ME459/659 S&V Measurements
Group Homework 2 due February 19 2019

Application of Experimental Modal Analysis

Estimation of (natural) mode shapes for a vibrating structure and comparison to predictions

The assignments has several parts, each *item* requires of you to read the attached documentation to learn the background and fundaments of various procedures conducted to estimate physical parameters, to measure system natural frequencies and to conduct measurements of system response due to (say) an impact, to process the system response to produce amplitudes and phase of motion at distinctive natural frequencies to build (natural) mode shapes, and also to produce predictions of system modal response for verification of the analytical tool via comparisons to test data.

The documents you must read are labeled as README in the attached zip file. Note that prior ME UG students prepared the documents and purposely use an *easy to read and learn* format.

1. The physical system for analysis.

Figure 1 depicts a cross-sectional view of a solid rotor made of common steel. Most of the rotor has a uniform outer diameter D_R=10.06 cm*;* and its right end has a fitted steel *cap* for installation of imbalance holes (used during rotordynamic tests). The overall mass and length of the rotor equal m_R=29.12 kg and L_R=47.63 cm (uncertainty 10 gram and 0.1 mm, respectively).

A pair of gas bearings supports the rotor. Both bearings are equidistant of the rotor center of mass. The simple rotor is mainly used to verify the performance of the gas bearings (hereby not of further interest).

Complex rotors are made of many components which include, for example, hubs for coupling connection, thrust collars, bearing sleeves, and shrunk impellers with many thin blades. Finding the mass moment of inertia of complex rotors is not a simple analytical task. One must devise a method to obtain the polar mass moment of inertia (I_P) and the transverse mass moment of inertia (I_T) that determine the moment reactions for rotor turning about its spinning axis and a pitching axis, respectively. Note that turning a rotor at its rated speed (say Ω) requires at minimum for a drive moment proportional to the angular acceleration.

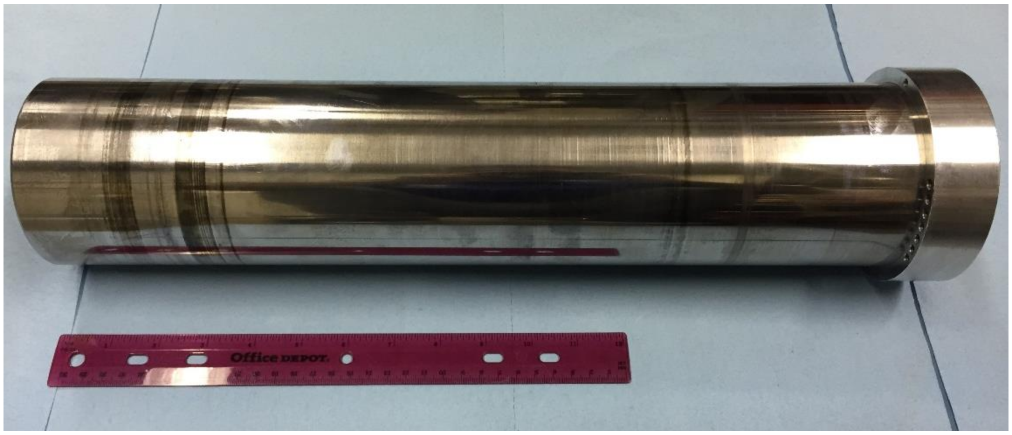

Fig. 1 Photograph of test rotor.
Rotor mass m_R=29.12 kg, overall length L_R=47.63 cm and diameter D_R=10.06 cm (left section). Left: drive end; Right: Free end with cap for imbalances insertion.

2. How to measure mass moments of inertia

The mass moment of inertia properties of a rigid body (complex in shape) are typically obtained by suspending the body from cables and forcing its rotation about a particular axis. To this end, (inextensible) wires or cables of length *l* hold the rotor at a distance *b* from its cg. These wires provide little restraint along the direction of rotor angular dislacement. Figure 2 depicts typical dispositions for measurement of the transverse (I_T) and polar (I_P) mass moments of inertia.

