Notes 5

Sensor calibration & uncertainty in measurements and engineering analysis

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Resources

Experimentation and Uncertainty Analysis for Engineers,
H. Coleman & W. Steele, Wiley Pubs, 1989

MEASUREMENT SYSTEMS: APPLICATION & DESIGN,
Doebelin, E., 5th ed, ISBN-13: 978-0072990720

Product specifications and technical notes from
PCB, Bruel & Kjaer, etc

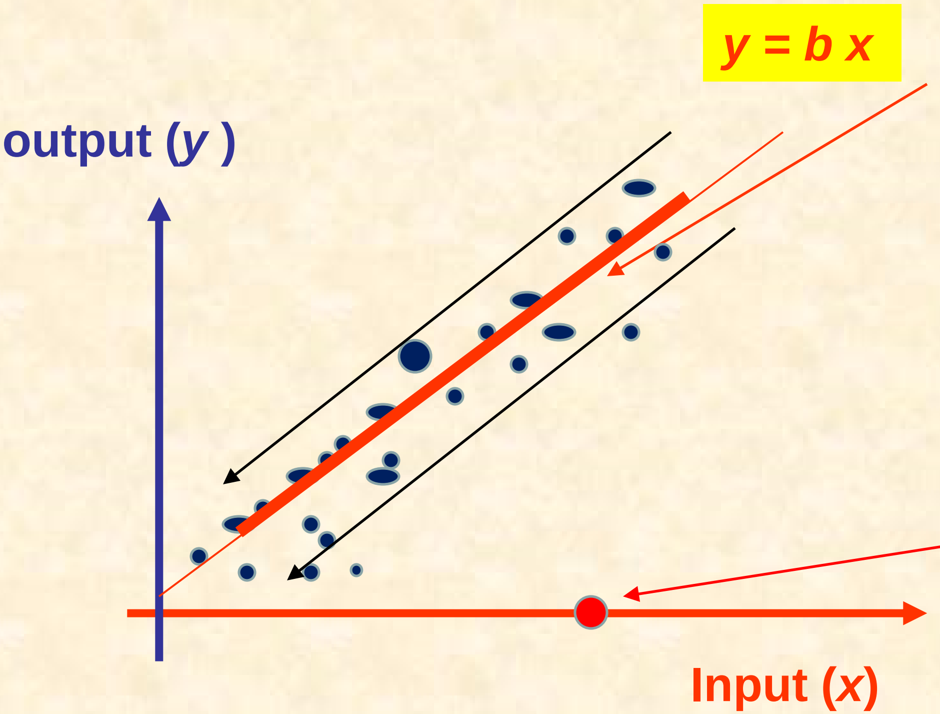
+ 35 y of experience performing measurements

Sensors

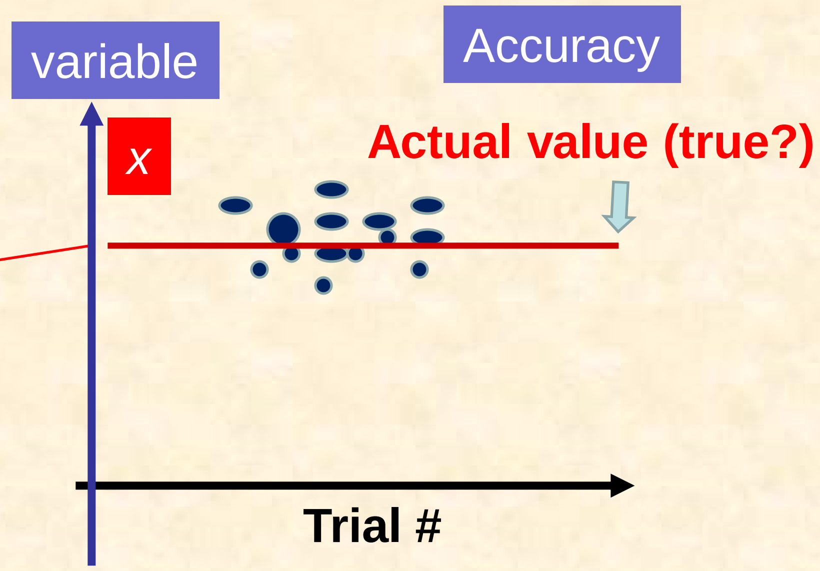
Selecting, installing and using a sensor are not trivial tasks! **Costly errors are frequent because the user (you) does not deem necessary to read a manual or to follow instructions for installation or operation. Sensors for vibration measurement are not plug & play.**

Sensor & static calibration

Typically measure variable (*x*) by recording an output (*y*) (~voltage) that is proportional.

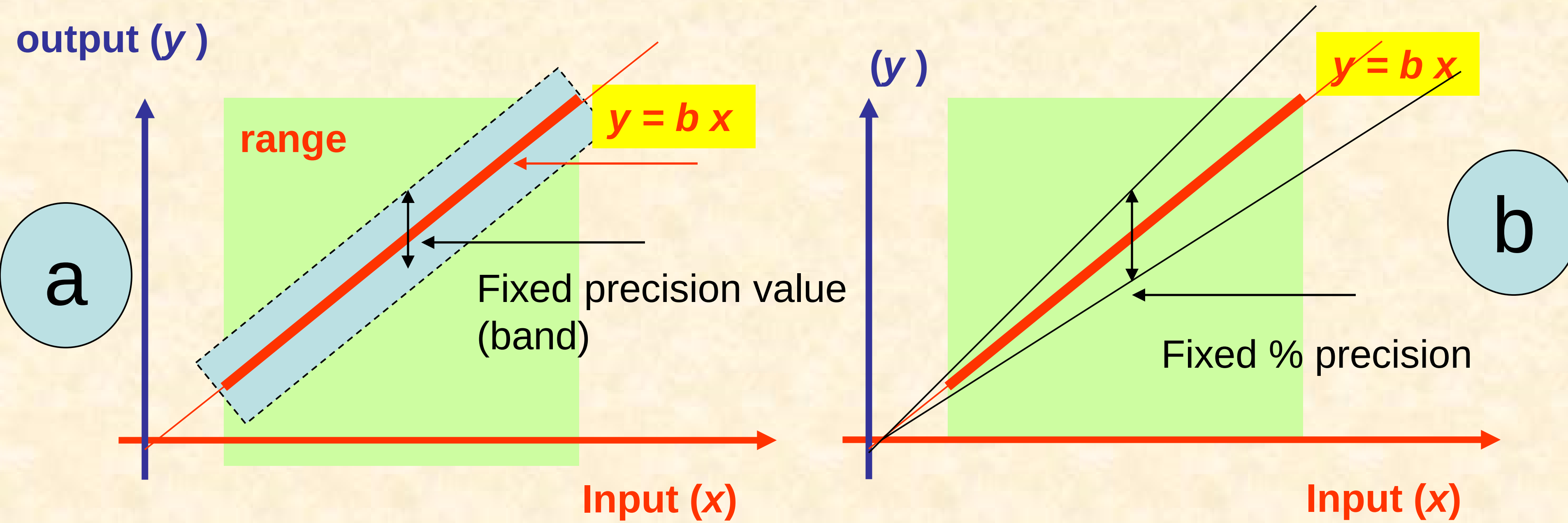


b is gain or sensitivity
obtained from (say) least square method to recorded values



