S&V measurements

Notes 7: Rotor Imbalance Response and Balancing of a Rigid Rotor (one plane)

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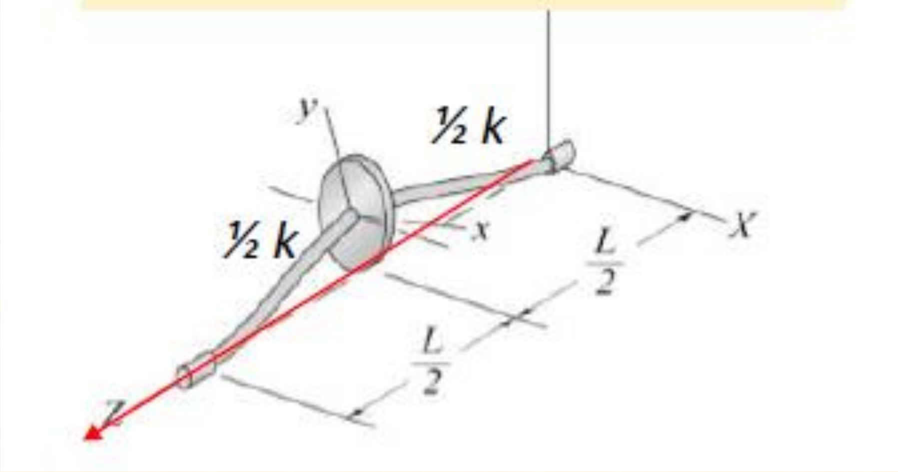

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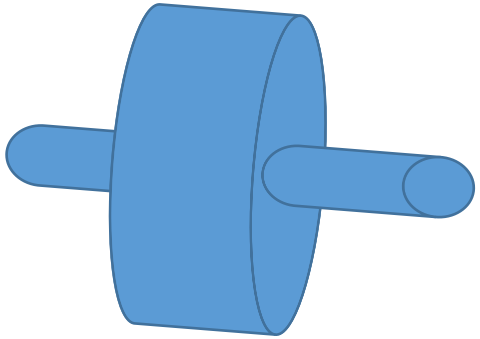
http://rotorlab.tamu.edu/me459/default.htm

Notes 7. Rotor Imbalance Response and Balancing of a Rigid Rotor (one plane)

Application of Vibration Measurements

- Response (amplitude and phase) of a simple rotor-bearing system to mass imbalance.
- A method for balancing a rigid rotor in one plane.
- Limits for rotor vibration.



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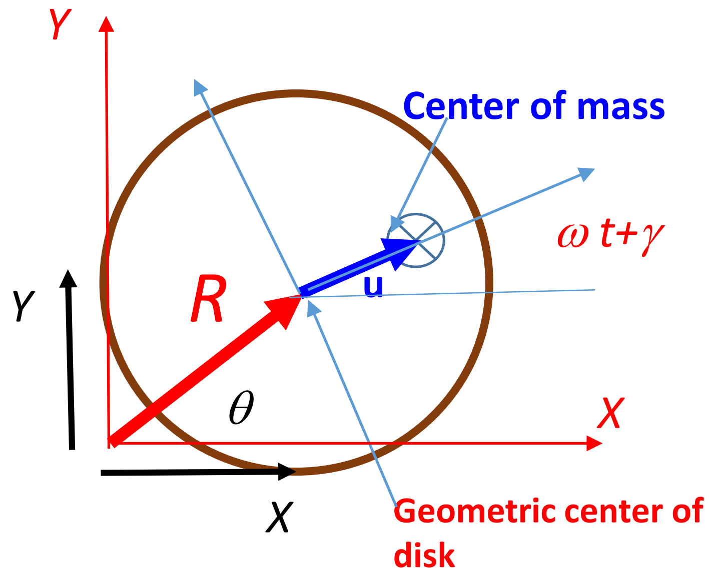
Mast-Childs Chair Professor

ME459/659 S&V Measurements

Please watch
https://www.youtube.com/watch?v=R2hO--TIjjA

Simplest rotor models (Flöpp-Jeffcott)

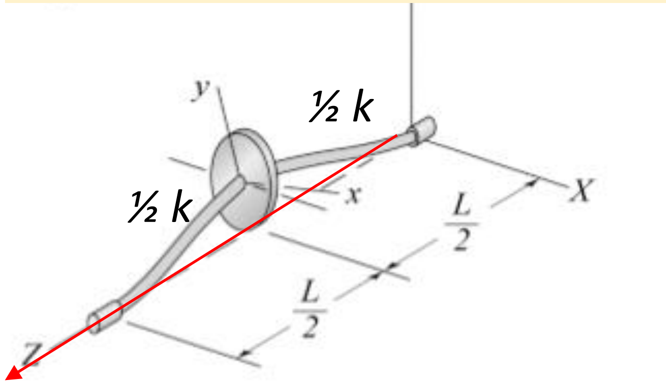
Disk and shaft rotate with constant angular speed ω



