Given the real and imaginary parts of a mechanical complex stiffness, H=F/X, find the system parameters (K,C,M)

TEST DATA

$f =$

	1
1	60
2	120
3	180
4	240
5	300
6	360
7	420
8	480
9	540
10	600
11	660
12	720
13	780
14	840
15	900
16	...

$\frac{1}{s}$

$H_R =$

	1
1	$2.964\cdot10^{7}$
2	$2.858\cdot10^{7}$
3	$2.685\cdot10^{7}$
4	$2.45\cdot10^{7}$
5	$2.156\cdot10^{7}$
6	$1.807\cdot10^{7}$
7	$1.406\cdot10^{7}$
8	$9.573\cdot10^{6}$
9	$4.642\cdot10^{6}$
10	$-6.944\cdot10^{5}$
11	$-6.4\cdot10^{6}$
12	$-1.244\cdot10^{7}$
13	$-1.877\cdot10^{7}$
14	$-2.537\cdot10^{7}$
15	$-3.218\cdot10^{7}$
16	...

$\frac{lbf}{in}$

$H_I =$

	1
1	$3.519\cdot10^{6}$
2	$3.506\cdot10^{6}$
3	$3.945\cdot10^{6}$
4	$4.497\cdot10^{6}$
5	$5.094\cdot10^{6}$
6	$5.713\cdot10^{6}$
7	$6.346\cdot10^{6}$
8	$6.986\cdot10^{6}$
9	$7.632\cdot10^{6}$
10	$8.282\cdot10^{6}$
11	$8.934\cdot10^{6}$
12	$9.588\cdot10^{6}$
13	$1.024\cdot10^{7}$
14	$1.09\cdot10^{7}$
15	$1.156\cdot10^{7}$
16	$1.222\cdot10^{7}$
17	$1.288\cdot10^{7}$
18	$1.354\cdot10^{7}$
19	$1.42\cdot10^{7}$
20	...

$\cdot\frac{lbf}{in}$

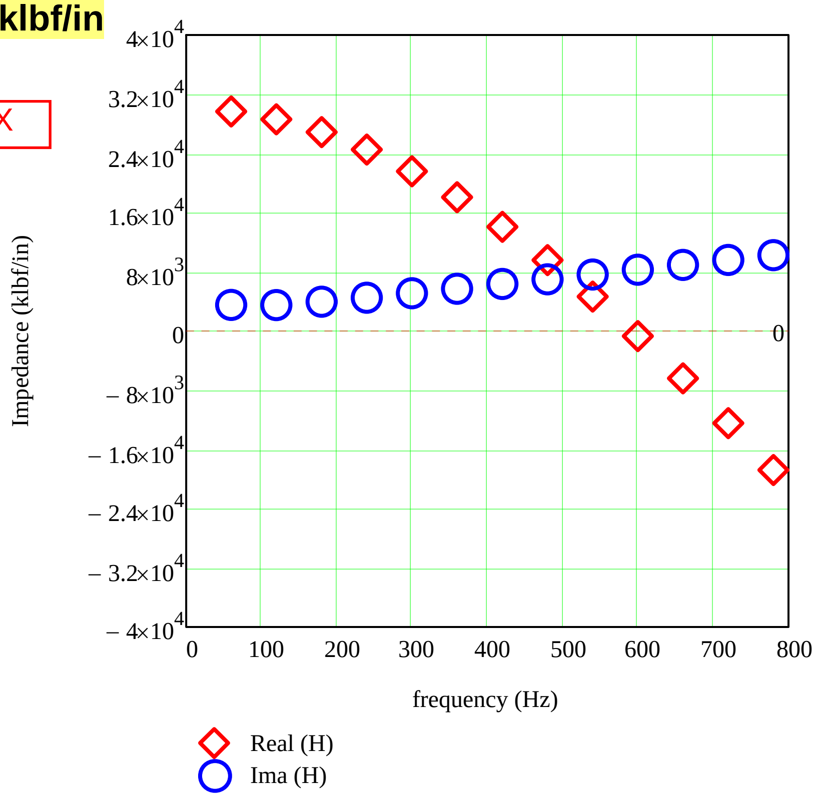
Identification of parameters in a SDOF system - ME617 SP2018

Given the real and imaginary parts of a mechanical complex stiffness, F/X, find the system parameters (K,C,M)