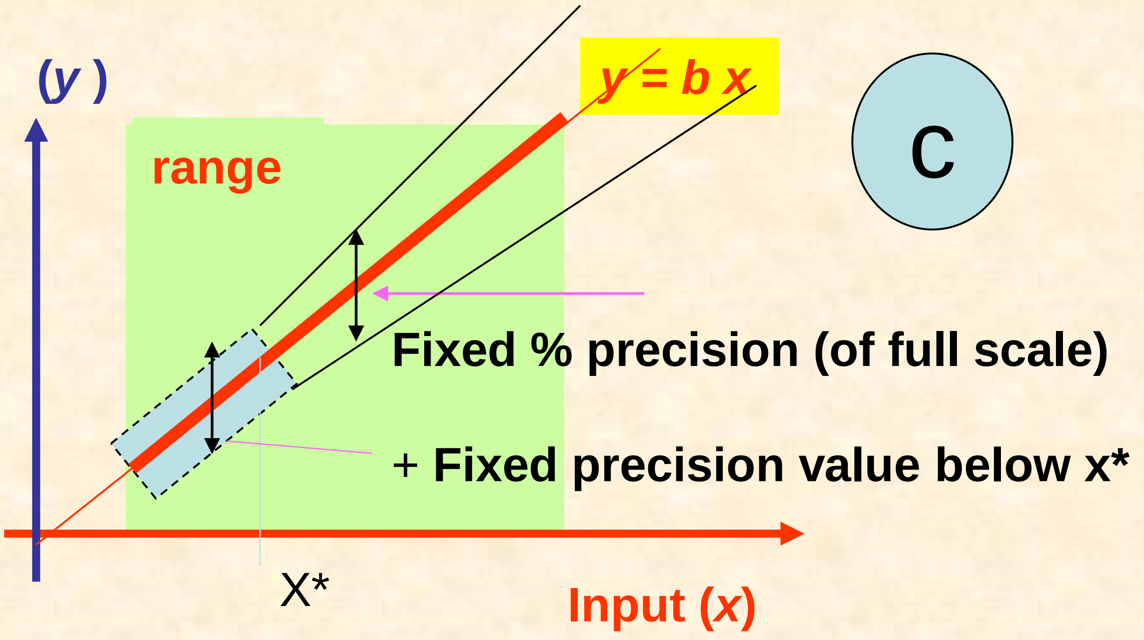
Sensors

Sensor sensitivity, range and uncertainty (precision) must be well known



Sensor (a) is reliable for measurements at large *x* *(upper end of range)*
Sensor (b) can be used for small *x* (lower end of range)

Sensor – preferred



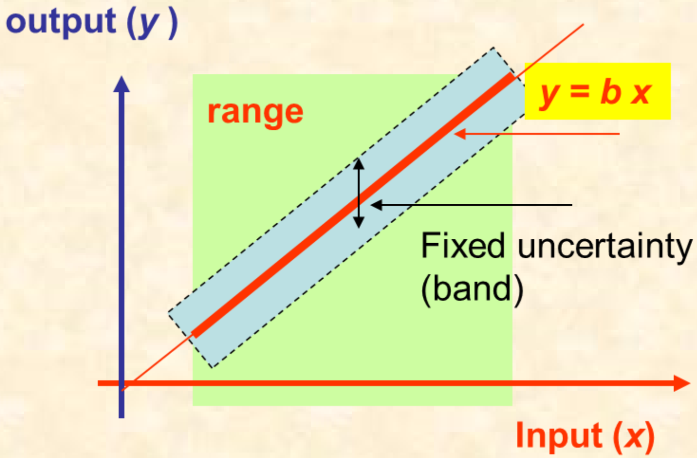
Sensor with definite bands for both low and high values

Sensor measurement error

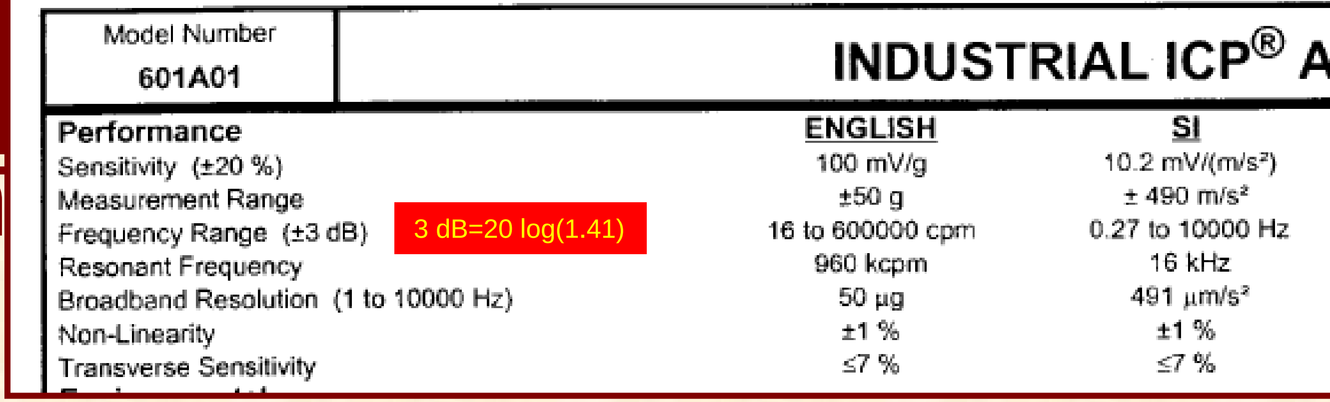
Uncertainty = bias limit + imprecision

Bias: **difference b/w reference (actual) and recorded value. Sensor calibration removes large known biases.**

Imprecision= $t_{95,N-1}\,\sigma \rightarrow$ (1.96 x σ) for N=inf.
σ is standard-deviation for 95% confidence

