathbf{C}_{(\omega)}$$

Proportional to Damping

$$C_{\text{eff}} = C - k/\omega = [\text{Im}(H_{xx}) - \text{Re}(H_{xy})]/\omega$$

Effective Damping

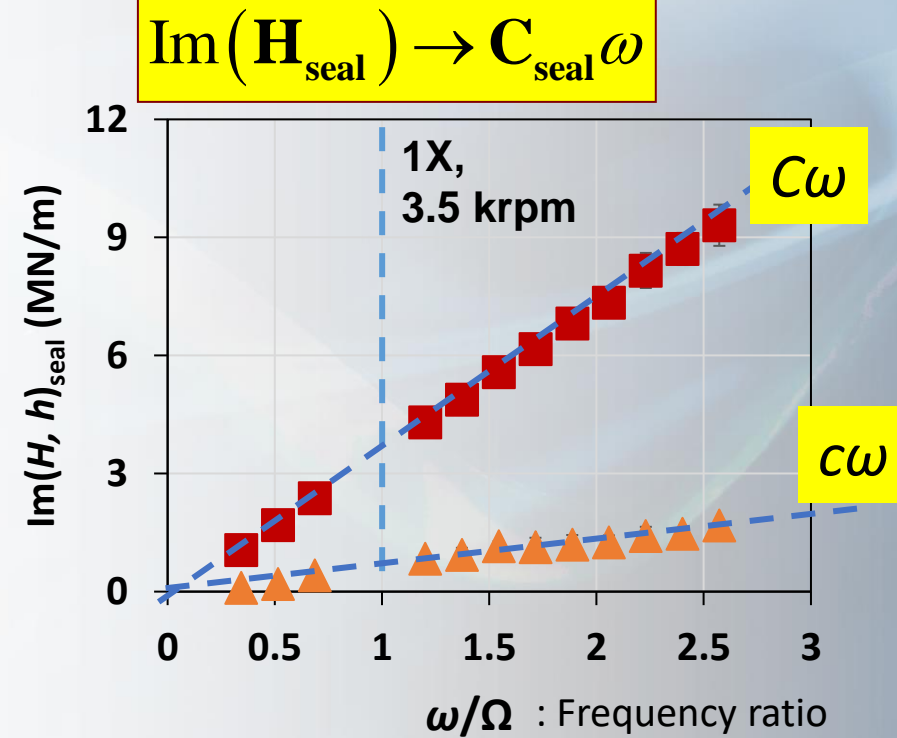
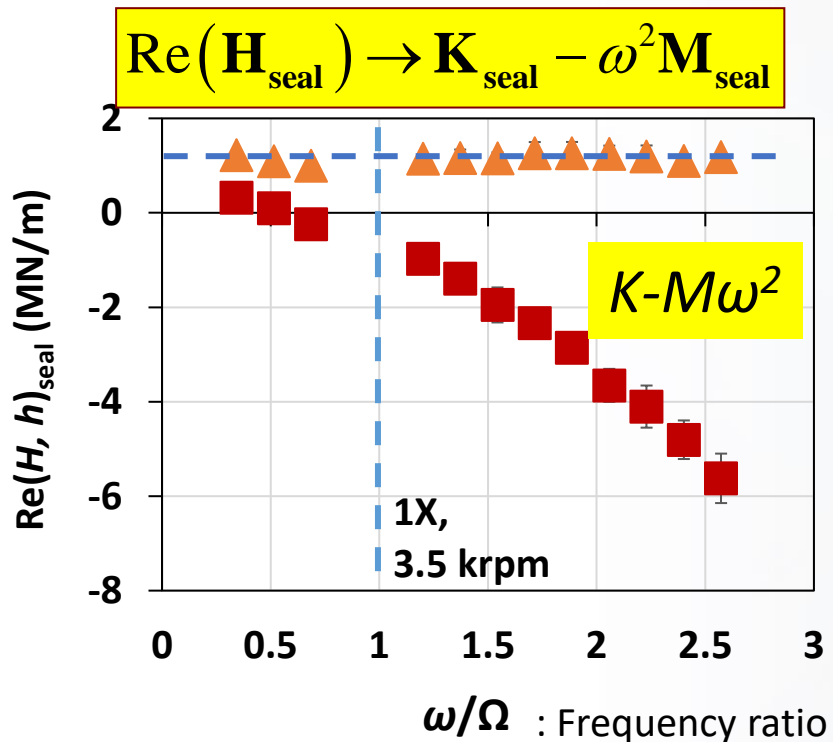


Grooved Seal **H**

Unidirectional single frequency load,
motion amplitude = 0.05 c

$$P_s/P_a = 3.9$$

LVF=1 : liquid only



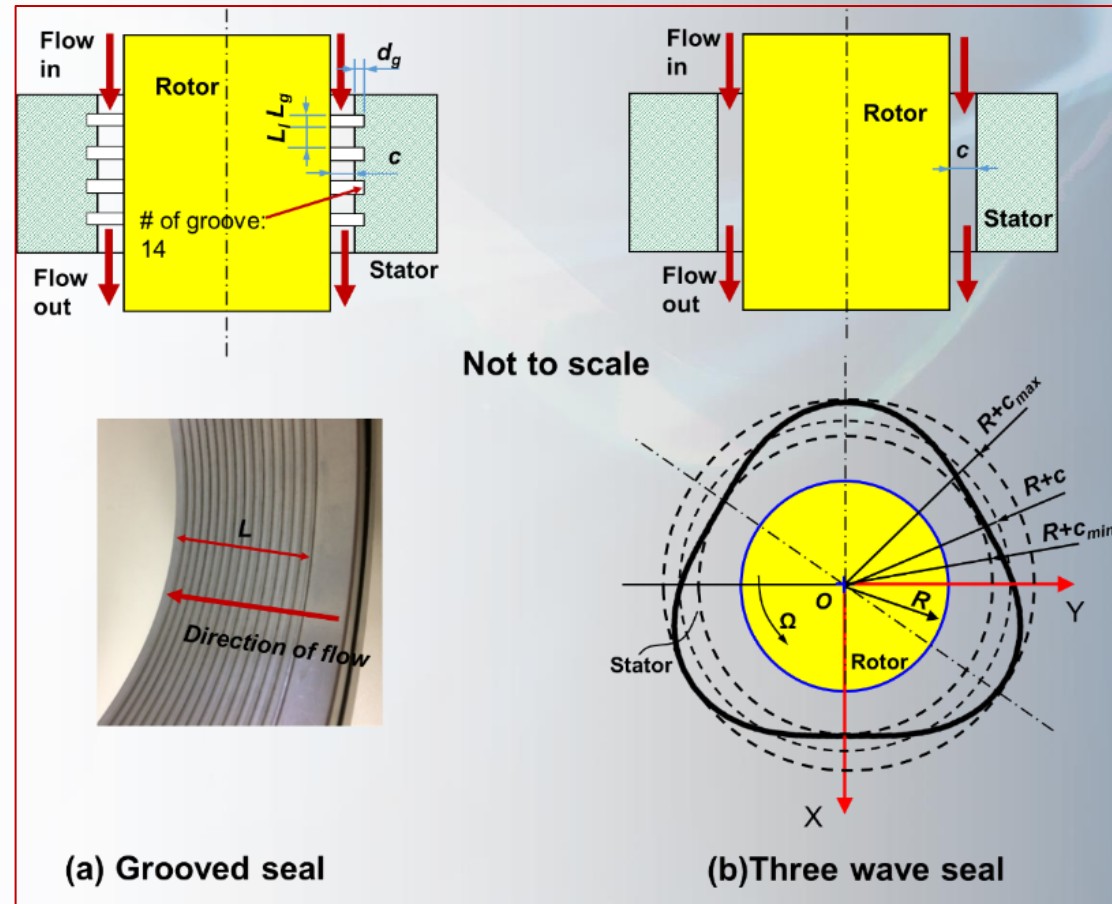
Curve Fit \rightarrow force coefficients

$K = 0.3 \text{ MN/m}$, $k = 1.2 \text{ MN/m}$, $C = 10 \text{ kNs/m}$, $c = 1.8 \text{ kNs/m}$, $M = 6.6 \text{ kg}$, $m = 0 \text{ kg}$



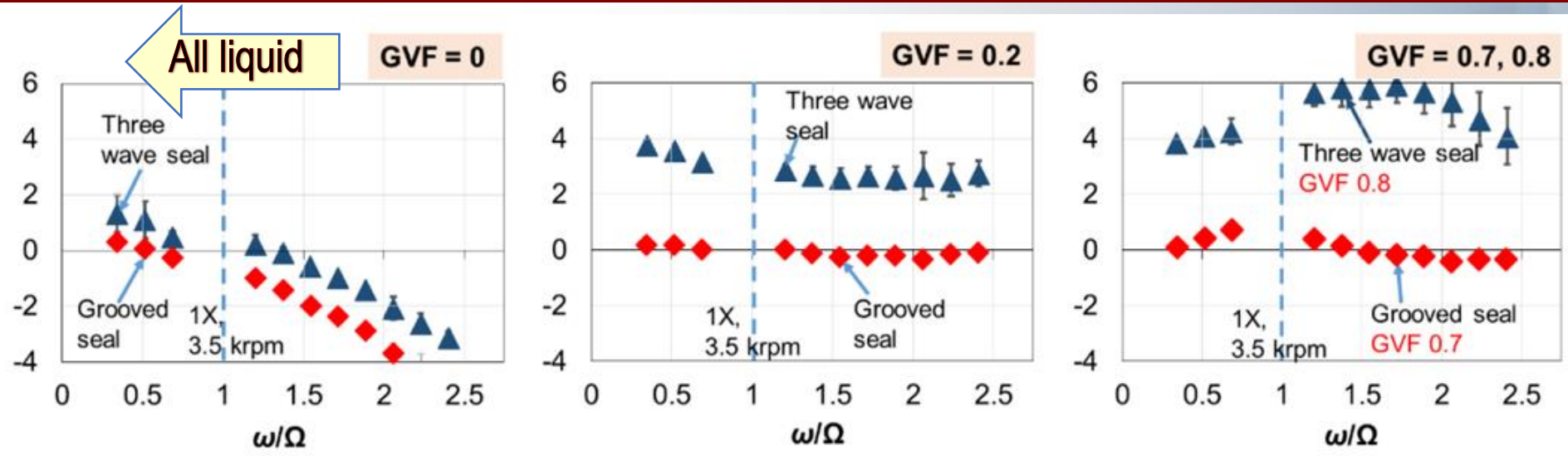


Compare force coefficients: grooved seal vs. wave seal



Direct stiffness

K (MN/m)



GVF = 0 (liquid): Both seals : K reduces with frequency \rightarrow large added mass.