You may recall (ME363) this ad-hoc set up is called a bifilar pendulum (Read *how to inertias*). Recording with a stop watch the period of rotor motion (T) aids to determine the mass moment of inertia from:

$$I = m_R\, g \left(\frac{T\, b}{2\pi}\right)^2 \frac{1}{l} \tag{1}$$

Above m_R is the rotor mass.

Item 1. Demonstrate from basic principles [Newton's EOM] that Eq. (1) is correct. In addition, assume the rotor is a uniform cylinder and state (from known literature) the formulas for calculation of I_P and I_T as a function of its mass, diameter and length.

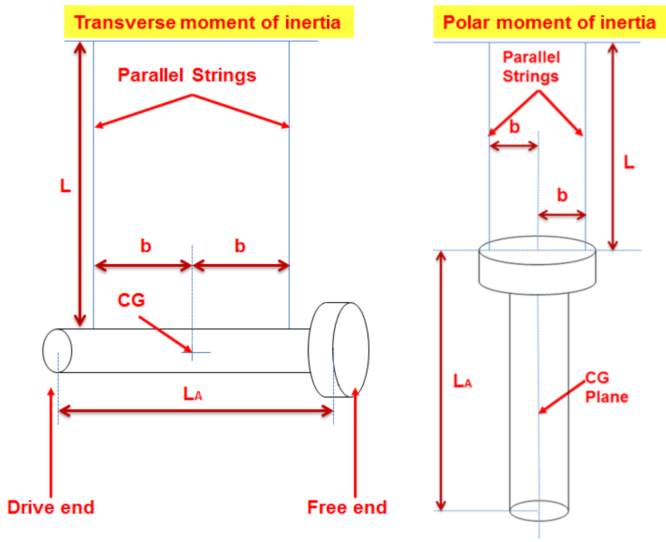

Fig. 2 Schematic views of ad-hoc setups for measurement of a body mass moment of inertia (transverse and polar).

Figure 3 depicts photographs of the rotor suspended from wires to record the natural period of motion (*T*) by turning slightly the rotor and clocking a number of oscillations. The procedure repeats several times to produce an accurate average of the natural period of motion. Table 1 lists distinctive wire length *l* and distance *b* and the recorded period *T*. In the Table, *T* and *P* stand for transverse and polar designations. The uncertainty in the estimation of the clocked period of motion is ¼ s.

Table 1 Dimensions and recorded periods for identification of rotor mass moments of inertia

		T	*P*	
b	Distance from strings to rotor center of gravity	8.89	5.40	cm
l	Length of wire	114.30	69.85	cm
T	Period of oscillation (average of 20 periods),	3.34	1.09	s