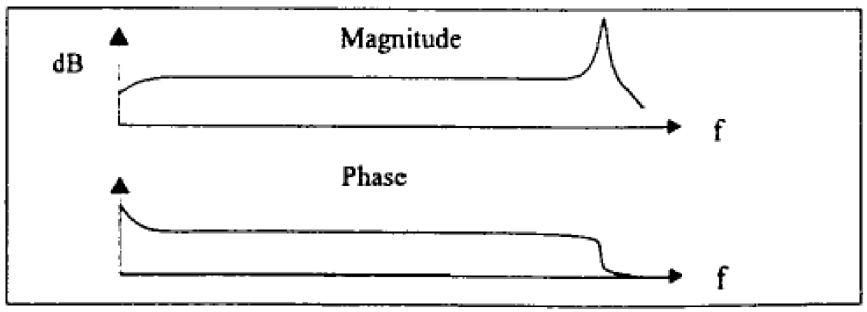


Typical calibration sheet for accelerometer



Model Number 601A01	INDUSTRIAL ICP® A	
Performance	**ENGLISH**	**SI**
Sensitivity (±20 %)	100 mV/g	10.2 mV/(m/s²)
Measurement Range	±50 g	± 490 m/s²
Frequency Range (±3 dB)	16 to 600000 cpm	0.27 to 10000 Hz
Resonant Frequency	960 kcpm	16 kHz
Broadband Resolution (1 to 10000 Hz)	50 µg	491 µm/s²
Non-Linearity	±1 %	±1 %
Transverse Sensitivity	≤7 %	≤7 %

3 dB=20 log(1.41)



Typical FRF – amplitude & phase

Effect of mount on FRF

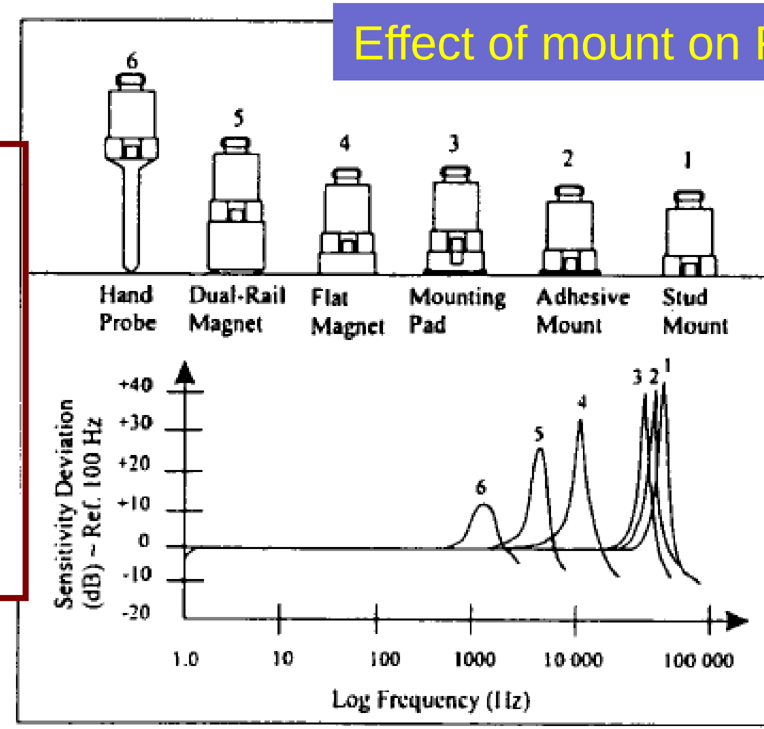
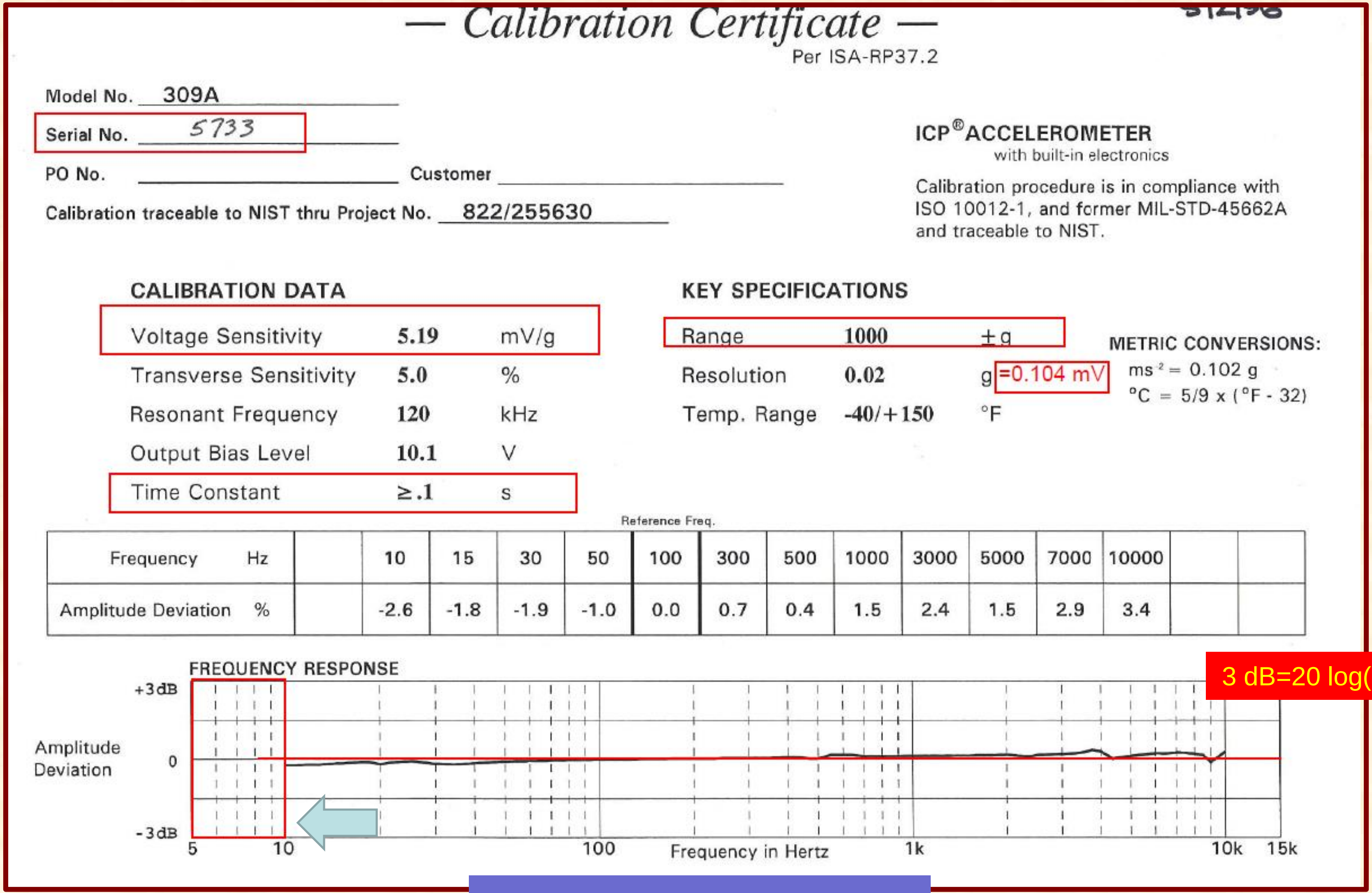


Figure 4. Assorted Mounting Configurations and Their Effects on High Frequency

https://www.pcb.com/Resources/Technical-information/tech_gen

Typical calibration certificate



— Calibration Certificate —

Per ISA-RP37.2

Model No. 309A

Serial No. 5733

PO No. ______ Customer ______

Calibration traceable to NIST thru Project No. 822/255630

ICP® ACCELEROMETER
with built-in electronics

Calibration procedure is in compliance with ISO 10012-1, and former MIL-STD-45662A and traceable to NIST.