GVF 0.7 & 0.8:

Grooved seal shows small change in $K \sim 0$ for all frequencies.

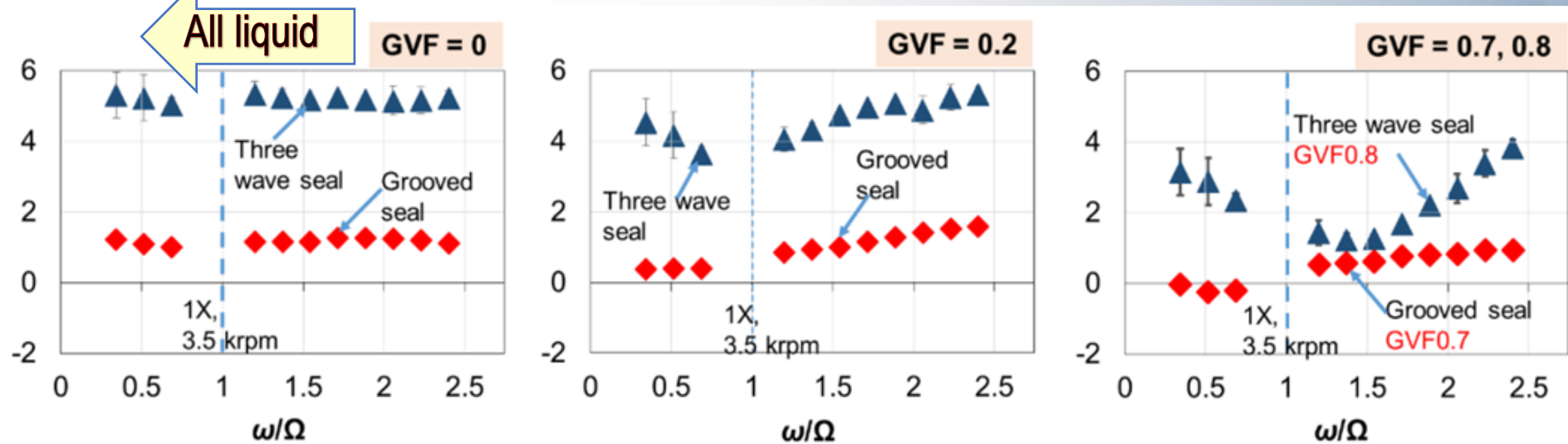
Three-wave seal produces significant stiffening ($K \gg 0$)

$K > 0$ desirable for system static stability and to increase its natural frequency. \rightarrow Most useful in vertical pumps.



Cross-coupled stiffness

k (MN/m)



GVF = 0 (liquid) & 0.2: Wave seal shows larger cross-stiffness → affects stability.

GVF > 0.7:

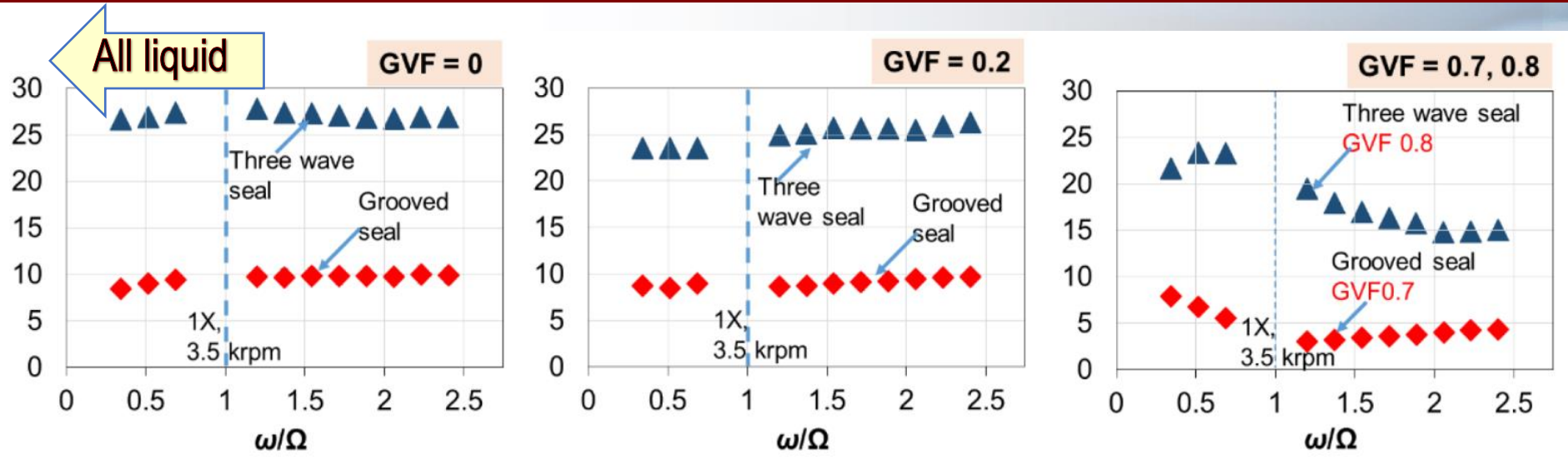
Grooved seal shows small changes in $k \sim 0$ for all frequencies.

Three-wave seal has k lowest for synchronous frequency.

$k \rightarrow 0$ desirable for system dynamic stability.



Direct damping C (kNs/m)



GVF = 0 (liquid) and 0.2:

Wave seal has ~ 2.5 more damping & not affected by frequency.

GVF > 0.7, 0.8

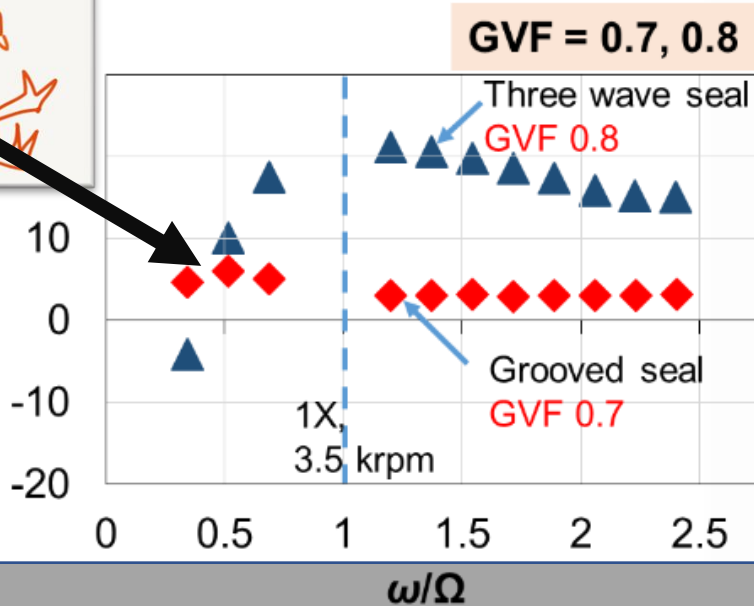
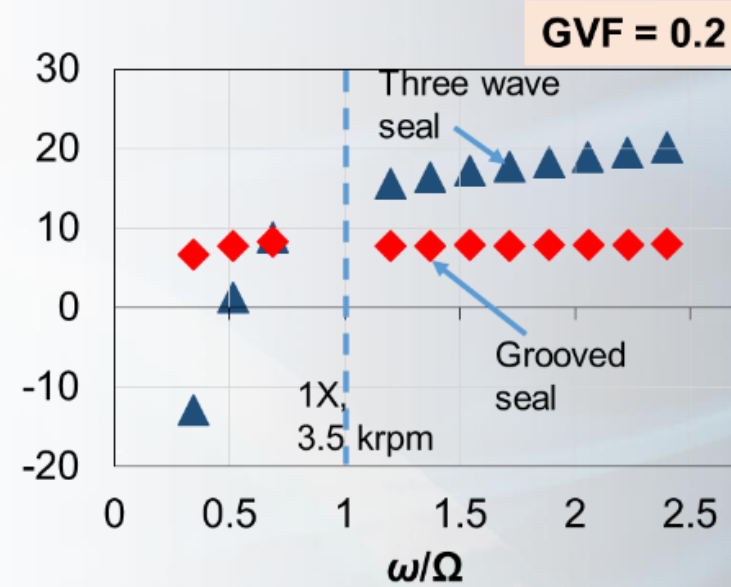
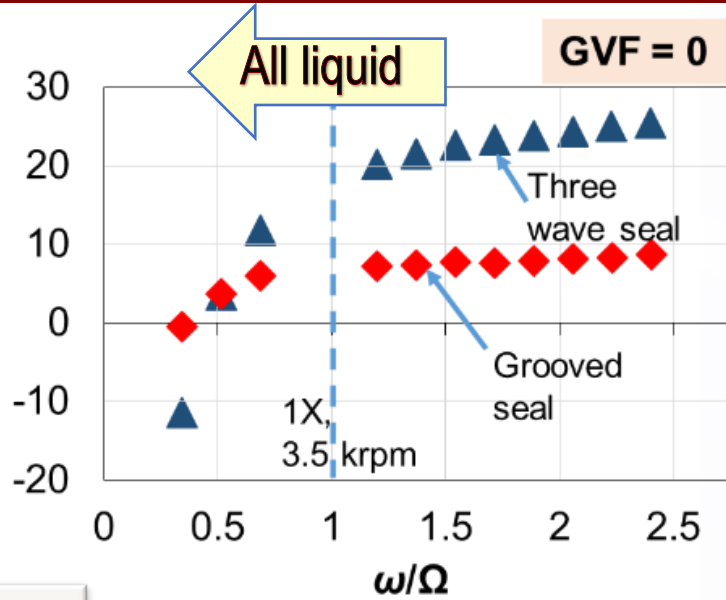
Wave seal still shows large C , albeit decreasing with frequency above shaft speed.

