FREQUENCY RESPONSE FUNCTIONS FOR SDOF systems

Table 1. Frequency Response Function Names			
Dimension	Displacement / Force	Velocity / Force	Acceleration / Force
Name	Admittance, Compliance, Receptance	Mobility	Accelerance, Inertance

Table 2. Frequency Response Function Names			
Dimension	Force / Displacement	Force / Velocity	Force / Acceleration
Name	Dynamic Stiffness	Mechanical Impedance	Apparent Mass, Dynamic Mass

$$M \cdot \frac{d^2}{dt^2} X + D \cdot \frac{d}{dt} X + K \cdot X = F_o \cdot e^{i \cdot \omega \cdot t}$$

EOM for periodic load

$$i = \sqrt{-1}$$

Let $X(t) = Z \cdot e^{i \cdot \omega \cdot t}$ and substitute into EOM to obtain

Z E complex #

$$\left[\left(K - M \cdot \omega^2\right) + i \cdot \omega \cdot D\right] \cdot \left(Z \cdot e^{i \cdot \omega \cdot t}\right) = F_o \cdot e^{i \cdot \omega \cdot t}$$

Hence,

$$\frac{Z}{F_o} = \frac{1}{\left[\left(K - M \cdot \omega^2\right) + i \cdot \omega \cdot D\right]}$$

OUTPUT/INPUT
is the Transfer function

Define $M := 10 \cdot kg$ $K := 3.948 \cdot 10^{6} \cdot \frac{N}{m}$

$$\omega_n := \left(\frac{K}{M}\right)^{.5} = 628.331 \frac{rad}{s}$$ natural frequency

$$f_n := \frac{\omega_n}{2 \cdot \pi} = 100.002\ Hz$$

Set damping ratios:

$\zeta_1 := 0.05$ $\zeta_2 := 2 \cdot \zeta_1$ $\zeta_3 := 4 \cdot \zeta_1$

$$D_1 := \zeta_1 \cdot 2 \cdot \sqrt{K \cdot M} = 628.331\ N \cdot \frac{s}{m}$$

Since Z E C, then $Z = X_A \cdot e^{-i \cdot \varphi}$: amplitude and phase φ

Thus $$X(t) = X_A \cdot e^{i \cdot (\omega \cdot t - \varphi)}$$

where $$\frac{X_A}{F_o} = \frac{1}{\sqrt{\left[\left(K - M \cdot \omega^2\right)^2 + (\omega \cdot D)^2\right]}}$$ and $$\tan(\varphi) = \frac{\omega \cdot D}{K - M \cdot \omega^2}$$

Introduce parameters

NATURAL frequency: $$\omega_n = \sqrt{\frac{K}{M}}$$

DAMPING ratio $$\zeta = \frac{D}{2 \cdot \sqrt{K \cdot M}}$$

Operating frequency ratio: $$r = \frac{\omega}{\omega_n}$$

The transfer function **DISPLACEMENT/FORCE** equals

COMPLIANCE, ADMITTANCE or FLEXIBILITYT FN.

$$\frac{X_A}{F_o} = \frac{1}{\sqrt{\left[\left(1 - r^2\right)^2 + (2 \cdot \zeta \cdot r)^2\right]}} \cdot \frac{1}{K}$$

$$\tan(\varphi) = \frac{2 \cdot \zeta \cdot r}{1 - r^2}$$

displacement/force

units of 1/stiffness

DYNAMIC STIFFNESS FN : input/output.