Luis San Andres (c)

klbf/in

H=F/X



Impedance (real and imaginary) TEST

Identification procedure

select whole frequency range

$C_I := \mathrm{slope}(\omega, H_I)$ → $C_I = 1.674 \times 10^3 \cdot \frac{\mathrm{lbf}\cdot\mathrm{s}}{\mathrm{in}}$

identified damping, stiffness and mass

$M_I := -\mathrm{slope}(\omega2, H_R)$

$K_I := \mathrm{intercept}(\omega2, H_R)$

→ $M_I = 663.401 \cdot \mathrm{lb}$

$K_I = 2.601 \times 10^7 \cdot \frac{\mathrm{lbf}}{\mathrm{in}}$

natural frequency and damping ratio

$$f_{n\_} := \left(\frac{K_I}{M_I}\right)^{.5} \cdot \frac{1}{2 \cdot \pi}$$

$f_{n\_} = 619.174 \cdot Hz$

$$\zeta := \frac{C_I}{2 \cdot \left(K_I \cdot M_I\right)^{.5}}$$

$\zeta = 0.125$

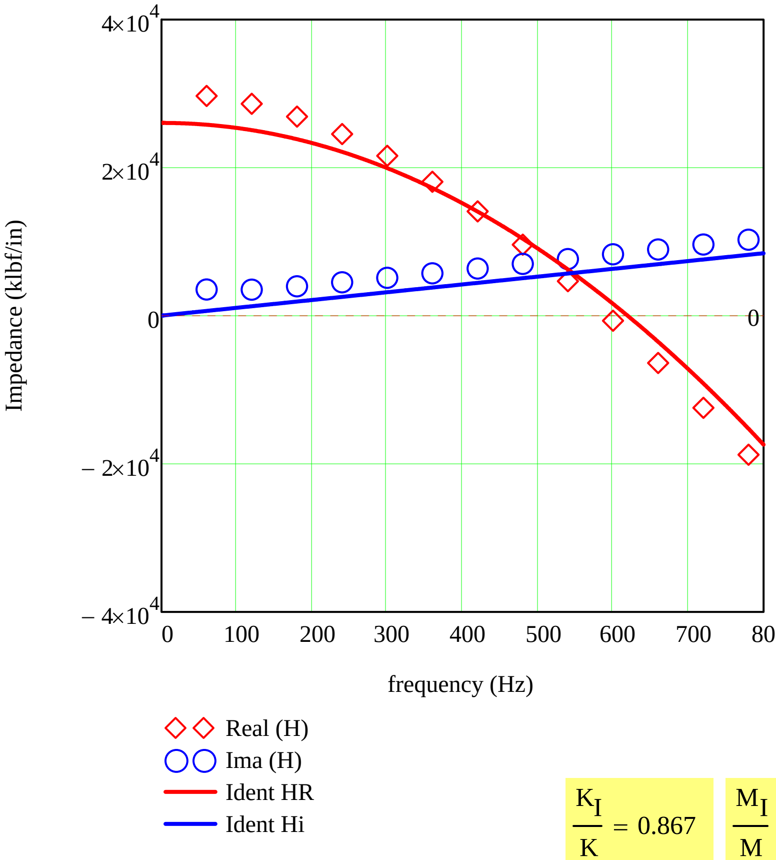
build complex stiffness with parameters found

$H_{I\_R}(\omega) := K_I - M_I \cdot \omega^2$

klbf/in

TEST DATA and CURVE FITS

$H_{I\_i}(\omega) := C_I \cdot \omega$



$C_I = 1.674 \times 10^3 \cdot \frac{lbf \cdot s}{in}$

$M_I = 663.401 \cdot lb$

$K_I = 2.601 \times 10^7 \cdot \frac{lbf}{in}$

correlation coefficients

$\mathrm{corr}\left(\omega, H_I\right)^2 = 0.997$

$\mathrm{corr}\left(\omega 2, H_R\right)^2 = 0.993$

Is K correct = K(ω=0)

$\frac{K_I}{K} = 0.867$ $\frac{M_I}{M} = 0.663$ $\frac{C_I}{C} = 0.949$

$$\mathrm{Fles}(\omega) := \frac{1}{\left[\left(K_I - M_I \cdot \omega^2\right)^2 + \left(C_I \cdot \omega\right)^2\right]^{.5}}$$

buld flexibility function (for later)