$C > 0$ desirable for system dynamic stability.



Effective damping

$$C_{eff} = (C - k/\omega) \text{ (kN.s/m)}$$

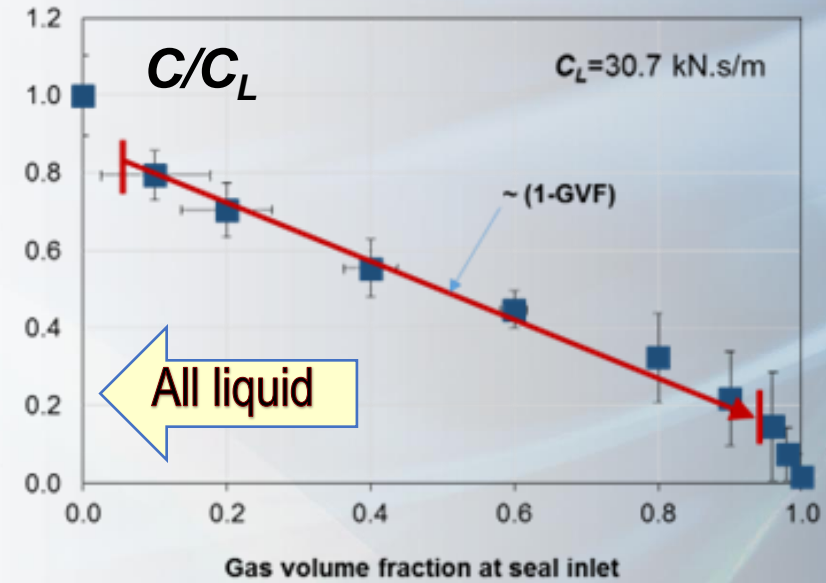
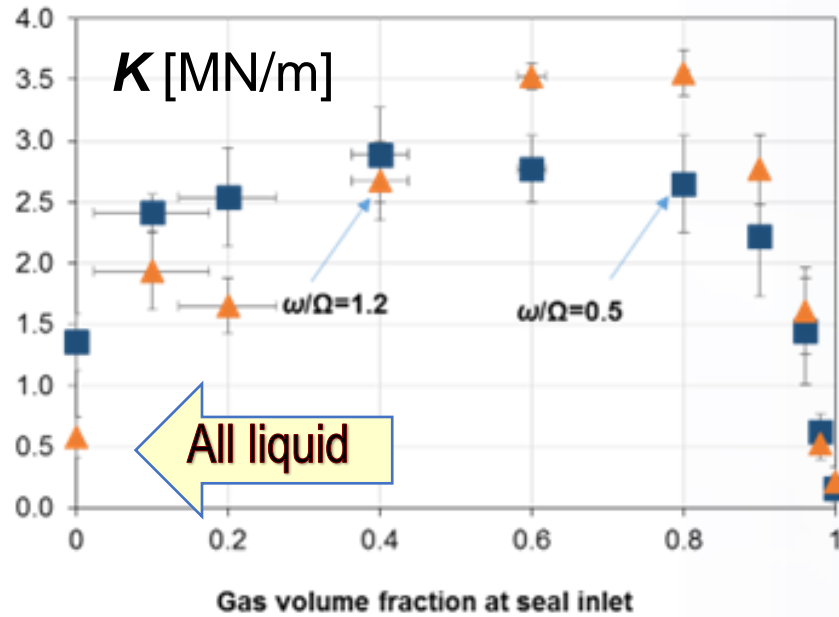


Wave seal shows much larger effective damping for most frequencies above break frequency.

Whirl frequency ratio WFR =
Three wave seal: 0.47
Grooved seal: 0.33



Three-wave seal stiffness and damping



Wave seal quickly stiffens ($K \gg 0$) when supplied with a small amount of gas.

GVF	0	0.10	0.5	0.7
GMF	0	0.05%	0.43%	1.00%

Damping decreases in proportion to $GVF = 1 - LVF$



Conclusion Grooved vs Three Wave Seals

(a) Grooved seal leaks more compared to other seals
← **laminar flow**. NOT a sound choice for a seal!

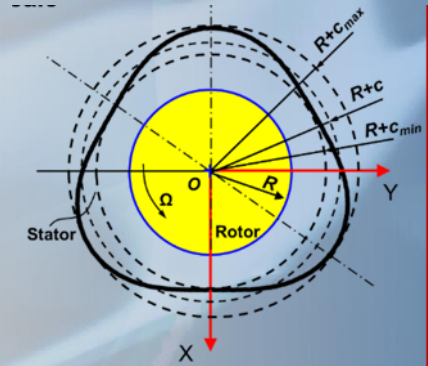
(b) Leakage drops continuously with an increase in gas volume fraction (GVF).

(c) Force coefficients are frequency dependent ← **gas/oil mixture**

(d) For pure liquid (**GVF=0**), seals produce a small centering stiffness (K) and a large added mass (M).

(e) Wave seal produces large direct stiffness $K (>0)$ even for small GVFs (> 0.05).

(f) Wave seal offers 2.5 to 3 x more effective damping than grooved seal!



Unexpected phenomenon

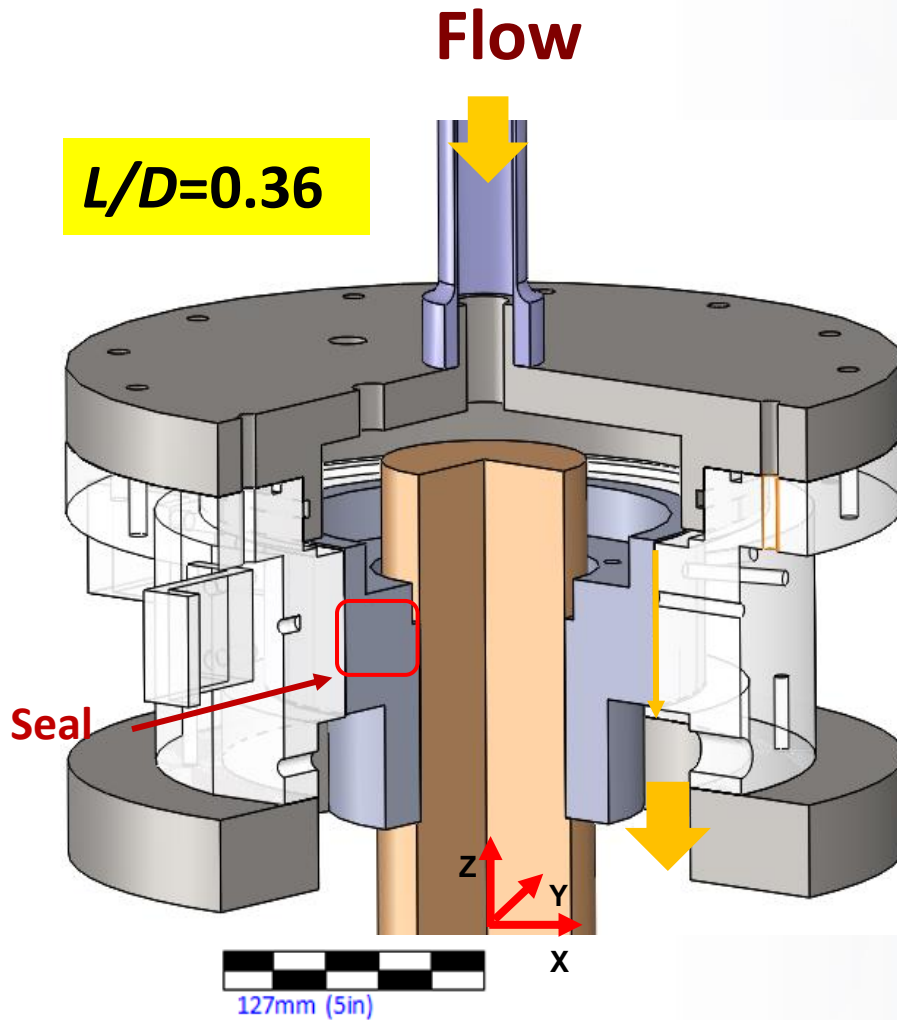
Observed (and quantified) low frequency seal cartridge motions exacerbated by GVF.



