MEEN 459/659 GROUP COMPUTATIONAL ASSIGNMENT #4 (DUE APRIL 12, 2019)

A twist into torsional vibrations

BACKGROUND

Feese and Kokot [1,2] present an analysis of the torsional vibrations in a system comprising a (Variable Frequency Drive) VFD motor, elastic coupling, and a reciprocating compressor processing a residue gas. Figure 1 displays two photographs of one compressor unit. During installation and commissioning of four identical compressor trains, the compressors exhibited excessive torque fluctuations. The issues prompted the facility to commission an analysis of the installed units.

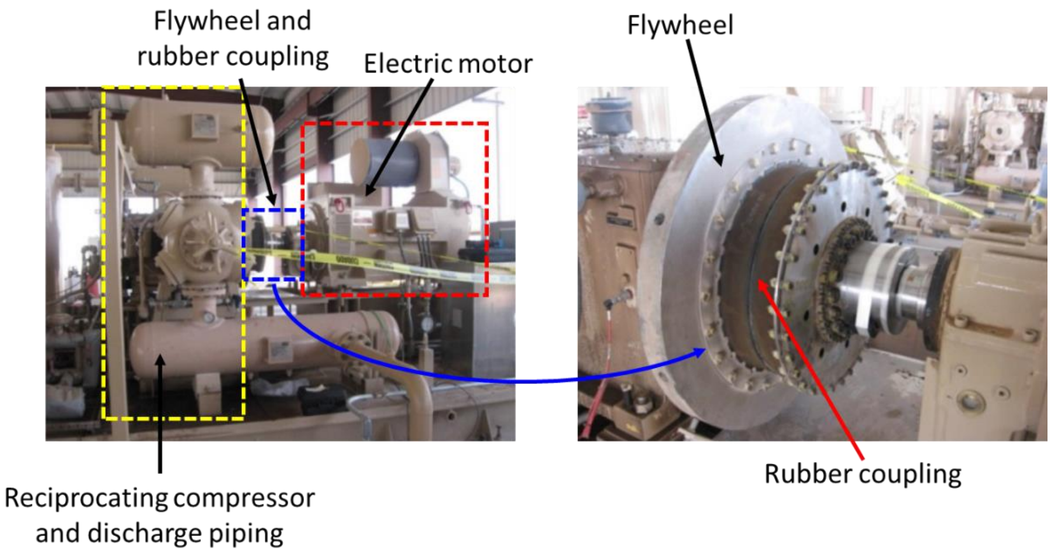


Figure 1. Photographs of a motor driven reciprocating compressor [1].

Figure 2 displays a rotational dynamics model of the motor-compressor system including rotational stiffnesses and dissipative elements (i.e. viscous damping) from each of the system components. A rotating magnetic field applies a torque (T_{motor}) to the motor shaft (mass moment of inertia J_1), driving it at an angular speed $\dot{\theta}_1$. An elastomeric coupling, with torsional stiffness (K_2) and damping (C_2), connects the motor to a massive flywheel and compressor crankshaft (J_2). The flywheel turns with angular speed $\dot{\theta}_2$. The small air gap between the rotor armature and the motor stator windings produces an electro-magnetic (EM) torsional stiffness (K_1) and viscous damping (C_1). K_1 connects to ground.

When performing its function (i.e. compressing the residue gas), the gas in the pistons of the reciprocating compressor provides a resistive load torque (T_{laod}) on the crankshaft.

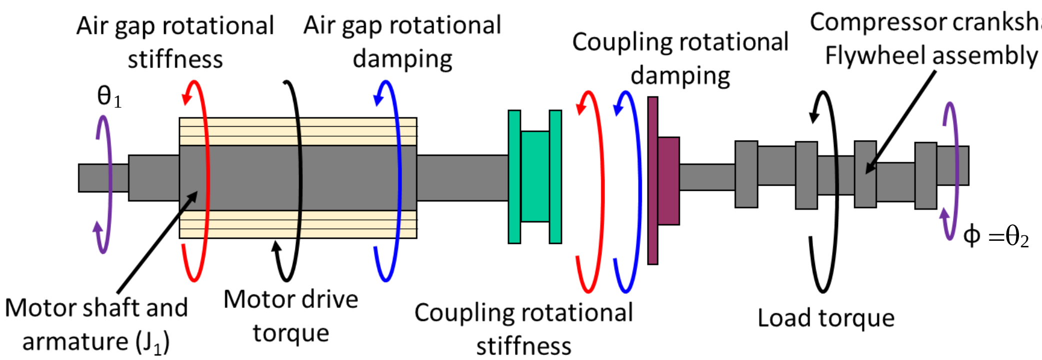


Figure 2. Torsional dynamics model of an electric motor and reciprocating gas compressor

Table 1 details the operating speed and mechanical parameters (stiffness, damping and inertias) of the motor, coupling and compressor. Note that Table 1 states the parameters in SI units, whereas Ref. [1,2] report the system parameters in US units.