CALIBRATION DATA

Voltage Sensitivity	5.19	mV/g
Transverse Sensitivity	5.0	%
Resonant Frequency	120	kHz
Output Bias Level	10.1	V
Time Constant	≥.1	s

KEY SPECIFICATIONS

Range	1000	±g
Resolution	0.02	g =0.104 mV
Temp. Range	-40/+150	°F

METRIC CONVERSIONS:

ms^{-2} = 0.102 g

°C = 5/9 x (°F - 32)

Reference Freq.

Frequency Hz		10	15	30	50	100	300	500	1000	3000	5000	7000	10000		
Amplitude Deviation %		-2.6	-1.8	-1.9	-1.0	0.0	0.7	0.4	1.5	2.4	1.5	2.9	3.4		

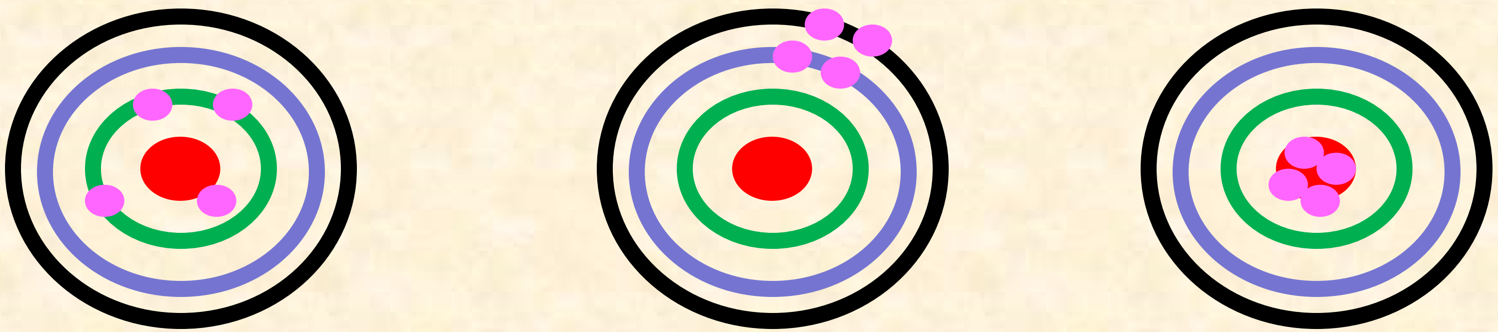
3 dB=20 log(1.41)

Typical FRF – amplitude

Question

Are precision and accuracy the same thing?

NO. Accuracy is how close a measured value is to the **actual (true) value**. **Precision** is how close the measured values are **to each other**.



High Accuracy Low Precision	Low Accuracy **High Precision**	**High Accuracy** **High Precision**

Rules for sensor keeping

a) **Always request sensor calibration certificate from vendor (if possible for same test conditions as yours). Calibration must be traceable.**

b) **Always keep serial # of sensor & its gain labeled near sensor location.**

c) **Maintain sensor calibration certificates in a safe place (and with multiple copies).**

d) **If possible, conduct own calibration (under test conditions *P,T*).**

e) **Re-certify calibration after a few years or after unusual test conditions (cost $$).**

f) **Always use sensor for its intended use and do not exceed manufacturer specifications (range and usage conditions).**

MOST IMPORTANT: Read manual for installation and operation of sensor to avoid costly mistakes.

Statistics review

Brief review of Statistics

Mean Value:

The arithmetic mean $\bar{y}$ of a set of *n* measurements $\{y_1, y_2, ..., y_n\}$ of a variable *y* is the sum of the measurements divided by the total number of measurements

$$\bar{y} = \frac{\sum y_i}{n}$$

Important notes

The mean is the arithmetic average of the measurements in a data set.

There is only one mean for a data set.

The mean is, in some sense, the best estimate of the true value of *y*.

The **value of the mean** is influenced by extreme measurements, trimming can help reduce the degree of influence.

Brief review of Statistics

Variance:

The variance s^2 of a set of *n* measurements $\{y_1, y_2, \ldots, y_n\}$ with mean $\bar{y}$ is the sum of the square deviations divided by (*n* - 1), i.e.

$$s^2 = \frac{\sum (y_i - \bar{y})^2}{(n-1)}$$

Important notes

Brief review of Statistics

Standard deviation $\sigma = s$

of a set of *n* measurements is the positive square root of the variance. σ is a measure of the tendency of the measurements to cluster around the mean value.