ASME GT2017-63254



Seal Geometry and Fluid Properties



Rotor and seal

UNIFORM CLEARANCE Seal

Diameter (D)	127 mm (5 in)
Length (L)	46 mm (1.8 in)
Clearance (c)	0.203 mm (8 mil) at 34 °C
Supply pressure (P_s)	1.0~3.5 bar (abs)
Oil ISO VG 10 density (ρ_l)	830 kg/m³
viscosity (μ_l)	10.6 cP at 34 °C
Air density (ρ_{ga})	1.2 kg/m³ at 1bar
viscosity (μ_{ga})	0.02 cP at 20 °C, 1 bar (abs)
Shaft speed (Ω_{max})	3.5 krpm
Rotor surface speed $\frac{1}{2} D\Omega_{max}$	23 m/s

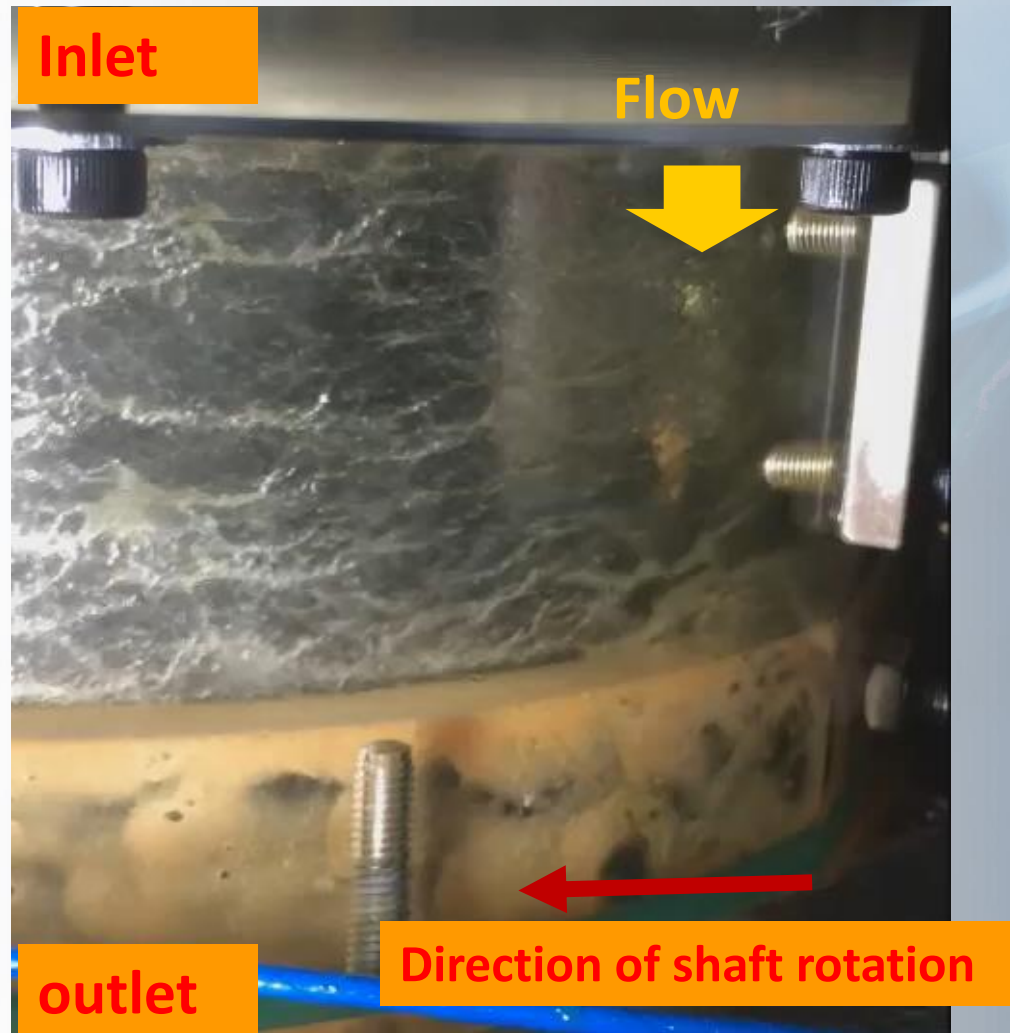


Visualization of Mixture

$P_s/P_a = 2$, speed 1.8 krpm

Stroboscope light
with frequency 30 Hz
freezes shaft motion

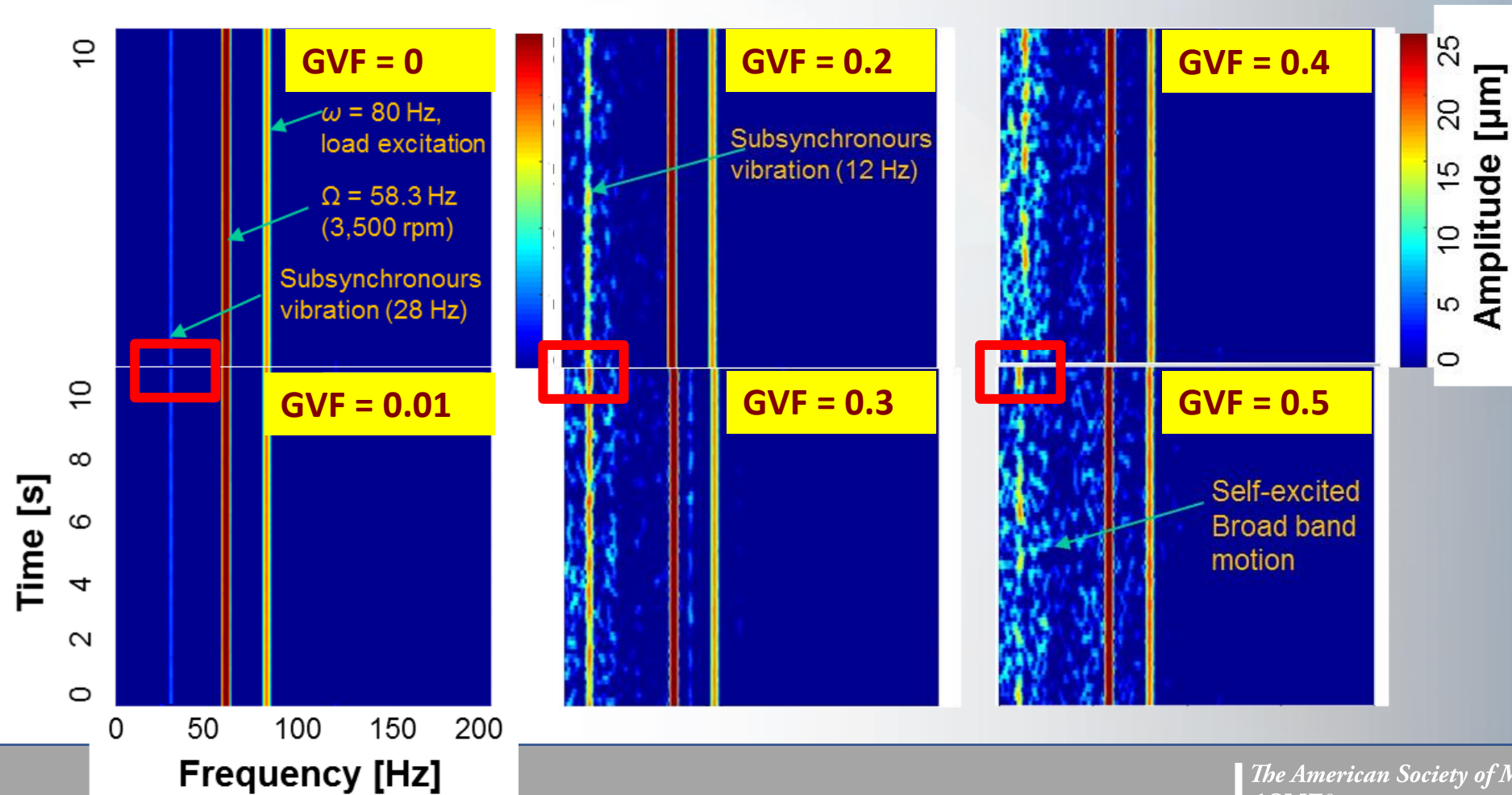
Air bubbles
coalesce and
liquid merges to
make streamlets