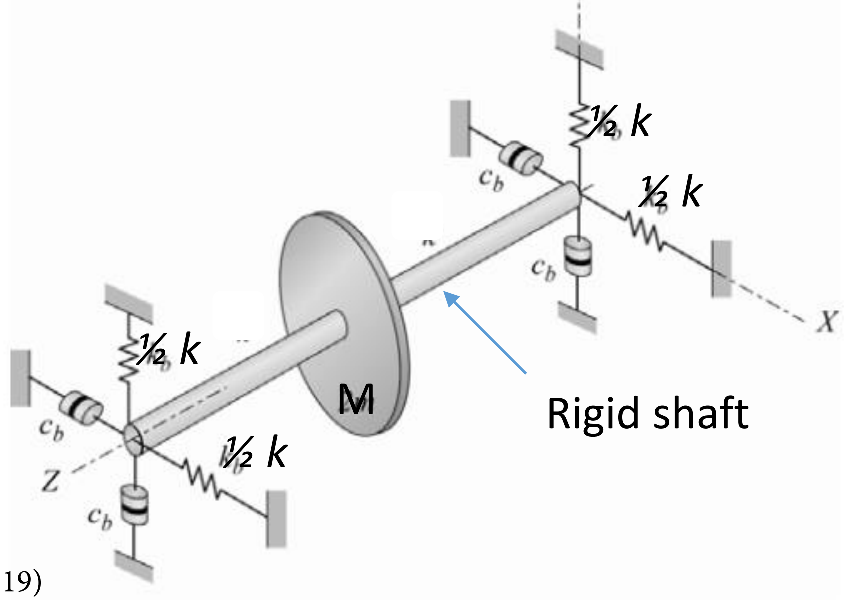
$$R = |\rho|\, u$$

$$Z = X + i \cdot Y = R \cdot e^{i \cdot \theta} \qquad (1)$$

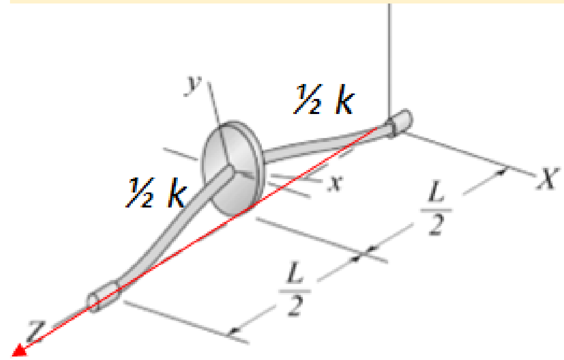
(a) Flexible rotor on rigid supports



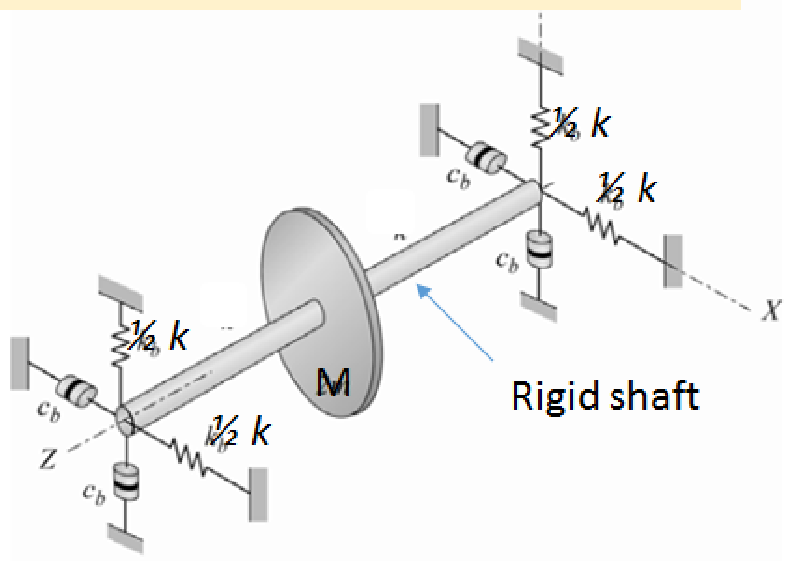
(a) Rigid rotor on flexible supports (bearings)



(a) Flexible rotor on rigid supports



(a) Rigid rotor on flexible supports (bearings)



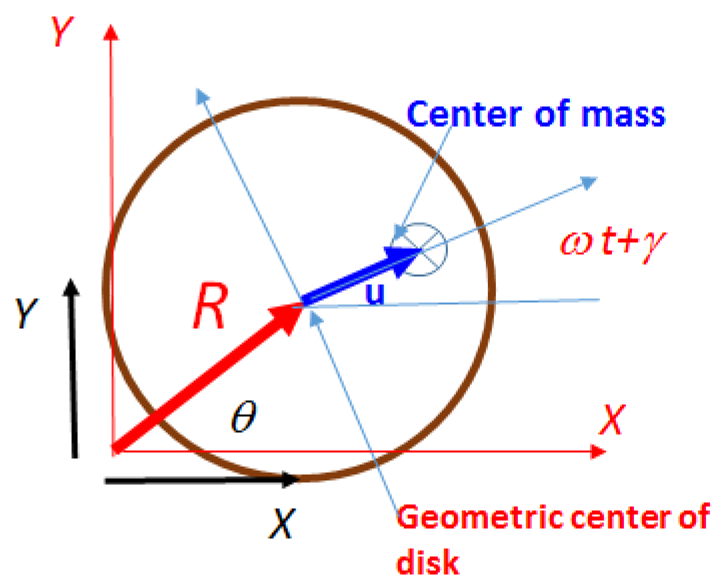
Nomenclature

M; disk mass

1/2 K: shaft stiffness (a),
support stiffness (b)

C: damping coefficient

IMBALANCE RESPONSE and Balancing of Rotors



$$R = |\rho|\, u$$

$$Z = X + i \cdot Y = R \cdot e^{i \cdot \theta} \qquad (1)$$

u: imbalance (cg offset)

Position of disk cg:

$$X_g = X + u \cdot \cos(\omega \cdot t + \gamma)$$

$$Y_g = Y + u \cdot \sin(\omega \cdot t + \gamma)$$

X, Y are coordinates of disk center

Define $M := 100 \cdot kg$ $K := 3.948 \cdot 10^{7} \cdot \frac{N}{m}$

$$\omega_n := \left(\frac{K}{M}\right)^{.5} = 628.331 \cdot \frac{rad}{s}$$ natural frequency

Set damping ratios:

$$f_n := \frac{\omega_n}{2 \cdot \pi} = 100.002 \cdot Hz$$

$\zeta_1 := 0.05$ $\zeta_2 := 2 \cdot \zeta_1$ $\zeta_3 := 4 \cdot \zeta_1$

$$D_1 := \zeta_1 \cdot 2 \cdot \sqrt{K \cdot M} = 6.283 \times 10^{3} \cdot N \cdot \frac{s}{m}$$