$$s = \sigma = \sqrt{\frac{\sum (y_i - \bar{y})^2}{(n-1)}}$$

Important notes

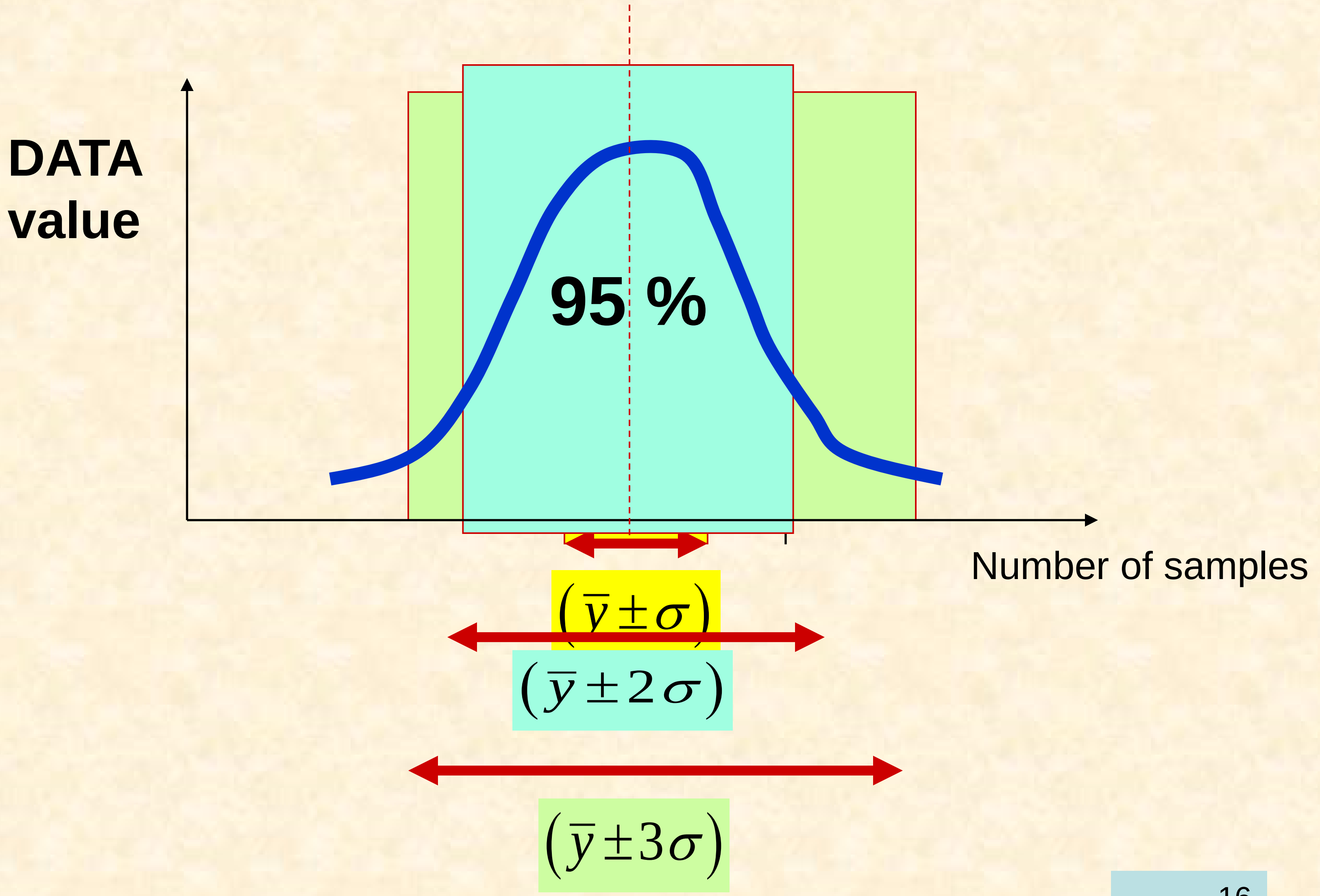
Give a set of *n* measurements possessing a mound-shaped histogram (Gaussian distribution about the mean value), then

the interval $(\bar{y} \pm \sigma)$ contains ~ 68% of the measurements, then

the interval $(\bar{y} \pm 2\sigma)$ “” ~ 95% of the measurements, and

the interval $(\bar{y} \pm 3\sigma)$ “” ~ nearly all the measurements (100%)

Standard deviation $\sigma = S$



Demonstrative only – graph not to scale

Exercise

Find class average (mean) age and its standard deviation

Uncertainty Analysis

review

About Uncertainty Analysis

In many cases direct measurement of a variable or experimental result is not possible. Instead, one measures the values or magnitudes of several variables or parameters, and combines them in a data reduction equation to obtain the desired result.

There is no (ready) SENSOR to do the job!

End measurement depends on the accuracy of other instruments measuring secondary (other) parameters

Introduction to Uncertainty Analysis

In many cases direct measurement of a variable or experimental result is not possible. Instead, one measures the values or magnitudes of several variables or parameters, and combines them in a data reduction equation to obtain the desired result.

For example, consider an experiment to answer the question:

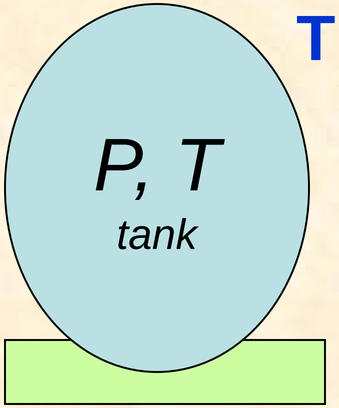
What is the density of air in a pressurized tank?

Not having a density meter, one must resort to physical principles and determine the density indirectly by using, for example, the equation of state of an ideal gas,

Experimentation and Uncertainty Analysis for Engineers,
H. Coleman & W. Steele, Wiley Pubs, 1989

Introduction to Uncertainty Analysis

What is the density of air in a pressurized tank?

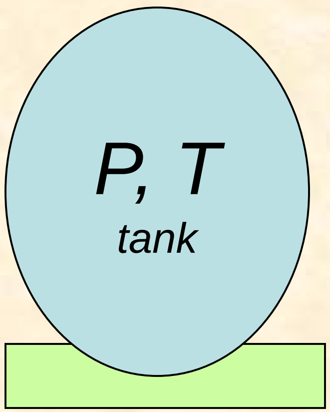


The equation of state of an ideal gas is

$$P = \rho \Re_g T$$

The gas composition is usually well known; hence, the particular gas constant R_g is easily found from Tables (books). If one could measure the gas pressure (P) and absolute temperature (T) within the tank, then one could <u>estimate</u> the magnitude of the gas density (ρ).

Answer: Uncertainty Analysis



The measurement of each of the physical variables (*P, T*) has an associated uncertainty. Published values of material properties, $\Re_g$ for example, also have an uncertainty.

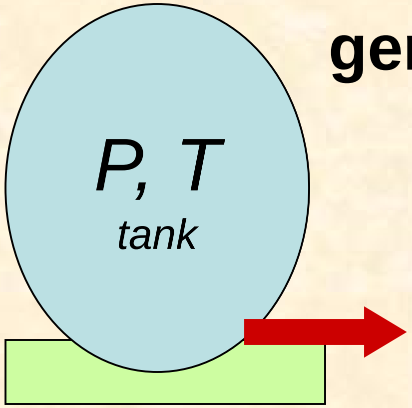
$$\rho = P \big/ \left(\Re_g T\right)$$

The key question in measurements or experimentation is:

How do the uncertainties in the individual variables <u>propagate</u> through a data reduction equation into a final (estimated) result?