===

$C = 1.763 \times 10^{3} \cdot \text{lbf} \cdot \frac{s}{in}$ $K = 3 \times 10^{7} \cdot \frac{\text{lbf}}{in}$ $M = 1 \times 10^{3} \cdot \text{lb}$

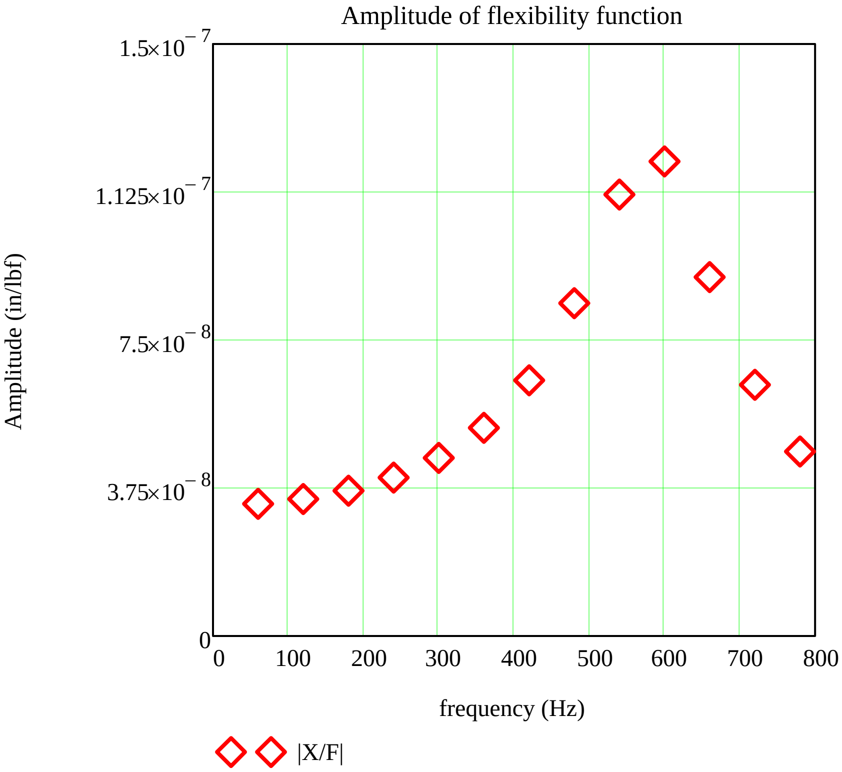
In spite of good correlation, the identified parameters are NOT near the actual ones.
WHY?

Amplitude of transfer function

Flexibility or compliance = $\left|\frac{X}{F}\right|$

in/lbf TEST DATA

.
|x/F|



USE MATHCAD function for generic fit: function of parameters and its derivatives

$K_{guess} := \frac{175}{5}\cdot 10^{6}$ N/m $M_{guess} := 100$ $C_{guess} := 0.1\cdot\left(K_{guess}\cdot M_{guess}\right)^{.5}$

3 parameters $u_1 = K$ $u_2 = M$ $u_3 = C$

$$u_{guess} := \begin{pmatrix} K_{guess} \\ M_{guess} \\ C_{guess} \end{pmatrix}$$

G is a function that returns an n + 1 element vector containing the function f and its partial derivatives with respect to its n parameters

$$G(\omega,u) := \begin{bmatrix} \dfrac{1}{\left[\left(u_1 - u_2\cdot\omega^2\right)^2 + \left(u_3\cdot\omega\right)^2\right]^{.5}} \\ \dfrac{-2\cdot\left(u_1 - u_2\cdot\omega^2\right)}{\left[\left(u_1 - u_2\cdot\omega^2\right)^2 + \left(u_3\cdot\omega\right)^2\right]^{1.5}} \\ \dfrac{-2\cdot\left(-\omega^2\right)\cdot\left(u_1 - u_2\cdot\omega^2\right)}{\left[\left(u_1 - u_2\cdot\omega^2\right)^2 + \left(u_3\cdot\omega\right)^2\right]^{1.5}} \\ \dfrac{-2\cdot\left(u_3\cdot\omega^2\right)}{\left[\left(u_1 - u_2\cdot\omega^2\right)^2 + \left(u_3\cdot\omega\right)^2\right]^{1.5}} \end{bmatrix}$$