Imbalance $em := 0.001 \cdot kg \cdot m$

and cg offset $u := \frac{em}{M} = 1 \times 10^{-5} m$

$$RPM := \frac{1}{60} \cdot Hz$$

Let $Diam_{rot} := 0.20 \cdot m$

$$f_n = 6 \times 10^{3} \cdot RPM$$

actual mass imbalance
as if located at outer radius

$$m_u := \frac{em}{\frac{1}{2} \cdot Diam_{rot}} = 0.01 \, kg$$

EQUATION of MOTION for JEFFTCOT ROTOR:

$$M\cdot\frac{d^2}{dt^2}X + D\cdot\frac{d}{dt}X + K\cdot X = M\cdot u\cdot\omega^2\cdot\cos(\omega\cdot t + \gamma) \qquad \text{(a)}$$

$$M\cdot\frac{d^2}{dt^2}Y + D\cdot\frac{d}{dt}Y + K\cdot Y = M\cdot u\cdot\omega^2\cdot\sin(\omega\cdot t + \gamma) \qquad \text{(b)}$$

Define a complex vector $Z = X + i\cdot Y = R\cdot e^{i\cdot\theta}$ (1) $\quad i = \sqrt{-1}$

and add the two equations, (a)+ i(b), to obtain

$$M\cdot\frac{d^2}{dt^2}Z + D\cdot\frac{d}{dt}Z + K\cdot Z = M\cdot u\cdot\omega^2\cdot(\cos(\omega\cdot t + \gamma) + i\cdot\sin(\omega\cdot t + \gamma))$$

or Using Euler's identity

$$M\cdot\frac{d^2}{dt^2}Z + D\cdot\frac{d}{dt}Z + K\cdot Z = M\cdot u\cdot\omega^2\cdot e^{i\cdot(\omega\cdot t+\gamma)} \qquad (2)$$

Let $Z(t) = \rho\cdot u\cdot e^{i\cdot(\omega\cdot t+\gamma)}$ (3) be the solution, where $R = \rho\cdot u$, $\rho = A\cdot e^{-i\varphi}$ is the amplitude ratio and phase angle

$$Z = A\cdot u\cdot e^{i\cdot(\omega\cdot t+\gamma-\varphi)} = X + i\cdot Y$$

Eq(3) into Eq(2) gives

$$\left(K - M\cdot\omega^2 + i\cdot\omega\cdot D\right)\rho\cdot u = M\cdot u\cdot\omega^2$$

or

$$\left(\frac{K}{M} - \omega^2 + i\cdot\omega\cdot\frac{D}{M}\right)\rho = \omega^2$$

Then
the amplitude ratio

$$\rho = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\left[\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right] + i\cdot\frac{\omega\cdot D}{K}\right]} \quad (4)$$

Define
NATURAL frequency:

$$\omega_n = \sqrt{\frac{K}{M}}$$

DAMPING ratio

$$\zeta = \frac{D}{2\cdot\sqrt{K\cdot M}}$$

Define operating
frequency ratio:

$$r = \frac{\omega}{\omega_n}$$

$$\rho = \frac{r^2}{\left[\left(1 - r^2\right) + i\cdot 2\cdot\zeta\cdot r\right]} = A\cdot e^{-i\varphi} \quad (5)$$

$$R = \rho\cdot u = u\cdot\left(A\cdot e^{-i\varphi}\right)$$

Extract the **amplitude of rotor response A (with respect to u)** and **phase angle** φ as

$$A = \frac{r^2}{\sqrt{\left[\left(1 - r^2\right)^2 + (2\cdot\zeta\cdot r)^2\right]}} \quad (6)$$

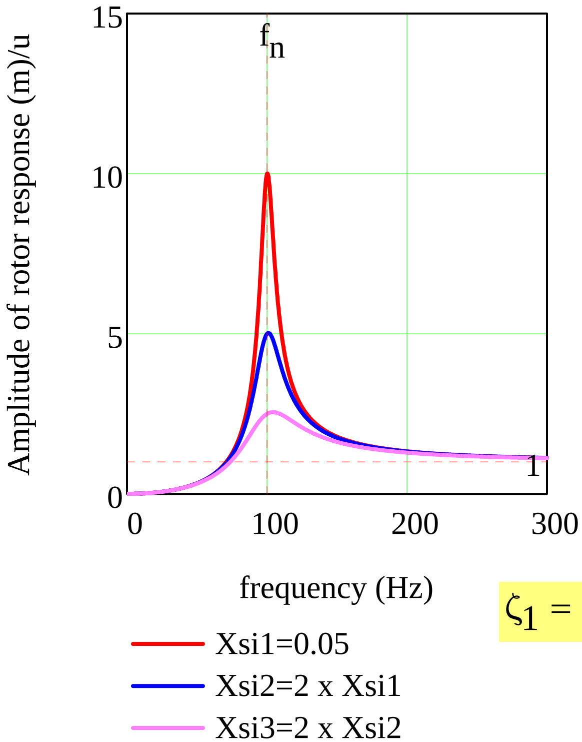
$$\tan(\varphi) = \frac{2\cdot\zeta\cdot r}{1 - r^2} \quad (7)$$

$$A(r,\zeta) := \frac{r^2}{\sqrt{\left[\left(1-r^2\right)^2 + (2\cdot\zeta\cdot r)^2\right]}}$$

$$\varphi(r,\zeta) := \left| \begin{array}{l} \varphi \leftarrow \operatorname{atan}\left(\dfrac{2\cdot\zeta\cdot r}{1-r^2}\right)\cdot\left(\dfrac{180}{\pi}\right) \\ \varphi \leftarrow \varphi + 180 \quad \text{if } r > 1 \\ \varphi \end{array} \right.$$