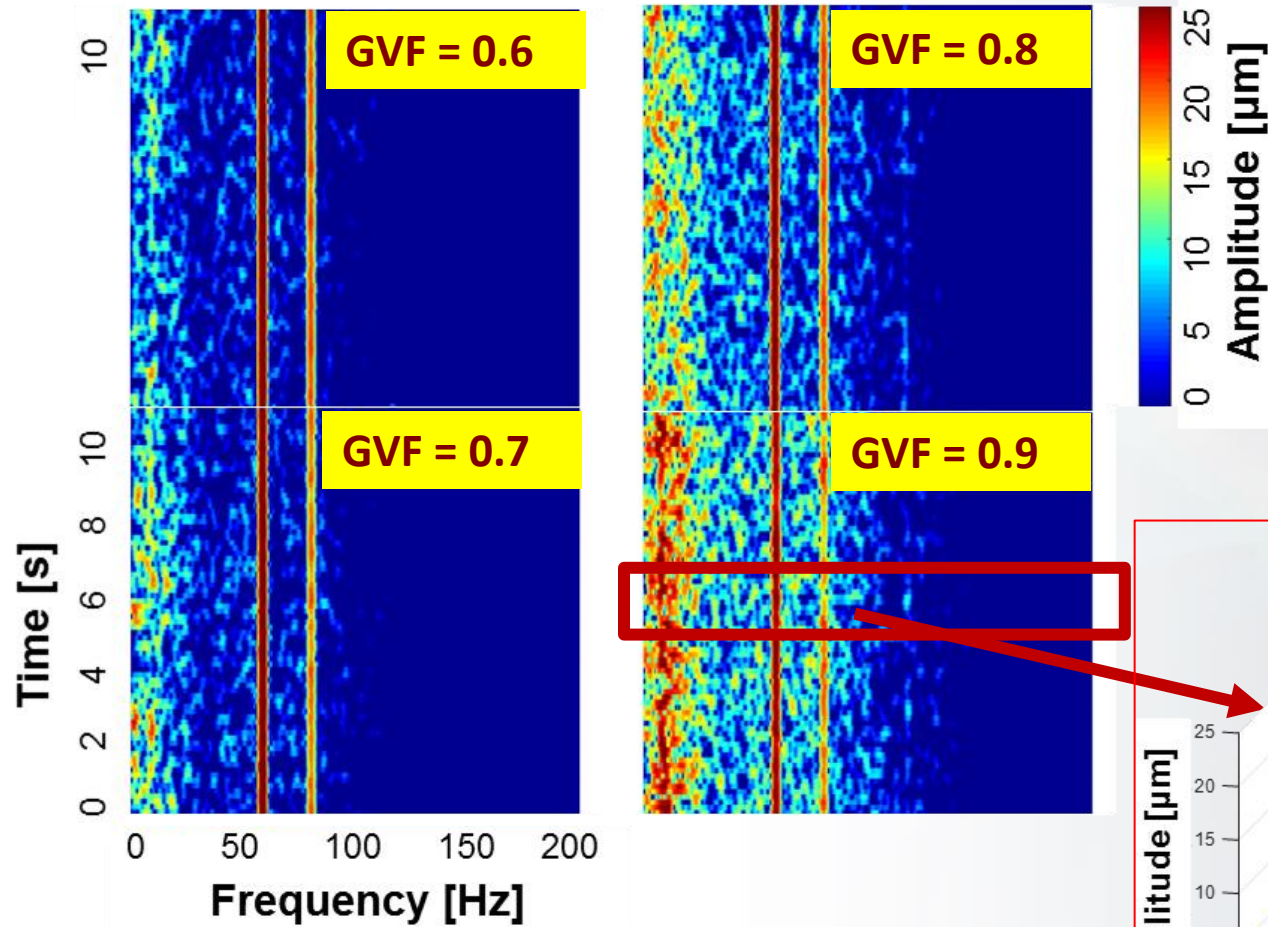
Low Frequency Self-Excited Vibrations

Small amplitude SSV at $\frac{1}{2}X$ for $GVF = 0$ and 0.01

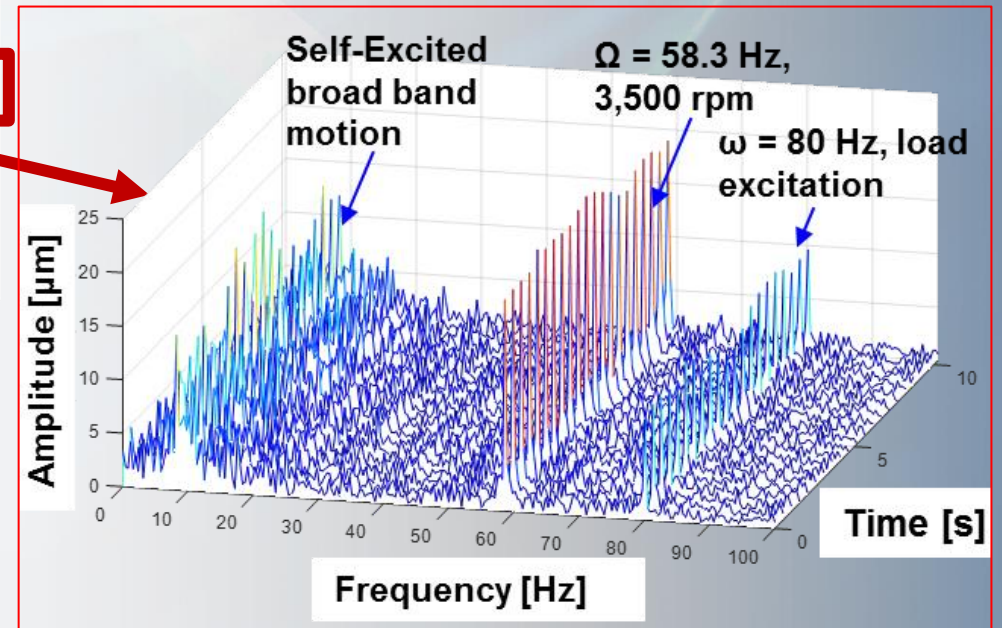
Low frequency at ~ 12 Hz for $GVF > 0.1$. Amplitude \rightarrow increases



Low Frequency Self-Excited Vibrations



As GVF increases
→ Low frequency motions increase in frequency and amplitude.



Low frequency self-excited motions due to acoustic resonance from sound speed in the mixture.

Sound speed in mixture:

$$V_s = \frac{1}{\sqrt{\rho_m \left(\frac{\alpha}{\rho_g V_{sg}^2} + \frac{1-\alpha}{\rho_l V_{sl}^2} \right)}}$$

V_{sg} : sound speed in air, **351 m/s** at 34 °C & 1 bar.

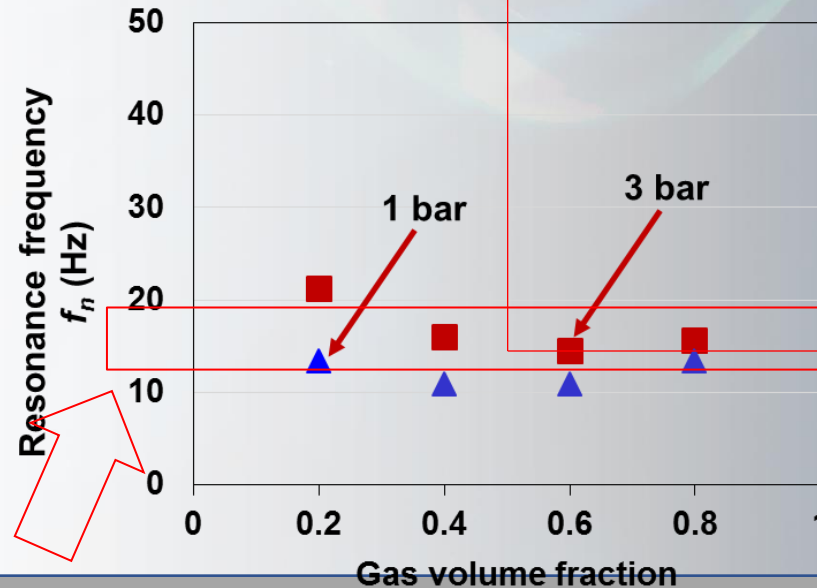
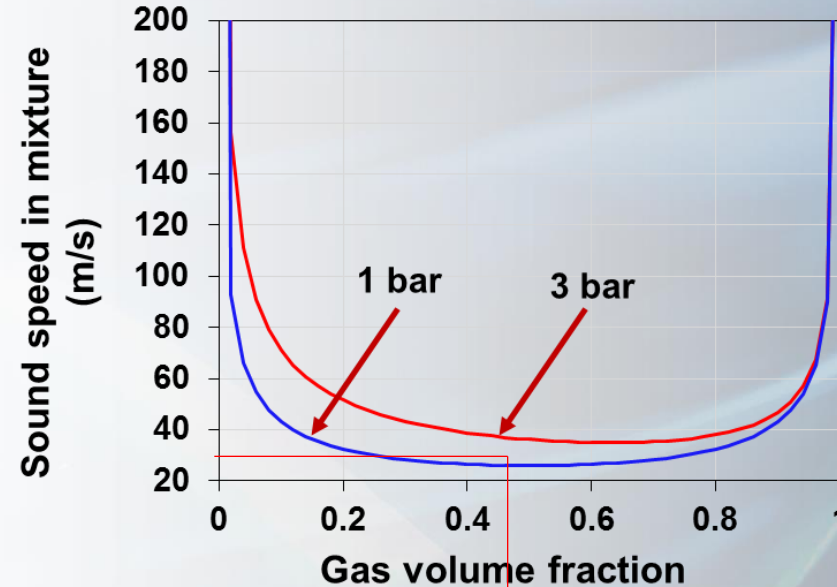
V_{sl} : sound speed in oil, **1,432 m/s**.

Acoustic resonance in a circular duct with one end open (*):

$$f_n = \frac{nV_s}{8\pi \cdot (L + \zeta D)} \text{ (Hz)}$$

$n = 1, 2, 3 \dots$

$n = 1$ for fundamental frequency, $\zeta = 0.4$ correction factor for tube with one end open.



Mixture low sound speed produces resonance → tests show it!