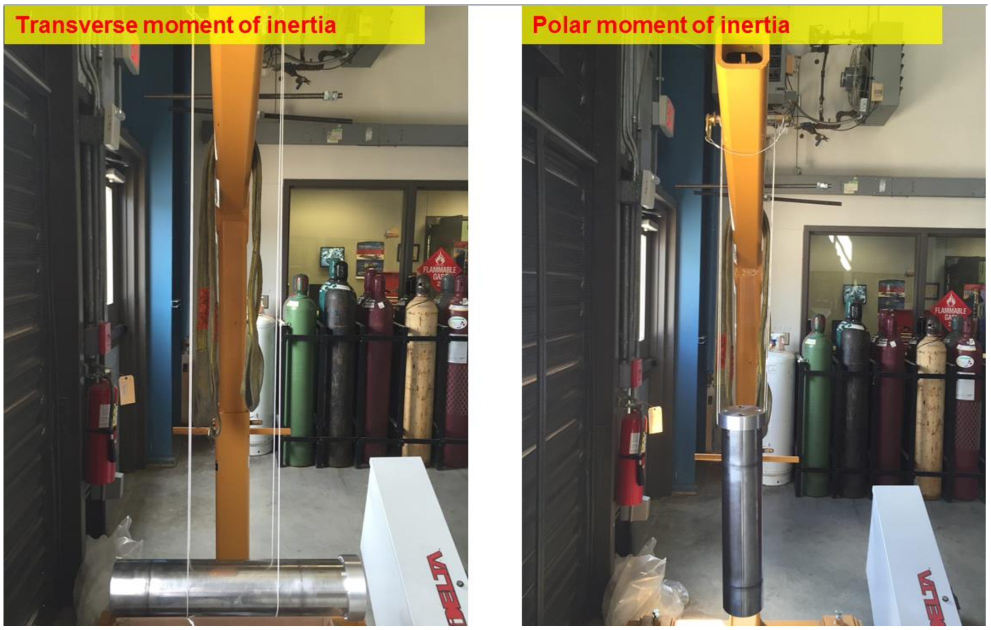

Fig. 3 Lab ad-hoc setups for recording period of natural motion (oscillation) of a suspended rotor.

Item 2. From data in Table 1 and other information stated calculate the rotor mass moment of inertia (transverse and polar) in **kg-cm²**. Produce an estimate of the uncertainty for I_P and I_T. Calculate using published formulas (uniform solid cylinder) I_P and I_T and compare to the parameter magnitudes obtained from measurement of the period of oscillation. Do the *calculated using published formulas* I_P and I_T have an uncertainty? If yes, produce the respective uncertainties; if not, explain why.

3. How to measure free-free mode modes and natural frequencies

Figure 4 shows the ad-hoc set-up for the modal identification of the free-free mode shapes of the rotor. These modes are unconstrained, i.e., have no support stiffness (from the bearings). The picture depicts the fixed location of a reference accelerometer at the rotor middle plane, and the location of a roaming accelerometer that is displaced (moved) manually from one end of the rotor to the other end.

For complete details on the procedure, please read the documents "*how to free-free modes,*" in particular the one written in 2008.

Fig. 4 Setup of rotor hanging from strings used for modal testing and identification of free-free modes.

Figure 5 shows a sample FFT amplitude of an acceleration signal obtained with the roaming (movable) accelerometer after an impulse is exerted on the rotor. The DFT is quite clean and shows two natural frequencies, one at 1,888 Hz and the other at 4,488 Hz (+/- 2 Hz). Note the first natural frequency is sharp with little damping while the second one is more damped with a double peak (likely due to hammer not impacting correctly). [More often single distinctive peaks appeared after an impact].

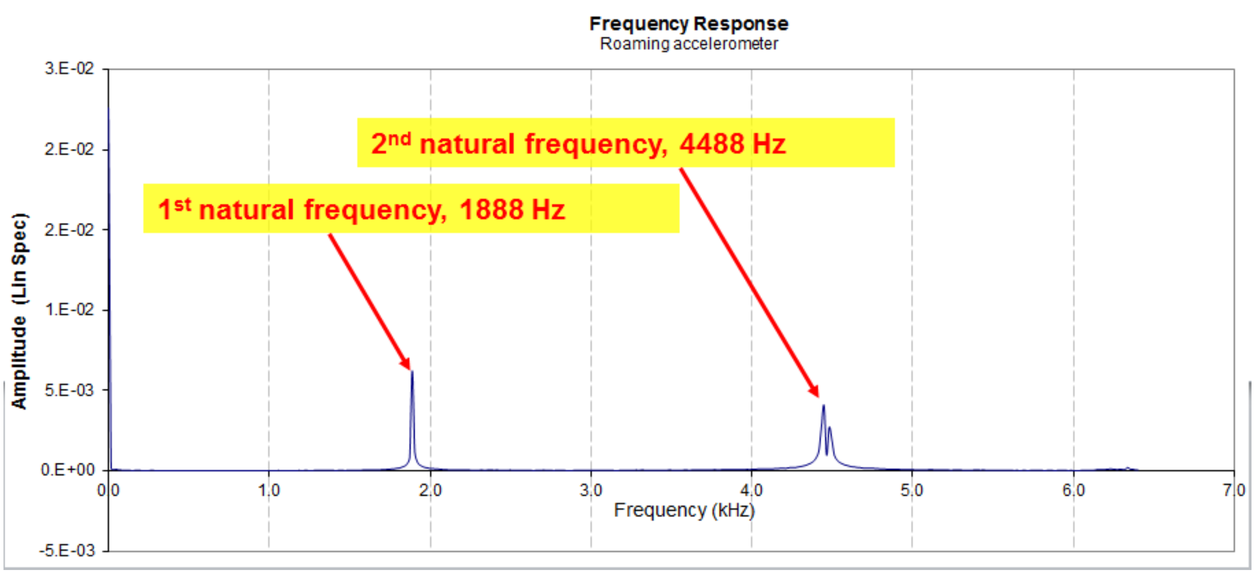

Fig. 5 Example amplitude of FFT of roaming accelerometer showing first and second natural (free-free mode) natural frequencies of test rotor.