Table 1. Electric motor and residue gas compressor mechanical parameters. Taken from Refs. [1,2]

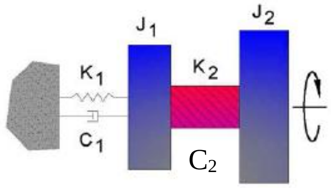
Motor operating speed range, $\dot{\theta}_1$	78.5 – 125.7 rad/s 750 RPM-1200 RPM
Motor armature and coupling inertia, J_1	27.34 kg.m^2
Compressor crankshaft and flywheel inertia, J_2	35.48 kg.m^2
Coupling torsional stiffness, K_2	55.36 kN.m/rad
“” viscous torsional damping, C_2	154 N.m.s/rad
Electromagnetic torsional stiffness, K_1	124.3 kN-m/rad
“” viscous torsional damping (air gap) C_1	88 N.m.s/rad

STATEMENT OF WORK

Deliver a technical report addressing to the technical queries below. Your report should contain well drawn figures and citations where appropriate.

1. **Read** critically the attached literature (full paper and magazine article) and write a concise **summary** (200 words or less) detailing the technical problem found, how it was analyzed and its resolution.

2. Let $\boldsymbol{\theta} = (\theta_1 \quad \theta_2)^T$ be a vector of angular displacements for the motor and compressor. Derive the equations of torsional motion for the mechanical system: $\mathbf{I}\ddot{\boldsymbol{\theta}} + \mathbf{C}\dot{\boldsymbol{\theta}} + \mathbf{K}\boldsymbol{\theta} = \mathbf{T} = \begin{Bmatrix} T_{motor} \\ -T_{load} \end{Bmatrix}$.



See (→) inset above & literature provided.

3. Perform an **undamped** modal analyses of the 2DOF system **without** and **with** the EM torsional stiffness (K_1 = 0 and 124.3 kN-m/rad). List the torsional natural frequencies [Hz] and (plot) the eigenvectors (modes of vibration) for the system. Compare your results to those in the literature provided [1,2]. Discuss the mode shapes found as per the motion of the two inertias (motor and flywheel-compressor). What is the effect of the electro-magnetic stiffness (K_1) on the natural frequencies, and in particular with reference to one of them being near the operating condition at 781 RPM (13.0 Hz)? Is the rigid body mode still present? How does the EM-stiffness work?

4. Perform analysis and derive the formula for the (elastic) natural frequency, shown on page 13 of Ref. [1].

5. At time $t > 0$, the electric motor starts the system **from rest**, driving the motor shaft with

$$T_M(t) = A \cdot t \left\{ 1 + 0.5\cos\left(4\pi \cdot L_f \left\{ t - \frac{\theta}{\Omega_s} \right\} \right) \right\} \text{ N.m} \qquad (1)$$

where A is a linear torque constant, L_f is the line frequency (60 Hz in the US), θ is the rotation angle of the motor shaft and Ω_s is the synchronous speed of the motor (781 rev/min). Note that the compression of the residue gas does create a resistive load at a fraction of the motor drive torque $T_L = (0.25 A \cdot t)$ (N.m). From the plot below, determine the linear constant A [(N.m)/s]. Let t^*=18 s