Basics of Uncertainty Analysis

An experimental result, say φ, is a function of the set of variables x_i. This can be described in the general functional form



$$\varphi = \varphi(x_1, x_2, x_3, x_4, \ldots, x_n)$$

$$\text{Example}: \varphi = \rho = \frac{P}{\Re_g T}, \text{where } x_1, = P, x_2 = T, x_3 = \Re_g, \quad n = 3$$

(2)

The uncertainty *U* of the end result (measured variable) is

$$U_\varphi = \left[\left(\frac{\partial \varphi}{\partial x_1} U_{x1} \right)^2 + \left(\frac{\partial \varphi}{\partial x_2} U_{x2} \right)^2 + \ldots + \left(\frac{\partial \varphi}{\partial x_n} U_{x_n} \right)^2 \right]^{1/2} \quad (3)$$

where *Uxi* is the uncertainty associated to the measured variable *xi.* The partial derivatives are known as absolute sensitivity coefficients.

MISO: multiple input → single output

The % uncertainty *U* of the estimated variable ϕ is

$$\varphi = \varphi(x_1, x_2, x_3, x_4, \ldots, x_n)$$

$$\frac{U_\varphi}{\varphi} = \left[\left(\frac{\partial \varphi}{\partial x_1} \frac{U_{x1}}{\varphi} \right)^2 + \left(\frac{\partial \varphi}{\partial x_2} \frac{U_{x2}}{\varphi} \right)^2 + \ldots + \left(\frac{\partial \varphi}{\partial x_n} \frac{U_{x_n}}{\varphi} \right)^2 \right]^{1/2}$$

The formulas:

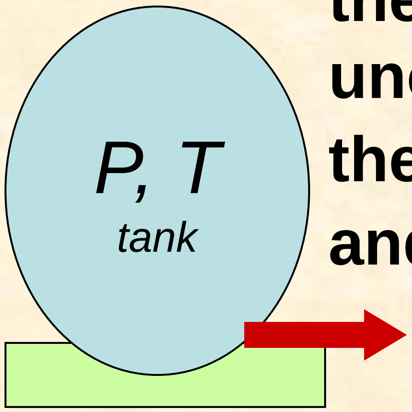
$$\frac{U_\varphi}{\varphi} = \left(\frac{U_{x_1}}{x_1} \right) + \left(\frac{U_{x_2}}{x_2} \right) + \ldots + \left(\frac{U_{x_n}}{x_n} \right)$$

$$\frac{U_\varphi}{\varphi} = \left| \frac{U_{x_1}}{x_1} \right| + \left| \frac{U_{x_2}}{x_2} \right| + \ldots + \left| \frac{U_{x_n}}{x_n} \right|$$

Are NOT correct on several counts

Formulas do not reflect equation used to find variable ϕ. First keeps directionality (signs) of *U* and *x* (<0)

Basics Rules of Thumb



RULE #1: Always solve the data equation for the experimental result before performing an uncertainty analysis, i.e., if the objective is to find the density by measuring (*P,T*), then $\rho = P/TR_g$ **and**

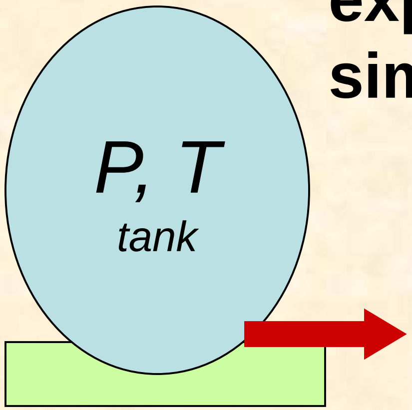
$$U_\rho^{\,2} = \left(\frac{\partial \rho}{\partial P}\right)^2 U_P^{\,2} + \left(\frac{\partial \rho}{\partial T}\right)^2 U_T^{\,2} + \left(\frac{\partial \rho}{\partial \Re_g}\right)^2 U_{\Re_g}^{\,2} \quad \text{(a)}$$

Performing operations

$$U_\rho^{\,2} = \left(\frac{\rho}{P}\right)^2 U_P^{\,2} + \left(-\frac{\rho}{T}\right)^2 U_T^{\,2} + \left(-\frac{\rho}{\Re_g}\right)^2 U_{\Re_g}^{\,2} \quad \text{(c)}$$

Basics Rules of Thumb

RULE #2: Always divide the uncertainty expression $U\varphi$ by the experimental result φ to simplify the algebraic simplification

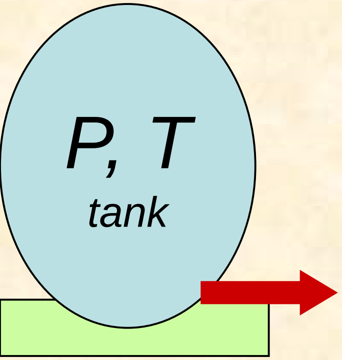


$$\left(\frac{U_\rho}{\rho}\right)^2 = \left(\frac{U_P}{P}\right)^2 + \left(\frac{U_T}{T}\right)^2 + \left(\frac{U_{\Re_g}}{\Re_g}\right)^2 \quad (*)$$

For this example, R_g is negligible (assumed zero) since the uncertainty for a universal constant is much less than those of the others, i.e., its measurement is performed with the greatest accuracy.

Example 1

Experimentation and Uncertainty Analysis for Engineers, H. Coleman & W. Steele, Wiley Pubs, 1989



A pressurized air tank is nominally at ambient temperature (25°C). How accurately can the air density be determined if the temperature is measured with an uncertainty of 1°C and the tank pressure is measured with an uncertainty of 3%.

$$U_{\Re_g} = 0$$

$$U_T/T = 1°K/(273.3+25)°K = 0.00335 = 0.33\% \quad \text{at } 25°C; \quad 1°K = 1°C$$

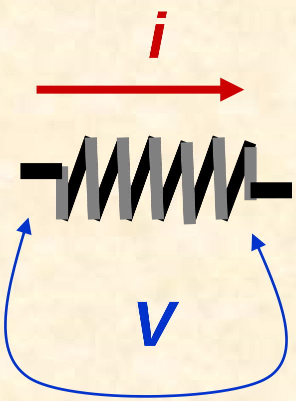
$$U_P/P = 0.03 = 3\% \text{ as stated}$$

$$\left(\frac{U_\rho}{\rho}\right)^2 = \left(\frac{U_P}{P}\right)^2 + \left(\frac{U_T}{T}\right)^2 + \left(\frac{U_{\Re_g}}{\Re_g}\right)^2 = (0.0300)^2 + (0.00335)^2 + 0 = 0.00091$$

$$U_\rho/\rho = 0.0301 = 3.01\% \text{ uncertainty}$$

The uncertainty in the measurement of pressure dominates the estimation of gas density. If the end result renders a too large uncertainty for the parameter of interest (density), then it shows the need to procure a method (or sensor) to measure the gas pressure more accurately

Example 2



Consider the calculation of electric power $P = V \times i$ from where the voltage V and current i are measured as

$$V = 100\ \text{V} \pm 2\ \text{V}; \qquad i = 10\ \text{A} \pm 0.3\ \text{A}$$

The nominal value of the power is

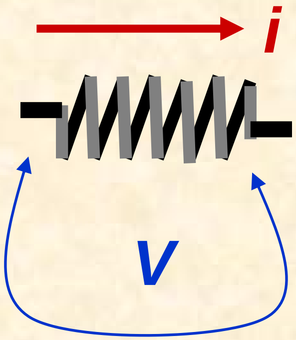
P = **100 V x 10 A = 1,000 W**.