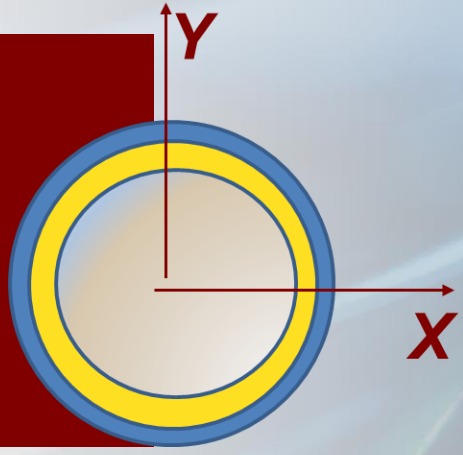
12.3 Hz





Useful for the practicing engineer

Air injection to increase stiffness

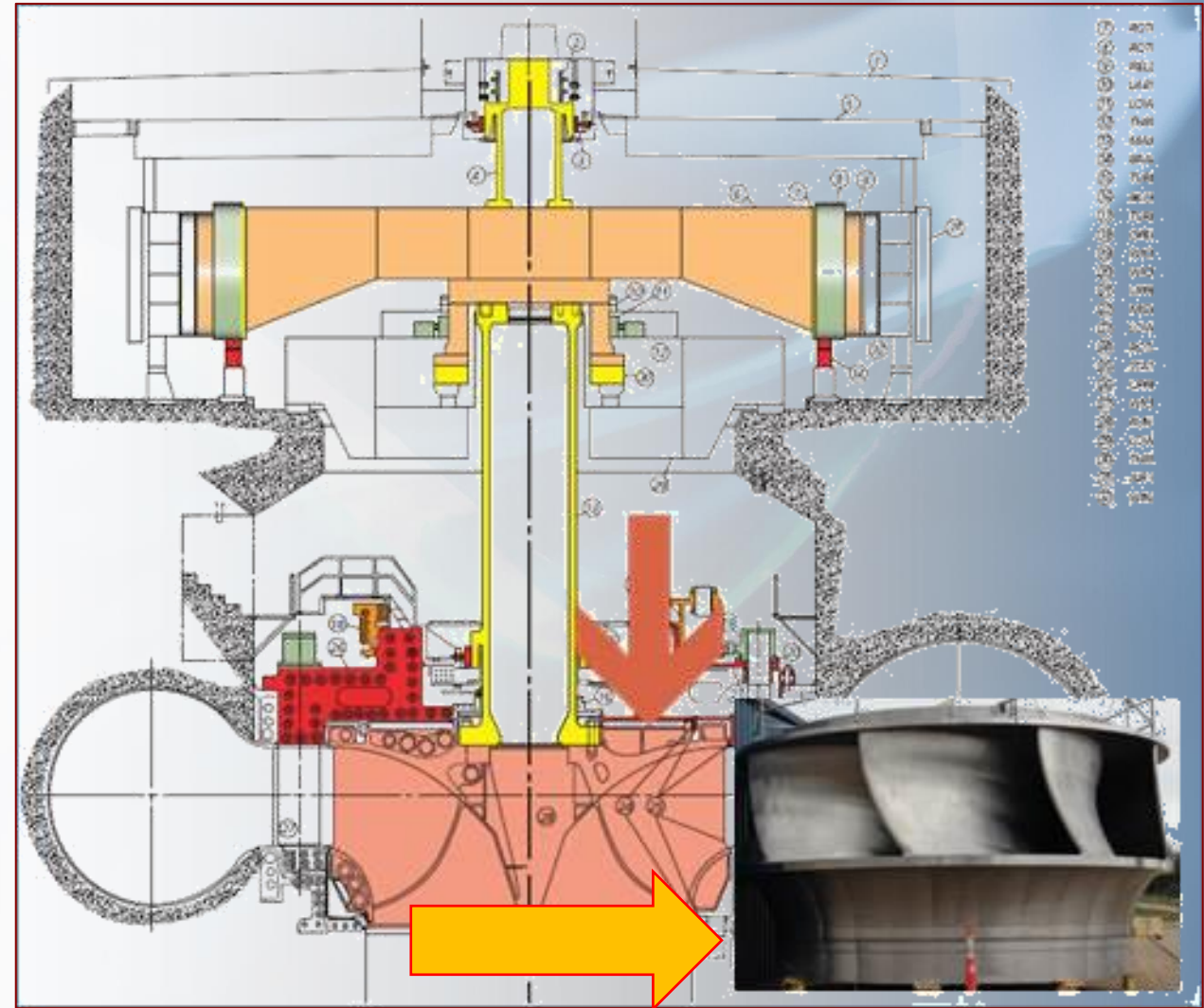


K



Often large vertical turbines/pumps show SSV (→ a resonance)

A common practice is to inject air into the band seal to reduce rotor motion amplitudes.



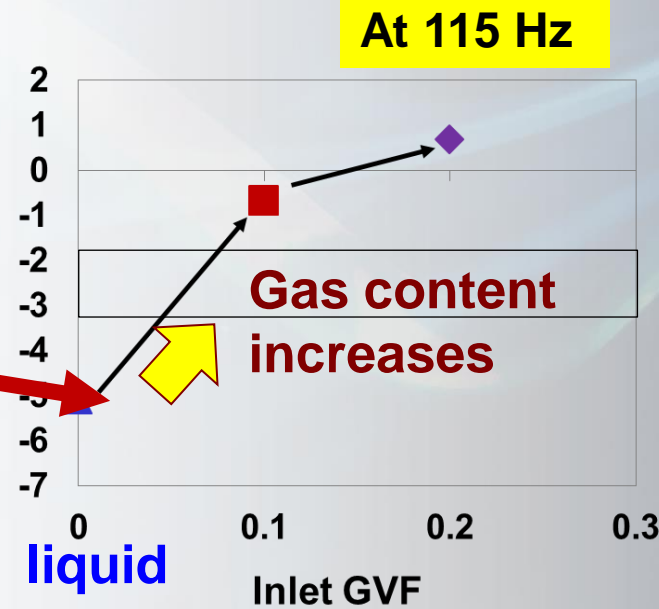
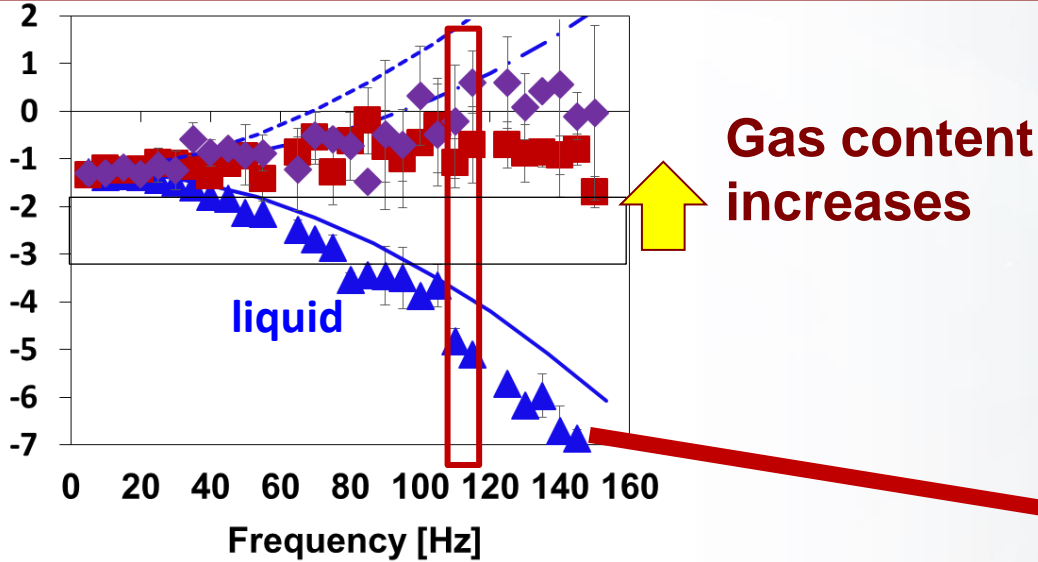
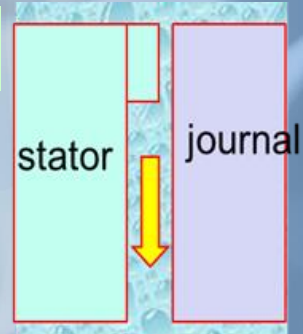
Air injection increases K

Step clearance seal

Air injection turns a negative stiffness into a positive one ($K > 0$ is centering).

0 rpm

$Q_i = 11.4$ L/min,
 $P_s = 2.9$ bara



- ▲ GVF=0
- GVF=0.1
- ◆ GVF=0.2

$K > 0$ brings static stability to vertical turbines.

Seal stiffness hardens due to quick drop in sound speed brought in by the small amount of gas and exacerbated by excitation frequency.