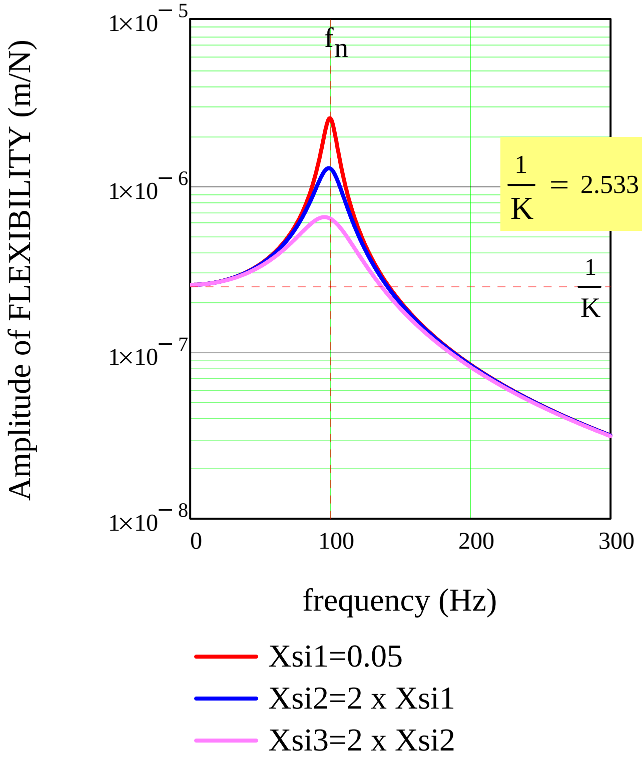
$$\frac{F_o}{X_A} = K \cdot \sqrt{\left[\left(1 - r^2\right)^2 + (2 \cdot \zeta \cdot r)^2\right]}$$

Flexibility function

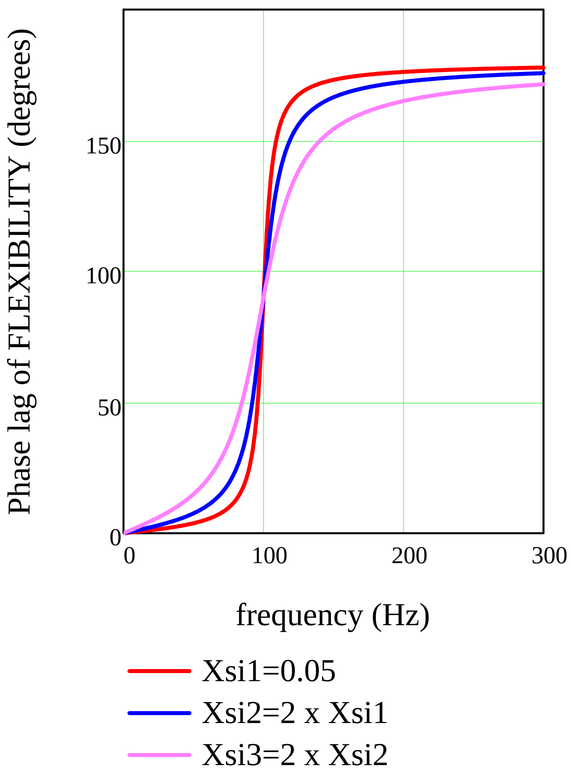
$$\frac{X_A}{F_o}$$

$$\mathrm{Flex}(r,\zeta) := \frac{K^{-1}}{\sqrt{\left[\left(1-r^2\right)^2+(2\cdot\zeta\cdot r)^2\right]}}$$

$$\varphi(r,\zeta) := \left|\begin{array}{l} \varphi \leftarrow \mathrm{atan}\left(\dfrac{2\cdot\zeta\cdot r}{1-r^2}\right)\cdot\left(\dfrac{180}{\pi}\right) \\ \varphi \leftarrow \varphi + 180 \ \text{ if } \ r > 1 \\ \varphi \end{array}\right.$$



$$\frac{1}{K} = 2.533\times10^{-7}\,\frac{m}{N}$$



At times the **velocity** is measured rather than the displacement, hence from

$$X(t) = Z \cdot e^{i \cdot \omega \cdot t}$$

obtain

$$V_X(t) = Z \cdot i \cdot \omega \cdot e^{i \cdot \omega \cdot t} = \omega \cdot Z \cdot e^{i \cdot \left(\omega \cdot t + \frac{\pi}{2}\right)}$$

$$V_X(t) = \omega \cdot X_A \cdot e^{i \cdot \left(\omega \cdot t + \frac{\pi}{2} - \varphi\right)} = v_A \cdot e^{i \cdot \left(\omega \cdot t - \varphi_V\right)}$$

and the **amplitude of the transfer function (velocity/force) is**

$$\frac{v_A}{F_o} = \frac{\omega}{\sqrt{\left[\left(K - M \cdot \omega^2\right)^2 + (\omega \cdot D)^2\right]}}$$

and

$$\varphi_V = \varphi - \frac{\pi}{2}$$

Using the nat. frequency and damping ratio (ωn, ζ) gives

MOBILITY fn.