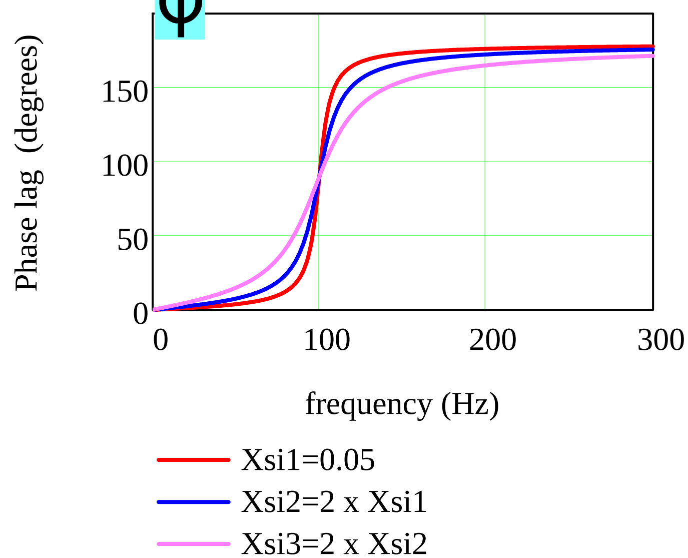
in degrees

A=|R/u|



$\zeta_1 = 0.05$

φ



Amplitude/u and phase of rotor imbalance response vs shaft speed (Hz)

Rotor response in fixed coordinates is

$$Z(t) = X + i \cdot Y = R \cdot e^{i \cdot (\omega \cdot t + \gamma)} = A \cdot u \cdot e^{i \cdot (\omega \cdot t + \gamma - \varphi)}$$

$$X(t) = A \cdot u \cdot \cos(\omega \cdot t + \gamma - \varphi)$$

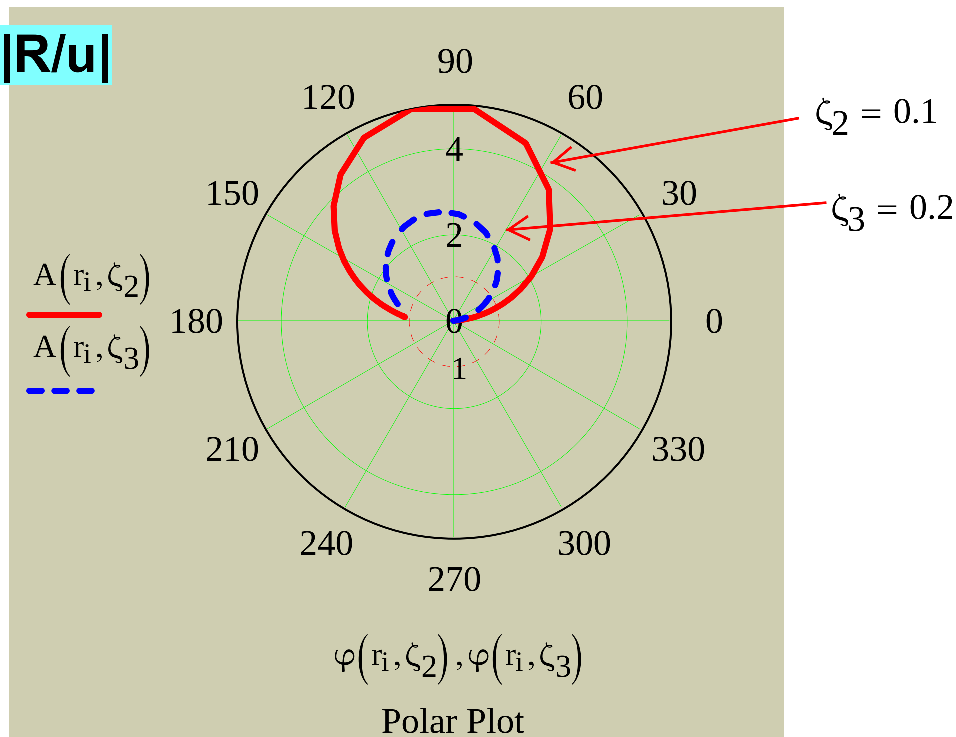
$$Y(t) = A \cdot u \cdot \sin(\omega \cdot t + \gamma - \varphi)$$

Polar plot: rotor amplitude vs phase angle

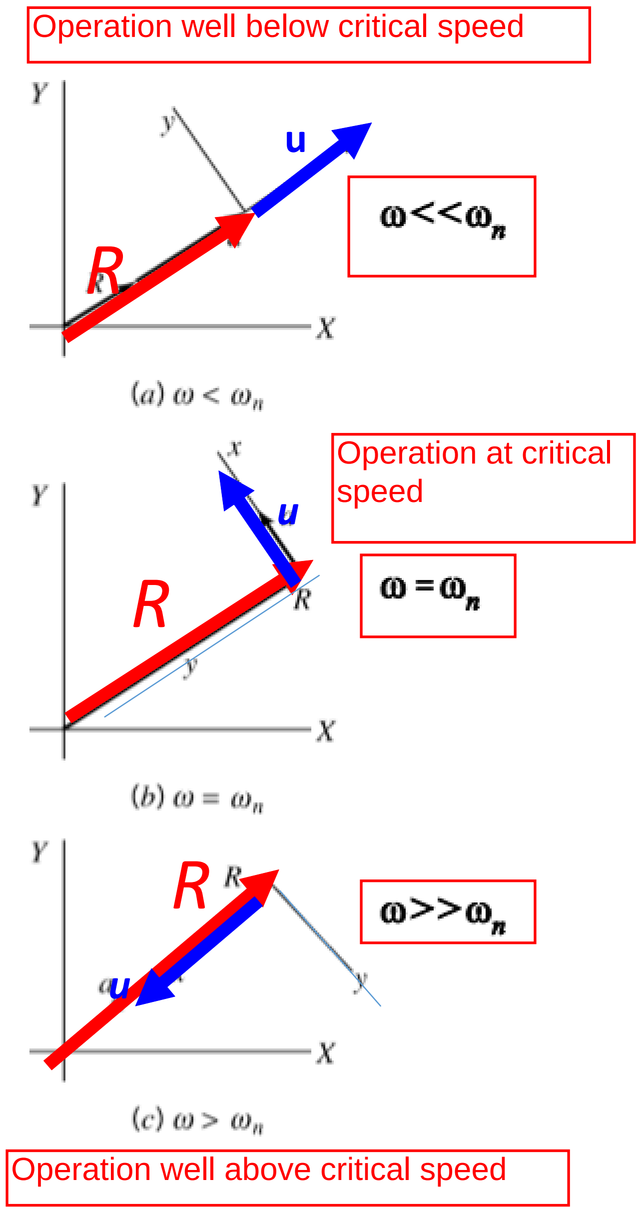
Shows path or locus of shaft center as it moves from low to high speed