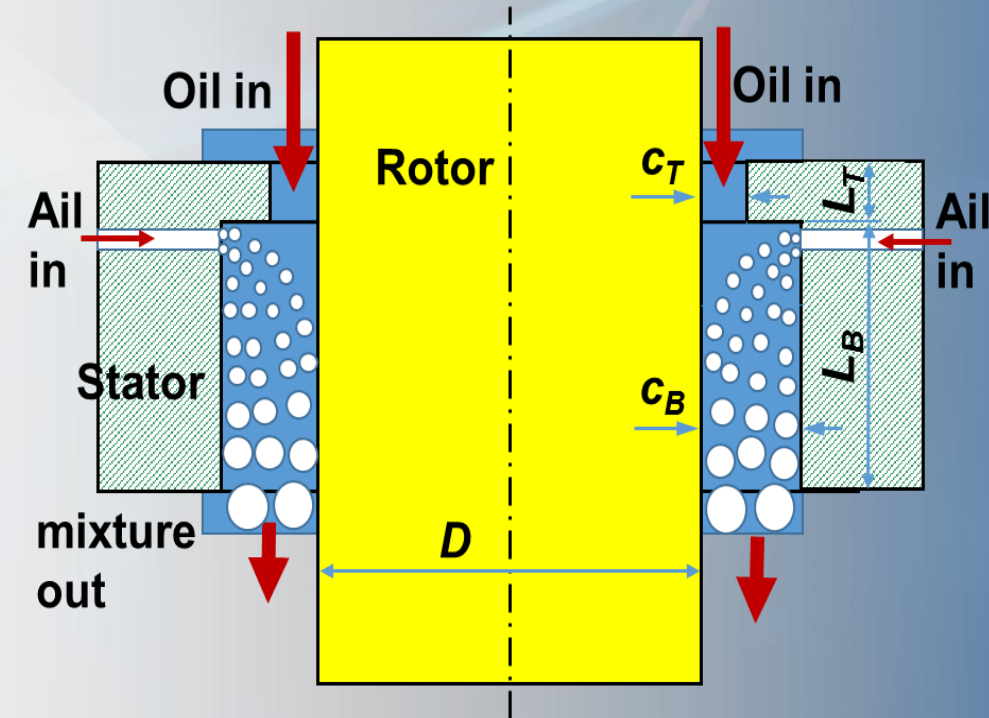




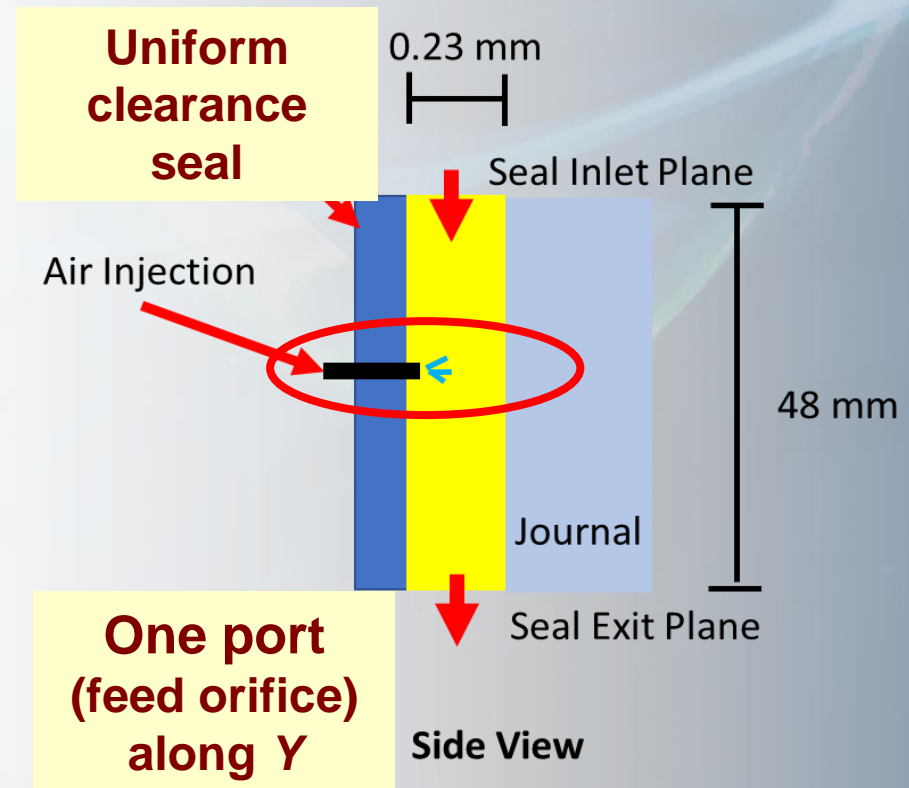
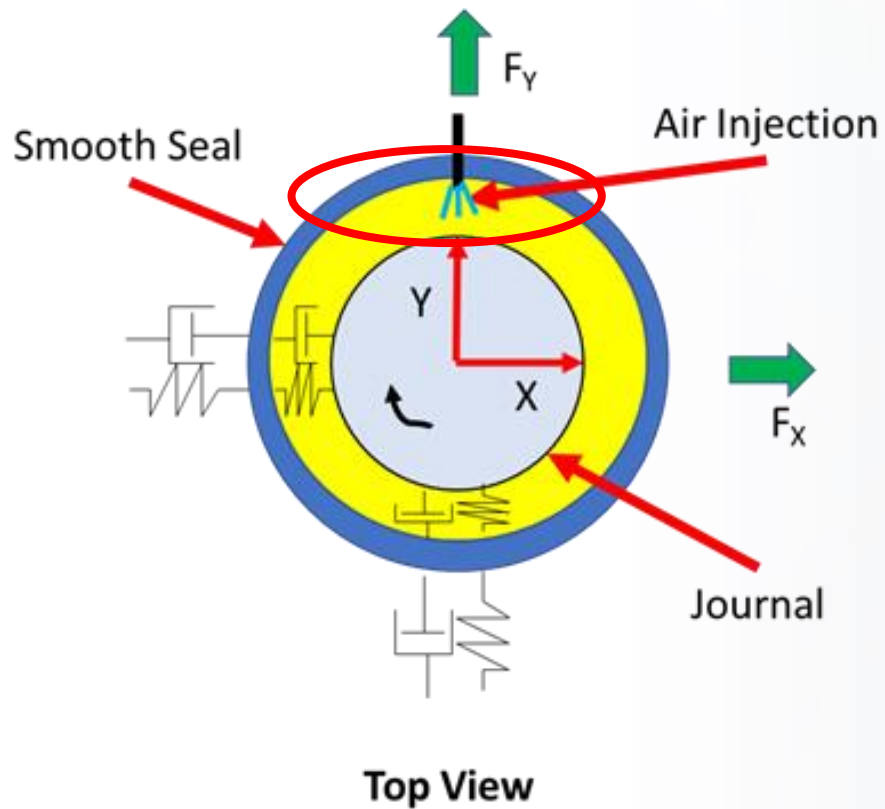
Idea!

LOCAL AIR INJECTION TO PROMOTE THE STATIC STABILITY OF VERTICAL PUMPS: TESTS & ANALYSIS

1. Design and manufacture a uniform clearance seal.
2. Measure leakage at various conditions of GVF, supply pressure, and shaft speed.
3. Conduct dynamic load tests and identify force coefficients vs. frequency and GVF.

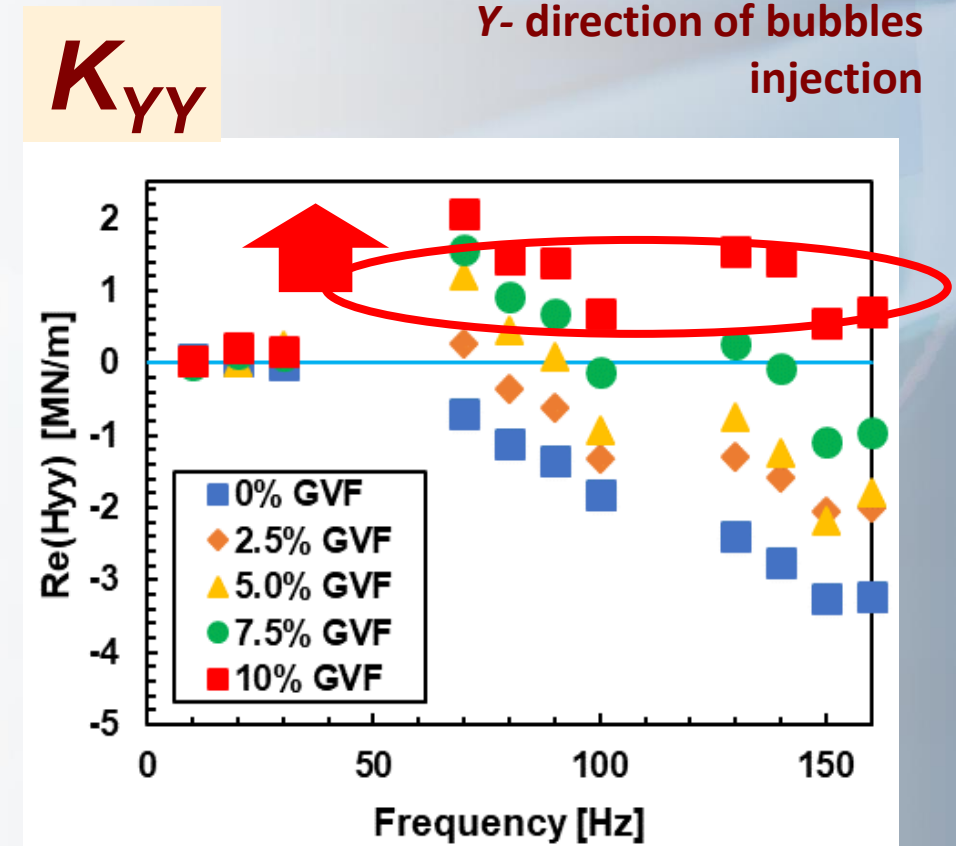
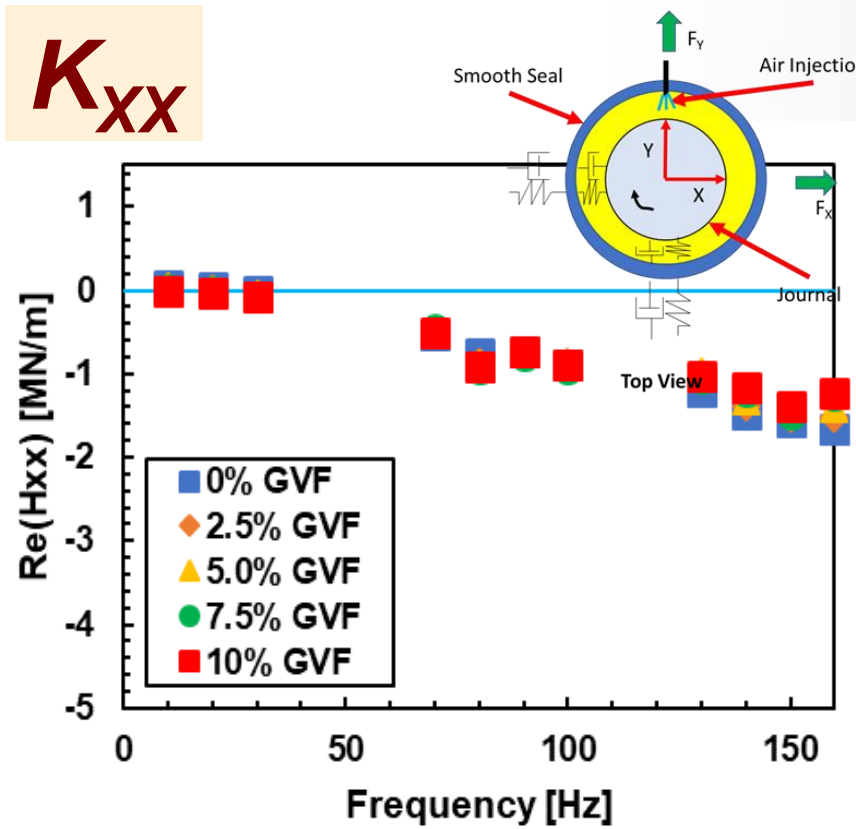


Local injection of bubbles to increase seal stiffness



Injection of bubbles increases K

0 rpm,
 $P_s/P_a=2.5$



$K_{xx} < 0$, $K_{yy} > 0$ as GVF increases

Stiffness asymmetry promotes rotor stability!