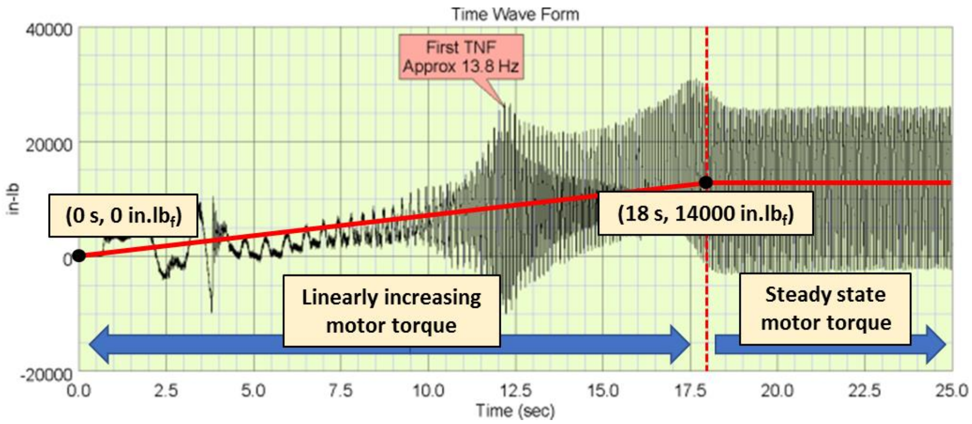


Figure 3. Measured motor torque during a cold-start of a compressor [1].

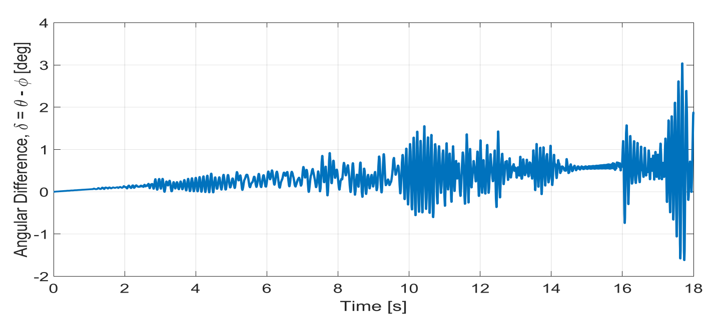
6. Assume a periodic torque (fluctuation) $\mathbf{T} = 0.1\begin{Bmatrix} t_* A \\ 0 \end{Bmatrix} e^{i\omega t}$ (N/m) and $\boldsymbol{\theta} = \boldsymbol{\theta}_o\, e^{i\omega t}$ where ω is an angular frequency ranging from 0 Hz to 20 Hz. Obtain $\left[\mathbf{K} - \omega^2\mathbf{I} + i\omega\mathbf{C}\right]\boldsymbol{\theta}_o = 0.1\begin{Bmatrix} t_* A \\ 0 \end{Bmatrix}$ and solve this algebraic equation to obtain the FRF amplitude and phase, for the motor and compressor, vs. frequency. Conduct the analysis with both K_1 = 0 and 124.3 kN-m/rad. Show plots of $|\theta_1|$ (motor) and $|\theta_2|$ (compressor) vs. frequency for both K_1. In addition plot the phase angle difference $(\theta_1 - \theta_2)$ vs. frequency. What is the effect of the EM K_1 on the amplitude of motions $|\theta_1|$ and $|\theta_2|$, and their phase angle difference, in particular at the operating condition of 781 RPM (13.0 Hz)?

7. Establish the state-space form (4 first order) of the EOMs for torsional vibrations of the electric motor/compressor system. Solve numerically[1] the EOMs over the time interval $0 \le t \le t_*$ and plot the angle difference $(\theta_1 - \theta_2)$ versus time. What happens to $(\theta_1 - \theta_2)$ as the compressor reaches steady state speed (and torque)? For this case, set K_1=0? (read [1])

8. Plot the motor speed versus time for $0 \le t \le t_*$ and compare your predictions to those in Figure 6 of Ref. [1].

9. State conclusions on what you have learned.

See below a (presumably) correct plot for the predicted $(\theta_1 - \theta_2)$



REFERENCES

[1] Feese, T., and Kokot, A., 2016, "Electromagnetic Effects on the Torsional Natural Frequencies of an Induction Motor Driven Reciprocating Compressor with a Soft Coupling," Proceedings of the 45th Turbomachinery & 32nd Pump Symposia, Houston, TX., September, pp. 1-22.

[2] Feese, T., and Kokot, A., 2017, "Electromagnetic Effects on the TNFS of a Reciprocating Compressor with a Soft Coupling," COMPRESSORTech[2], Jan-Feb, pp. 54-59.

Other reference

[3] Vance, J.M., 1988, *Rotordynamics of Turbomachinery*, John Wiley & Sons, New York, NY, ch. 3, pp. 77-79.

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[1] Recommend MATLAB® ODE solver ODE45.