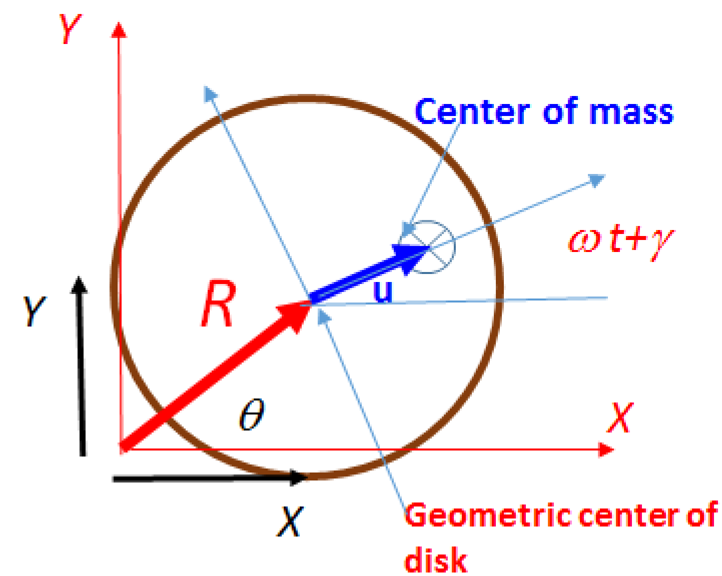
$$R = \rho \cdot u$$

$$A=|R/u|$$



Schematic views of rotor center and imbalance position at low and high shaft speeds (below and above) critical speed





watch

https://m.youtube.com/watch?v=h93Yn1ZoRjw

STEPS for rotor balancing - single plane & constant speed

unknown TBD (u_1, γ_1) >>> $U_1 = u_1 \cdot e^{i \cdot \gamma 1}$

In 100% cases, one does not know the state of rotor balancing

STEP 1: Measure ROTOR vibration at operating speed (< critical speed)

$Z_1 = B_1 \cdot e^{i \cdot \theta_1}$

imbalance U1 produces response(measured) : amplitude and phase

$Z_1 = \rho_\omega \cdot U_1$

STOP rotor and

STEP 2: ADD TRIAL weight or imbalance $(u_T, \gamma T)$ >>> $U_T = u_T \cdot e^{i \cdot \gamma_T}$

unknown ρ_ω

STEP 3: Measure ROTOR vibration at operating speed (< critical speed)

$Z_2 = B_2 \cdot e^{i \cdot \theta_2}$

trial imbalance produces response(measured) : amplitude and phase

which contains the effect of both the unknown imbalance U1 and the trial mass

$Z_2 = \rho_\omega \cdot U_2$

STEP 4: Stop rotor and determine influence cofficient CI

Subtract 2nd vibration vector from the first

$Z_{21} = Z_2 - Z_1$ $= \rho_\omega \cdot U_T$

Z21 is the amplitude and phase of rotor vibration due to trial imbalance only

find $C = \frac{Z_{21}}{U_T} = \rho_\omega$

is the influence coefficient determined from the measurements of Z2 and Z1 and the trial mass

Hence, the residual imbalance is

$U_1 = \frac{Z_1}{C_I} = u_1 \cdot e^{i \cdot \gamma 1}$

To balance, remove u1 at γ1 or add a counter-mass u1 at (γ1+180)

C = ρ is a vector containg the amplitude & angle $C \cdot e^{-i\varphi}$

README ME and KEEP ME
(tutorial available in Resources)

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ROTOR BALANCING TUTORIAL

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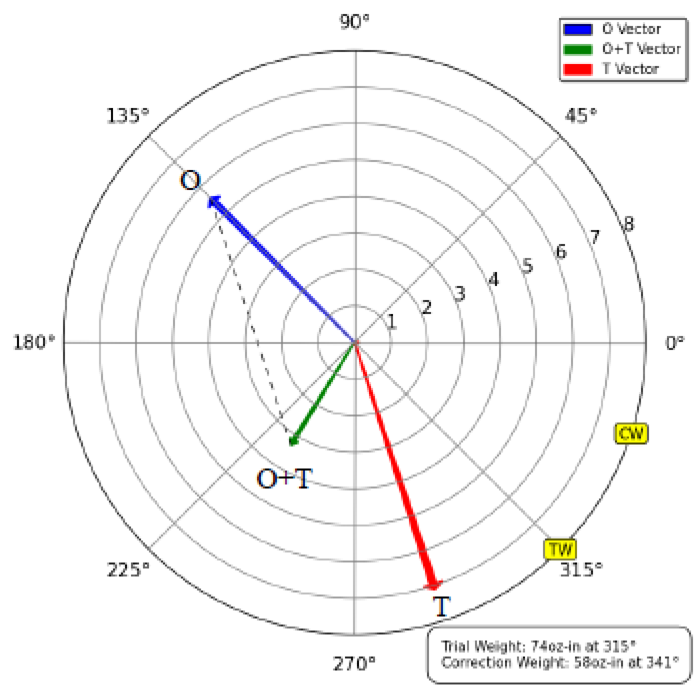