Table 2 lists the acceleration data collected by the accelerometers (amplitude and phase) at the first and 2nd natural frequencies of the rotor. The reference accelerometer is fixed at the middle of the rotor while the roaming accelerometer is displaced laterally (approximately) every two inches. The *position* listed in the table has origin or starts at the drive end of the rotor (end w/o cap). Please note that the actual magnitude of the recorded accelerations is NOT important. For modal analysis, the ratio of amplitudes and the difference in phase angles are important.

Table 2. Amplitude and phase of acceleration recorded at first and second natural frequency

Natural frequency: 1888 Hz →

Location	Position (mm)	A ref	A roaming	ϕ ref	ϕ roaming
	Roaming accel	mV	mV	degrees	degrees
1	0	4.05	61.12	126	-54
2	51	4.98	36.20	-175	5
3	102	4.37	3.86	97	-84
4	152	3.92	21.21	-80	-80
5	203	4.48	40.69	-32	-32
6	254	5.54	53.51	6	6
7	305	3.67	25.09	-9	-9
8	356	8.59	7.66	65	63
9	406	3.75	26.41	-174	6
10	451	3.57	45.23	-120	61

Natural frequency: 4,448 Hz →

Location	Position (mm)	A ref	A roaming	ϕ ref	ϕ roaming
	Roaming accel	mV	mV	degrees	degrees
1	0	0.00	3.40	-104	-160
2	51	0.01	0.29	-104	119
3	102	0.03	8.76	-115	-48
4	152	0.01	4.41	-114	-99
5	203	0.04	9.14	-115	-48
6	254	0.02	3.95	167	14
7	305	0.04	17.41	-153	12
8	356	0.02	8.21	-118	84
9	406	0.01	0.04	156	-108
10	451	0.07	8.44	-38	-38

Item 3. From data in Table 2 and other information stated produce plots depicting the natural mode shape of the rotor. For accurate understanding you may wish to display the rotor on the background of a plot. Discuss the nature of the two mode shapes, i.e., their physical meaning.

Item 4. Assume the rotor is a solid cylinder with uniform diameter and length, then using well-known formulas for the lateral vibration of beams (see ME617 Notes 14, for example) PREDICT the rotor (free-free) natural frequencies and mode shapes. Compare the predicted frequencies and mode shapes with the ones obtained experimentally. Quantify and discuss differences.

Item 5. Once installed in the test rig, the rotor will operate at a maximum speed of 18 krpm (300 Hz). For the purposes of a dynamic response (say to imbalance), can the rotor be regarded as rigid or flexible? Explain your answer.

Item6. (**Optional challenge**) Model the rotor using SolidWorks (or any other CAD program) and calculate (using for example FE methods), the rotor free-free mode natural frequencies. Showcase pictures of the free-free modes found by the software. Compare the numerical predictions (frequencies and mode shapes) against those experimentally identified and predicted using close formulas). Are there many more free-free mode natural frequencies (besides those at 0 Hz: rigid body modes)? What do these frequencies and mode shapes mean? Are they important for the purposes of the rotating system?

Working the homework (producing results) should not take you long (~ 2 h). You must, however, spend time reading and learning from the enclosed material. The assignment intends to show you a process for producing and analyzing test data and predictions in an engineering environment. Most times, **engineering starts when the results (or calculations) are available**. The instructor also spent a great deal of time describing with detail the whole process for measurement and analysis.

There may or may not be *exact* answers to the items noted. Please note that uncertainties are common in engineering practice.