$$\frac{v_A}{F_o} = \frac{1}{\sqrt{\left[\left(1 - r^2\right)^2 + (2 \cdot \zeta \cdot r)^2\right]}} \cdot \frac{\omega}{K}$$

velocity/force units of 1/Damping

While the

MECHANICAL IMPEDANCE

$$\frac{F_o}{v_A} = \frac{K}{\omega} \cdot \sqrt{\left[\left(1 - r^2\right)^2 + (2 \cdot \zeta \cdot r)^2\right]}$$

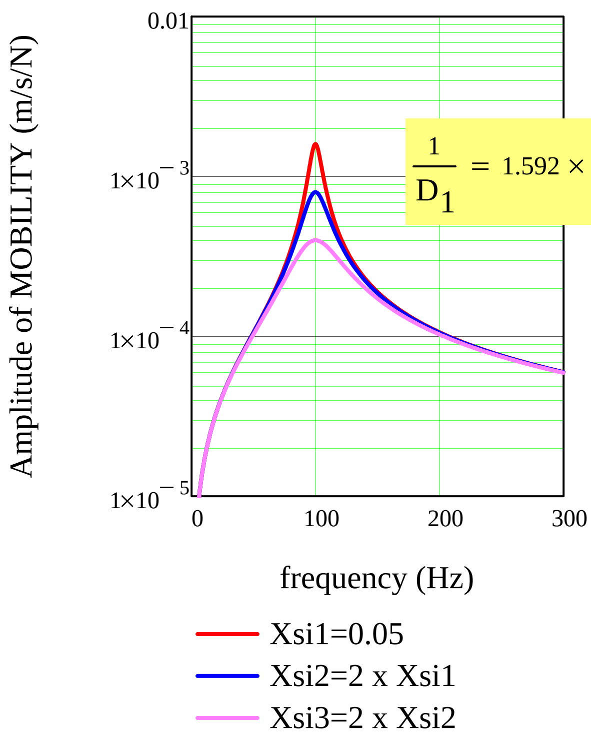
force/velocity

Mobility function

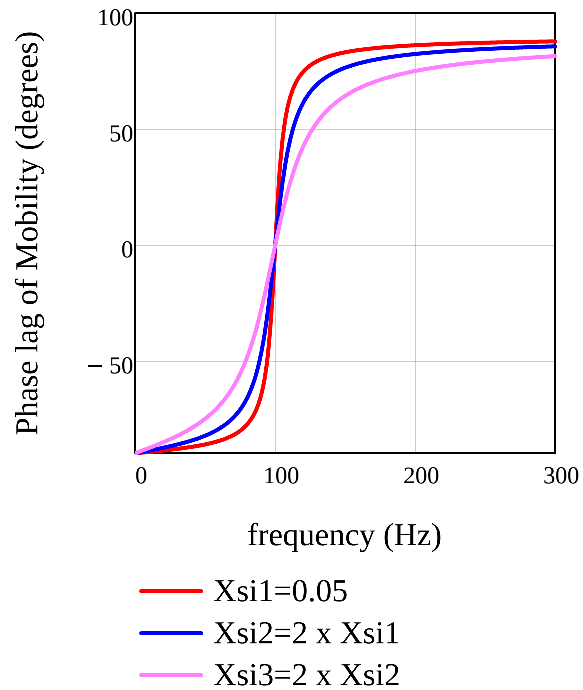
$$\frac{V_A}{F_o}$$

$$Mob(r,\zeta) := \frac{r \cdot \omega_n \cdot K^{-1}}{\sqrt{\left[\left(1 - r^2\right)^2 + (2 \cdot \zeta \cdot r)^2\right]}}$$

$$\varphi(r,\zeta) := \begin{array}{|l} \varphi \leftarrow \operatorname{atan}\left(\frac{2 \cdot \zeta \cdot r}{1 - r^2}\right) \cdot \left(\frac{180}{\pi}\right) \\ \varphi \leftarrow \varphi + 180 \quad \text{if } r > 1 \\ \varphi \leftarrow \varphi - 90 \end{array}$$