1 *Figure 4 - Single Plane Balance Vector Diagram*

Example

GRAPHICAL - VECTOR DIAGRAM for SINGLE Plane BALANCING

ROTOR BALANCING - ONE PLANE

Example from Pavelek

Conduct measurement of amplitude & phase of rotor vibration (with respect to a keyhasor) at a constant speed

STEP 0: RUNOUT vibration

$A_0 := 0 \cdot mil$ $\quad \varphi_o := 300 \cdot deg$ >>> $V_0 := A_0 \cdot e^{i \cdot \varphi_o}$

measurement NOT conducted at low rotor speed (SLOW ROLL)

STEP 1: Measure RESIDUAL vibration at operating speed (< critical speed)

Turn on rotor and bring it to rotor speed

$A_1 := 5.6 \cdot mil$ $\quad \varphi_1 := 135 \cdot deg$ >>>> $V_1 := A_1 \cdot e^{i \cdot \varphi_1}$

Subtract slow-roll from residual vibration

$V_{10} := V_1 - V_0$

This is vibration due to imbalance only

$|V_{10}| = 5.6 \cdot mil$

$\arg(V_{10}) = 135 \cdot deg$

STOP rotor!

STEP 3: ADD TRIAL weight

$m_{trial} := 74 \cdot oz \cdot in$ $\quad \varphi_m := 315 \cdot deg$

$m_T := m_{trial} \cdot e^{i \cdot \varphi_m}$

STEP 4: Record Vibration due to trial weight (and residual imbalance)

Turn on rotor and bring it to speed

$A_2 := 3.3 \cdot mil$ $\quad \varphi_2 := 238 \cdot deg$ >>>> $V_2 := A_2 \cdot e^{i \cdot \varphi_2}$

Subtract slow-roll from (trial weight +residual vibration)

$V_{20} := V_2 - V_0$

$|V_{20}| = 3.3 \cdot mil$

STOP rotor and remove trial mass!

This is vibration due to (original imbalance+trial mass) :

$$\arg(V_{20}) = -122 \cdot deg$$

STEP 5: Find influence coefficient

$$V_{21} := V_2 - V_1$$

$$|V_{21}| = 7.111 \cdot mil$$

$$\arg(V_{21}) = -71.884 \cdot deg$$

V21 is vibration vector containing influence of trial weight ONLY

Influence coefficient

$$C_I := \frac{V_{21}}{m_T}$$

$$|C_I| = 9.609 \times 10^{-5} \cdot \frac{1}{oz} \qquad \arg(C_I) = -26.884 \cdot deg$$

STEP 6: Find correction mass and its phase

Note negative SIGN as correction mass is to be placed opposite to the residual imbalance mass.

$$m_{correct} := \frac{-V_{10}}{C_I}$$

$$|m_{correct}| = 58.277 \cdot oz \cdot in \qquad \arg(m_{correct}) = -18.116 \cdot deg$$

add a correction mass at noted angle
OR
remove a mass at 180 deg away

ROTOR BALANCING - ONE PLANE

$deg := 1 \cdot \frac{\pi}{180}$ $\quad mil := 0.001 \cdot in$

Conduct measurement of amplitude & phase of rotor vibration (with respect to a keyhasor) at a constant speed

STEP 0: RUNOUT vibration $\quad A_0 := 0.95 \cdot mil \quad \varphi_o := 300 \cdot deg$ >>> $V_0 := A_0 \cdot e^{i \cdot \varphi_o}$

measurement conducted at low rotor speed (**SLOW ROLL**)

STEP 1: Spin rotor **to desired speed and measure RESIDUAL vibration at operating speed (< critical speed)**

Measure vibration $\quad A_1 := 2.7 \cdot mil \quad \varphi_1 := 348.3 \cdot deg$ >>>> $V_1 := A_1 \cdot e^{i \cdot \varphi_1}$

Subtract slow-roll from residual vibration

$$V_{10} := V_1 - V_0$$

This is vibration due to imbalance : $\quad |V_{10}| = 2.186 \cdot mil$

$$\arg(V_{10}) = 7.231 \cdot deg$$

STOP rotor!

STEP 2: ADD TRIAL weight $\quad m_{trial} := 0.001 \cdot kg \cdot m \quad \varphi_m := 225 \cdot deg$ from P-mark

$$m_T := m_{trial} \cdot e^{i \cdot \varphi_m}$$

STEP 3: Record Vibration due to trial weight (and residual imbalance) $\quad$ **Turn on** rotor and bring it to same operating speed

$A_2 := 3.05 \cdot mil \quad \varphi_2 := 317 \cdot deg$ >>>> $V_2 := A_2 \cdot e^{i \cdot \varphi_2}$

STOP rotor and remove trial mass!

Subtract slow-roll from (trial weight +residual vibration)

$$V_{20} := V_2 - V_0$$

This is vibration due to (original imbalance+trial mass) :

$$\left|V_{20}\right| = 2.159 \cdot mil$$

$$\arg\left(V_{20}\right) = -35.61 \cdot deg$$

STEP 5: Find influence coefficient

Subtract both vectors

$$V_{21} := V_2 - V_1$$

$$\left|V_{21}\right| = 1.587 \cdot mil$$

$$\arg\left(V_{21}\right) = -105.091 \cdot deg$$

V21 is vibration vector containing influence of trial weight ONLY

Influence coefficient

$$C_I := \frac{V_{21}}{m_T} = \left(9.908 \times 10^{-4} + 5.699i \times 10^{-4}\right) \cdot \frac{1}{oz}$$

$$\left|C_I\right| = 1.143 \times 10^{-3} \cdot \frac{1}{oz} \qquad \arg\left(C_I\right) = 29.909 \cdot deg$$

STEP 6: Find correction mass and its phase

Note negative SIGN as correction mass is to be placed opposite to the residual imbalance mass.

$$m_{correct} := \frac{-V_{10}}{C_I}$$

$$\left|m_{correct}\right| = 1.377 \times 10^{-3} m \cdot kg \qquad \arg\left(m_{correct}\right) = 157.323 \cdot deg$$

locate balancing weigh at same radius as trial mass was added

VIBRATION limits

API STD 617 - Axial and Cent Comp and Expanders for OG industry

$em = 1 \times 10^{-3} m \cdot kg$

Maximum peak to peak shaft displacement relative to stator at bearing locations

$u := \frac{em}{M} = 10\,um$

$A_{pp\_max}(kRPM) := 25 \cdot \left(\frac{12}{kRPM}\right)^{.5} \cdot um$

Shaft speed must be 20% less than any natural frequency and 15% above any natural frequency.