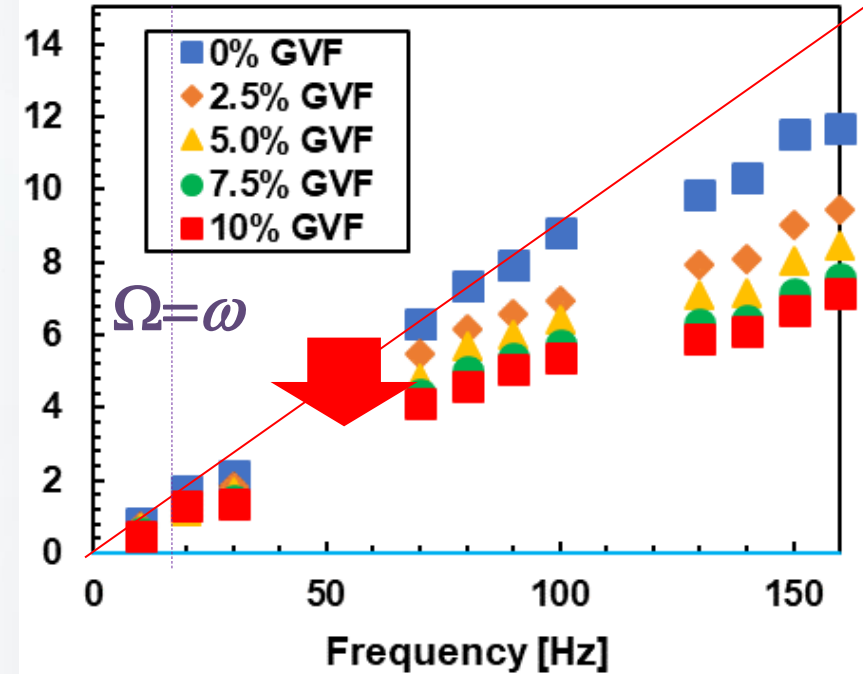
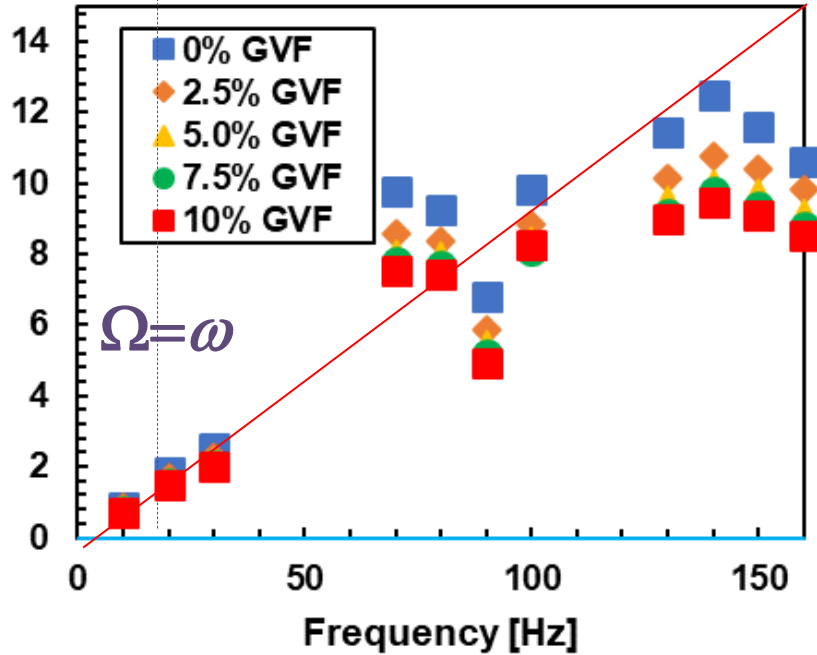
Injection of bubbles reduces damping

1 krpm,
Ps/Pa=2.5

$$\sim \omega C_{XX}$$

$$\sim \omega C_{YY}$$

Y- direction of bubbles injection



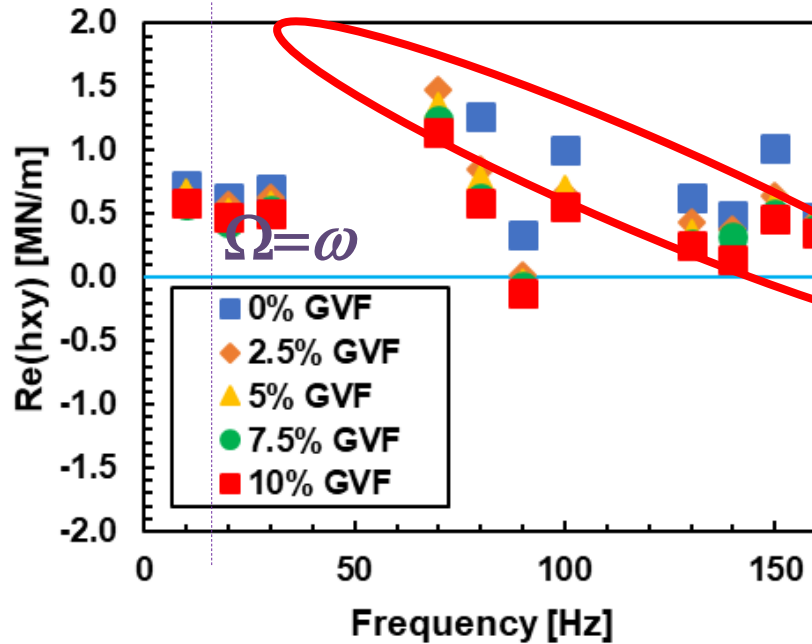
C_{XX} & $C_{YY} > 0$ decrease as GVF increases.



Injection of bubbles reduces k_{xy}

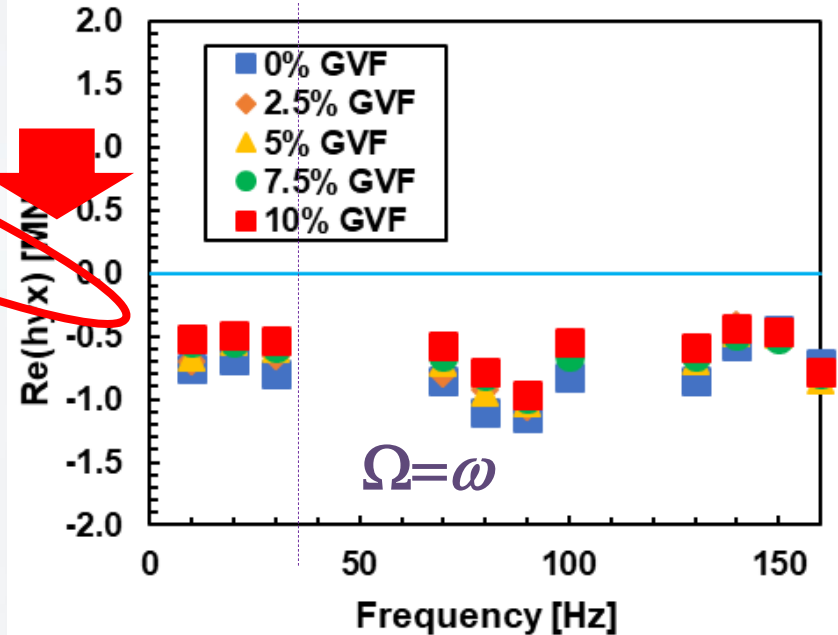
1 krpm
Ps/Pa=2.5

k_{xy}



k_{yx}

Y- direction of bubbles injection



$k_{xy} \sim -k_{yx}$ for $\omega < 2\Omega$. $|k_{xy} - k_{yx}|$ decreases as both GVF & frequency increase \rightarrow + stable.

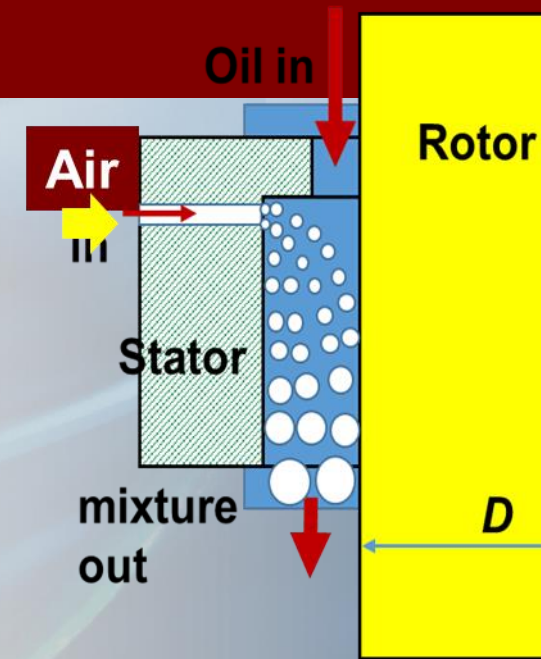
$$C_{eff,xx} = (C_{xx} - k_{xy}/\omega) \sim C_{xx}$$

$$C_{eff,yx} = (C_{yx} + k_{xy}/\omega)$$



Closure

A SIMPLE WAY TO CONTROL SEAL STIFFNESS IN VERTICAL PUMP CONFIGURATIONS



Local gas injection produces seal stiffness hardening & orthotropy → increases static stability (+ good for vertical systems). Even a small amount of gas injection helps!



The road ahead

To seal or not to seal....

As O&G demands increase, industry will continue to advance more efficient, reliable and cost-effective sealing solutions.





**Turbomachinery
Laboratory**

Texas A&M Engineering Experiment Station

Questions?



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Learn more:

<http://rotorlab.tamu.edu>

53th Turbomachinery & Pump Symposium, Aug 20-22, 2024, Houston
(<https://turbolab.tamu.edu/proceedings>)

“Wet (Bubbly Liquid) Seals for Multiphase Pumps: Leakage and Dynamic Force Coefficients of Two seals and a Simple Way to Control Seal Stiffness in Vertical Pump Configurations,” <https://hdl.handle.net/1969.1/1582500>

Questions to lsanandres@tamu.edu

and/or get pdf of this presentation





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