By taking the worst possible variations in voltage and current, calculate:

$$P_{max} = (100 + 2)\ \text{V}\ (10 + 0.3)\ \text{A} = 1050.6\ \text{W}$$

$$P_{min} = (100 - 2)\ \text{V}\ (10 - 0.3)\ \text{A} = 950.6\ \text{W}$$

Example 2



***V* = 100 V ± 2 V; *i* = 10 A ± 0.3 A**

P_{max} = (100 + 2) V (10 + 0.3) A = 1050.6 W

P_{min} = (100 - 2) V (10 - 0.3) A = 950.6 W

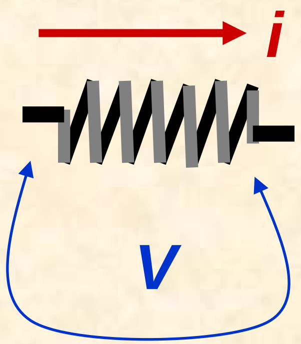
using a simple method of calculation, the maximum differences (error) in power are

$$\left(\wp_{\max} - \wp\right) \Big/ \wp = 5.06\,\%$$

$$\left(\wp_{\min} - \wp\right) \Big/ \wp = -\,4.94\%$$

It is unlikely that the power would be in error by the amounts because the voltmeter variations probably do not correspond with the ammeter variations. When the voltmeter reads an extreme "high," there is no reason why the ammeter must also read an extreme "high" at that particular instant. A very unlikely event!

Example 2



***V* = 100 V ± 2 V; *i* = 10 A ± 0.3 A**

P_{max} = (100 + 2) V (10 + 0.3) A = 1050.6 W

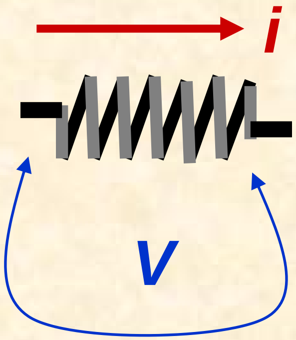
P_{min} = (100 - 2) V (10 - 0.3) A = 950.6 W

$$\left(\wp_{\max} - \wp\right) / \wp = 5.06\%$$

$$\left(\wp_{\min} - \wp\right) / \wp = -4.94\%$$

The simple calculation applied to the electric-power equation is a useful way of inspecting experimental data to determine what error could result in a final calculation; however, the test is too severe and should be used only for rough inspections of data

Example 2



***V* = 100 V ± 2 V; *i* = 10 A ± 0.3 A**

$\wp_{max}$ = (100 + 2) V (10 + 0.3) A = 1050.6 W

$\wp_{min}$ = (100 - 2) V (10 - 0.3) A = 950.6 W

The uncertainties in the measurement of current and voltage are equal to U_i=0.3 A and U_V=2 V. The uncertainty in the estimation (measurement) of the electrical power is

$$\left(\frac{U_\wp}{\wp}\right) = \left[\left(\frac{U_V}{V}\right)^2 + \left(\frac{U_i}{i}\right)^2\right]^{1/2}$$

$$= \left[\left(\frac{2\,\text{V}}{100\,\text{V}}\right)^2 + \left(\frac{0.3\text{A}}{10\,\text{A}}\right)^2\right]^{1/2} = \left[(0.02)^2 + (0.03)^2\right]^{1/2} = 0.036$$

= 3.6 % of measured value (1,000 W).

Questions on uncertainty

Notes on Uncertainty Analysis

- The uncertainty value $U\varphi$ is always a positive number (>0) with identical physical dimensions as the measured result φ.
- ASME Standards require $[U\varphi / \varphi] < 0.05$ (5%) for 95% coverage, i.e. uncertainty values larger than 5% of the measured value are discarded (unacceptable) in engineering practice.
- The uncertainty $[U\varphi / \varphi]$ could be > 100% when the variable measured is very small, i.e. $|\varphi| < U\varphi$. These measurements MUST be discarded. Find a better instrument!

Notes on Uncertainty Analysis

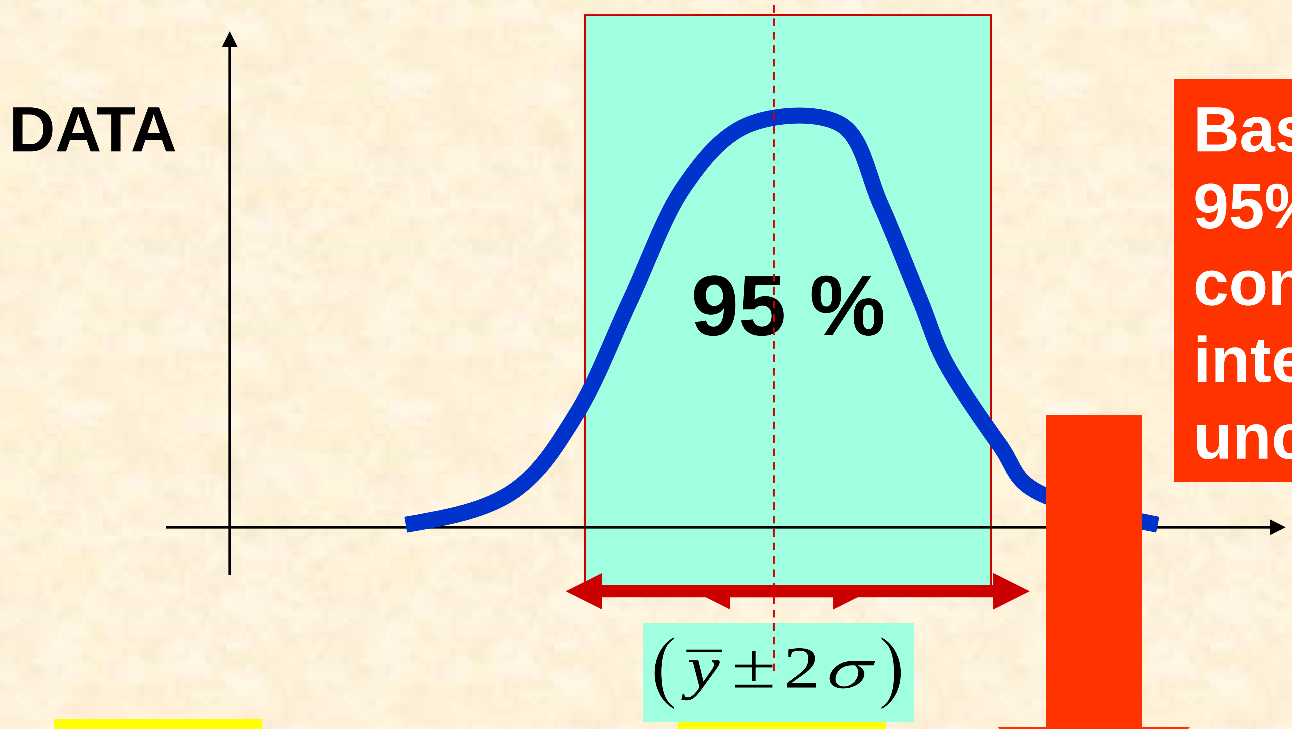
- The analysis above refers to static measurements, i.e. variables and parameters that do not change rapidly with time.
- The analysis introduced applies to MISO systems, i.e. multiple input (variable) and single output (one result). MIMO systems are common in industry!
- Uncertainty analysis is a must in modern engineering!