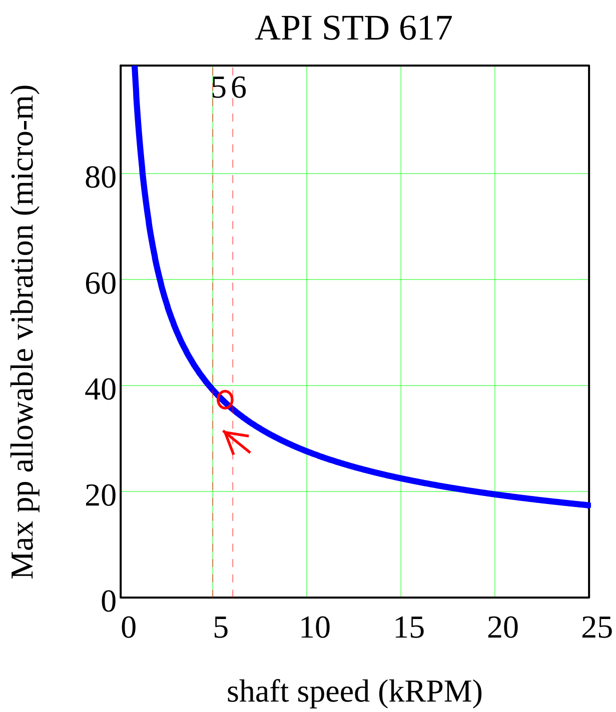
Let

$kRPM\_ := 5.4$

$f_n = 6 \times 10^3 \cdot RPM$

$f := kRPM\_ \cdot \frac{1000}{60} \cdot Hz$

$r := \frac{f}{f_n} = 0.9$



$A(r,\zeta_1) \cdot u = 38.521\,um$ for $\zeta_1 = 0.05$

$A_{pp\_max}(kRPM\_) = 37.268 \cdot um$

$A(r,\zeta_3) \cdot u = 19.897\,um$ for $\zeta_3 = 0.2$

$M \cdot u \cdot \omega_n^2 = 394.8\,N$ $\quad M \cdot g = 980.665\,N$

ratio of centrifugal load/weight $\frac{u \cdot \omega_n^2}{g} = 0.403$

MIN damping ratio if operating speed = critical speed

$$Q_{min} := \frac{A_{pp\_max}(kRPM\_)}{u} = 3.727$$

$$\zeta_{min} := \frac{1}{2 \cdot Q_{min}} = 0.134$$

ISO 1940-1973(E) **for process equipment :**
max speed = imbalance_cg_offset x frequency < 2.5 mm/s (~100 mil/s)

$$Max\_vibration\_speed := 2.5 \cdot \frac{mm}{s}$$

=0.1 inch/s

$$Max\_vibration\_speed = 0.098 \cdot \frac{in}{s}$$

Let $kRPM\_ = 5.4$

$$\omega := kRPM\_ \cdot 1000 \cdot \frac{\pi}{30 \cdot s} = 565.487 \frac{1}{s}$$

speed in rad/s

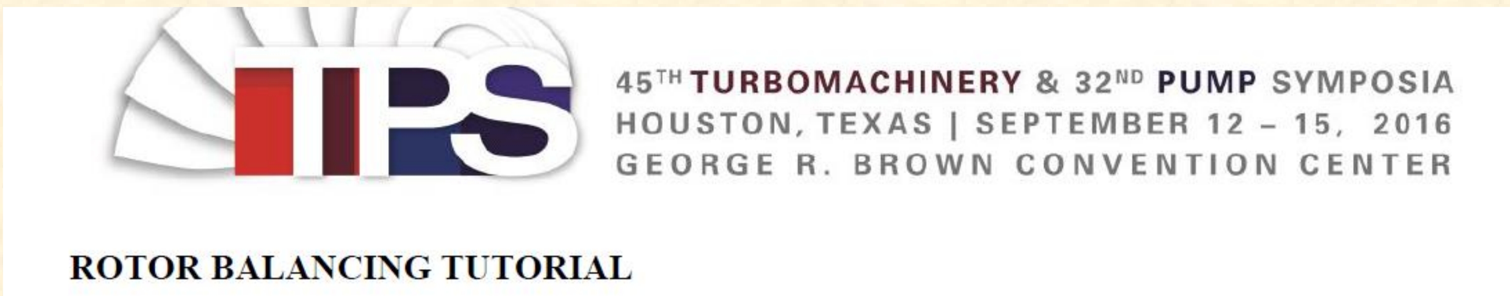
$$u_{max} := \frac{Max\_vibration\_speed}{\omega} = 4.421 \cdot um$$