$$\frac{1}{D_1} = 1.592 \times 10^{-3} \frac{m}{N \cdot s}$$



Other times the **acceleration** is measured rather than the displacement or velocity, hence from

$$X(t) = Z \cdot e^{i \cdot \omega \cdot t}$$

obtain

$$A_X(t) = Z \cdot i^2 \cdot \omega^2 \cdot e^{i \cdot \omega \cdot t} = -\omega^2 \cdot Z \cdot e^{i \cdot (\omega \cdot t)} = a_A \cdot e^{i \cdot (\omega \cdot t - \varphi_A)}$$

and the **amplitude of the transfer function (acceleration/force) is**

$$\frac{a_A}{F_o} = \frac{\omega^2 \cdot M^{-1}}{\sqrt{\left[\left(K - M \cdot \omega^2\right)^2 + (\omega \cdot D)^2\right]}}$$

and

$$\varphi_A = \varphi - \pi$$

Using the nat. frequency and damping ratio (ωn, ζ) gives

Accelerance fn
Inertance fn.

$$\frac{a_A}{F_o} = \frac{r^2 \cdot M^{-1}}{\sqrt{\left[\left(1 - r^2\right)^2 + (2 \cdot \zeta \cdot r)^2\right]}}$$

acceleration/force units of 1/M

While the

APPARENT Mass or Dynamic Mass fn:

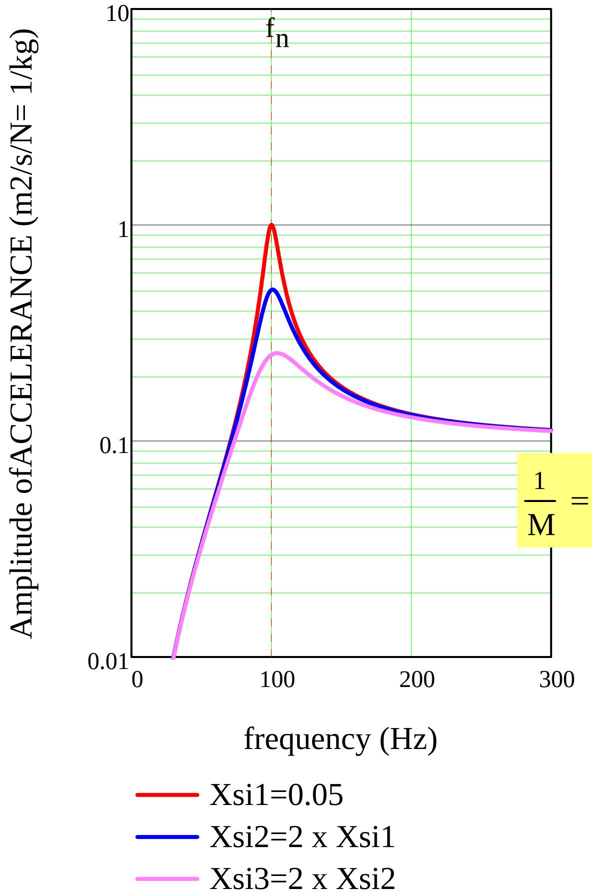
$$\frac{F_o}{a_A} = \frac{M}{r^2} \cdot \sqrt{\left[\left(1 - r^2\right)^2 + (2 \cdot \zeta \cdot r)^2\right]}$$

Accelerance function

$$\frac{A_A}{F_o}$$

$$\text{Accel}(r,\zeta) := \frac{r^2 \cdot M^{-1}}{\sqrt{\left[\left(1-r^2\right)^2 + (2\cdot\zeta\cdot r)^2\right]}}$$

$$\varphi(r,\zeta) := \left| \begin{array}{l} \varphi \leftarrow \text{atan}\left(\frac{2\cdot\zeta\cdot r}{1-r^2}\right)\cdot\left(\frac{180}{\pi}\right) \\ \varphi \leftarrow \varphi + 180 \quad \text{if } r > 1 \\ \varphi \leftarrow \varphi - 180 \end{array} \right.$$



$$\frac{1}{M} = 0.1\,\frac{1}{kg}$$