Question on Uncertainty Analysis

- **ASME Standards require [$U\varphi / \varphi$] < 0.05 (5%) for 95% coverage, i.e. uncertainty values larger than 5% of the measured value are discarded (unacceptable) in engineering practice.**

What is the rationale behind the 95% coverage? Is it rooted entirely on statistics or is there an engineering background?

Mean and standard deviation σ



Basis for 95% confidence interval in uncertainty

the interval $(\bar{y} \pm \sigma)$ **contains ~ 68% of the measurements, then**

the interval $(\bar{y} \pm 2\sigma)$ **"" ~ 95% of the measurements, and**

the interval $(\bar{y} \pm 3\sigma)$ **"" ~ nearly all the measurements (100%)**

Question on Uncertainty Analysis

How does one reduce uncertainty or imprecision (improve certainty) other than changing the sensor (measurement instrument) itself?

Correct. Repeating the measurement many times will not help. Hence, acquiring and using an appropriate sensor is a must for accurate measurements. In addition, the "formula" to find variable must be known to be correct.

$$\varphi = \varphi(x_1, x_2, x_3, x_4, ..., x_n)$$

Exercises

One of these exercises may appear as a problem statement in Exam 1

Exercise on Uncertainty Analysis

Why is there a need for Uncertainty Analysis in engineering measurements?

The natural frequency (w_n) of a one degree of freedom system is the frequency at which the system will oscillate under free vibration. The natural frequency is related to the mass (*m*) and stiffness (*k*) of the system through the equation $w_n = (k/m)^{1/2}$.

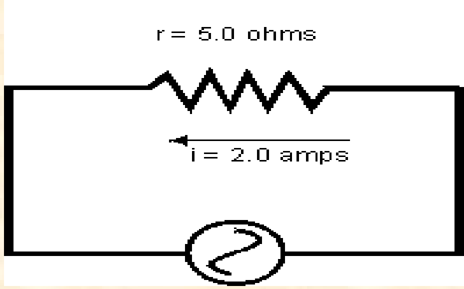
Set up the equation for the uncertainty in W_n assuming that the uncertainties of the measured values are U_m for *m* and U_K for *k*. Simplify your answer.

Assume that you have enough money to purchase equipment that would either reduce the uncertainty in *m* by a factor of 10 or equipment that would reduce the uncertainty in *k* by a factor of 10, but not both. Assuming both pieces of equipment are equally priced, how would you proceed? Why?

Exercises on Uncertainty Analysis

Industrial Engineers working on Quality Control usually use what is called the "6 sigma" control limit when examining data, where sigma is the standard deviation of the data. This control limit refers to the upper and lower bounds between which a data point is "in control." What is meant by " standard deviation?" Why do these engineers use six times the standard deviation to set their control range?

Suppose you are to calculate the power dissipated across the resistor in a circuit. The manufacturer of the resistor states that the resistor is rated at 5.0 (+/- 0.1) ohm. The current is measured with a calibrated ammeter which the manufacturer has rated +/- 0.02 A throughout the full scale. The ammeter consistently reads 2.0 A across the resistor. Based upon the given information, what is the dissipated power and the associated uncertainty of this value

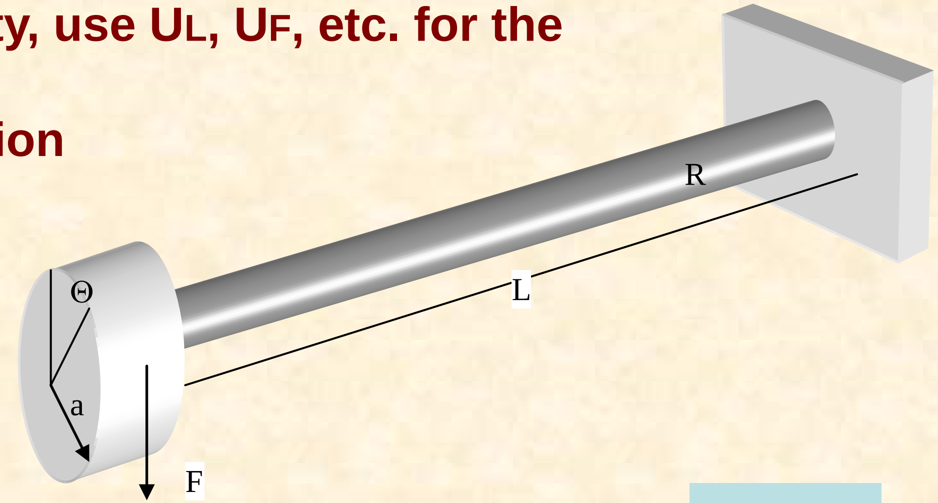


Exercise on Uncertainty Analysis

You're making big bucks working as an engineer at SlapOut, Inc and are asked to experimentally determining the shear modulus *G_s* of an alloy rod, where *L*=length of rod, *R*=radius of rod, *T*=applied torque = Force x moment arm, *a*=radius of disk, and *Θ*=angle of deflection. A co-worker, who happens to be a T-sipper, suggests that you buy a new force transducer because the one you were going to use has an uncertainty of 1%. The new transducer, being much more expensive, would have 0.01% uncertainty. ?

- **Solve for the data reduction equation, including any substitutions**
- **Write the general expression for uncertainty**
- **Determine all partial derivatives**
- **Write the new expression for uncertainty, use U_L, U_F, etc. for the uncertainties**
- **Comment on your co-worker's suggestion**

$$\Theta = \frac{2\,L\,T}{\pi\,R^4\,G_S}$$



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