$u = 10 um$

centrifugal force is $M \cdot u_{max} \cdot \omega^2 = 141.372 N$

$$\frac{u_{max} \cdot \omega^2}{g} = 0.144$$

See ISO & API standards for balancing and program for multiple plane – multiple speed balancing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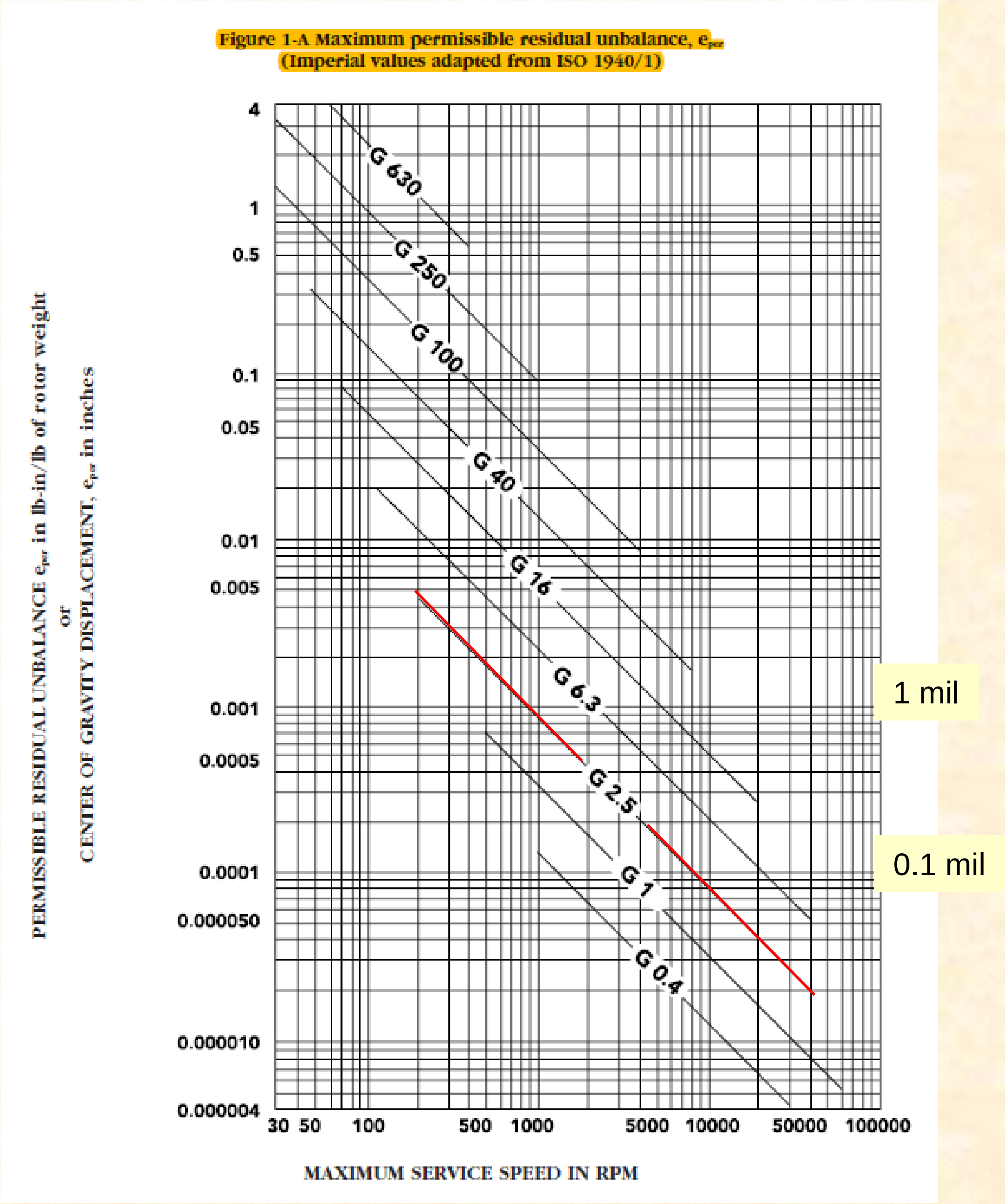
Texas A&M University
2016

ISO 1940 balancing table

Balance Quality Grade	Product of the Relationship ($e_{per} \times \omega$) [1] [2] mm/s	Rotor Types - General Examples
G 6.3	6.3	Parts of process plant machines Marine main turbine gears (merchant service) Centrifuge drums Paper machinery rolls; print rolls Fans Assembled aircraft gas turbine rotors Flywheels Pump impellers Machine-tool and general machinery parts Medium and large electric armatures (of electric motors having at least 80 mm shaft height) without special requirements Small electric armatures, often mass produced, in vibration insensitive applications and/or with vibration-isolating mountings Individual components of engines under special requirements
G 2.5	2.5	Gas and steam turbines, including marine main turbines (merchant service) Rigid turbo-generator rotors Computer memory drums and discs Turbo-compressors Machine-tool drives Medium and large electric armatures with special requirements Small electric armatures not qualifying for one or both of the conditions specified for small electric armatures of balance quality grade G 6.3 Turbine-driven pumps
G 1	1	Tape recorder and phonograph (gramophone) drives Grinding-machine drives Small electric armatures with special requirements
G 0.4	0.4	Spindles, discs and armatures of precision grinders Gyroscopes

ISO 1940 permissible balance



ISO 1940 permissible balance

DETERMINING PERMISSIBLE RESIDUAL UNBALANCE - U_{per}

$$U_{per} = e_{per} \times m$$

(m = rotor mass)

Permissible residual unbalance is a function of G number, rotor weight and maximum service speed of rotation. Instead of using the graph to look up the "specific unbalance" value for a given G number and service RPM and then multiplying by rotor weight (taking care to use proper units), U_{per} can be calculated by using one of the following formulae:

U_{per} (oz-in) = 6.015 x G x W/N (W in lb)

U_{per} (g-in) = 170.5 x G x W/N (W in lb)

U_{per} (g-mm) = 9549 x G x W/N (W in kg)

G = Balance quality grade from Table 1

W = Rotor weight

N = Maximum service RPM

compare

API 610 based on the following formula (using SI units):

T = 6350W/N

- W = rotor weight in kg
- n = speed in RPM
- T = tolerance in kg