G function=flexibility

$\frac{dG}{du_1}$

$\frac{dG}{du_2}$

$\frac{dG}{du_3}$

$$\begin{pmatrix} K_{II} \\ M_{II} \\ C_{II} \end{pmatrix} := genfit\left(w, FLEX, u_{guess}, G\right)$$

BUILD flexibility function

$K_{II} := K_{II}\cdot\frac{N}{m}$ $C_{II} := C_{II}\cdot N\cdot\frac{s}{m}$ $M_{II} := M_{II}\cdot kg$

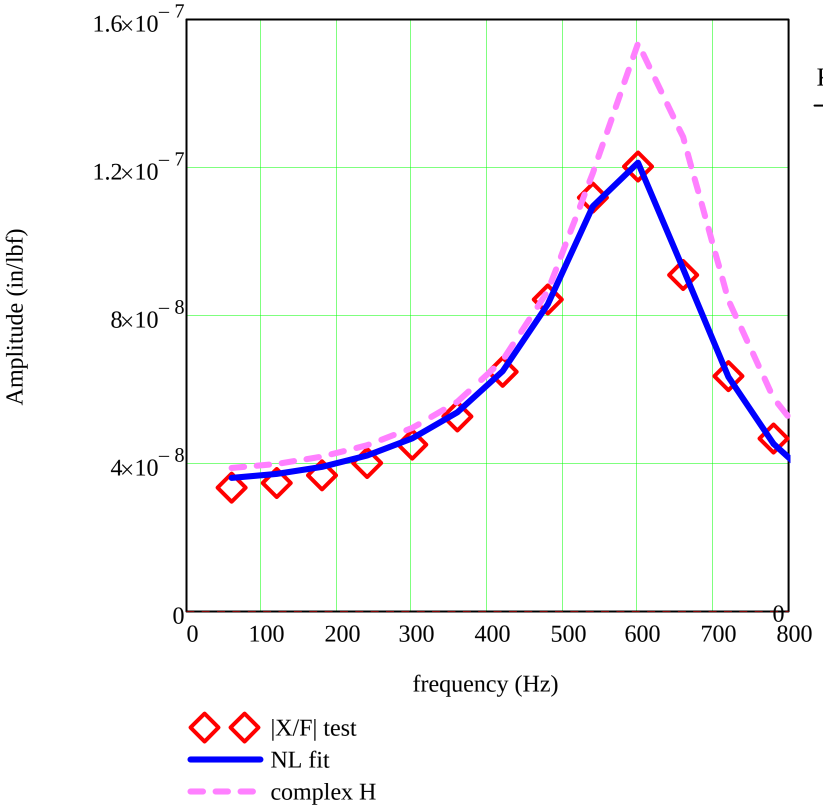
$$Flex(\omega) := \frac{1}{\left[\left(K_{II} - M_{II}\cdot\omega^2\right)^2 + \left(C_{II}\cdot\omega\right)^2\right]^{.5}}$$

$f_{nII} := \left(\frac{K_{II}}{M_{II}}\right)^{.5}\cdot\frac{1}{2\cdot\pi} = 600.126\ Hz$ $\zeta_{II} := \frac{C_{II}}{2\cdot\left(K_{II}\cdot M_{II}\right)^{.5}}$

Identified coefficients are:

$K_{II} = 2.795 \times 10^{7} \cdot \frac{lbf}{in}$ $M_{II} = 758.942 \cdot lb$ $C_{II} = 2.185 \times 10^{3} \cdot \frac{lbf \cdot s}{in}$

in/lbf TEST DATA and NL CURVE FIT



$\frac{K_{II}}{K} = 0.932$

natural frequency & damping ratio

$f_{nII} = 600.126 \cdot Hz$

$\zeta_{II} = 0.147$

Impedance (real and imaginary) TEST

Identified parameters from two methods

$\frac{K_{II}}{K} = 0.932$ $\frac{M_{II}}{M} = 0.759$ $\frac{C_{II}}{C} = 1.239$ NL curve fit

$\frac{K_{I}}{K} = 0.867$ $\frac{M_{I}}{M} = 0.663$ $\frac{C_{I}}{C} = 0.949$ curvet fit to H

$K := 64 \cdot 10^{6} \cdot \frac{lbf}{in}$ $M := 2000 \cdot lb$ $\zeta := 0.10$