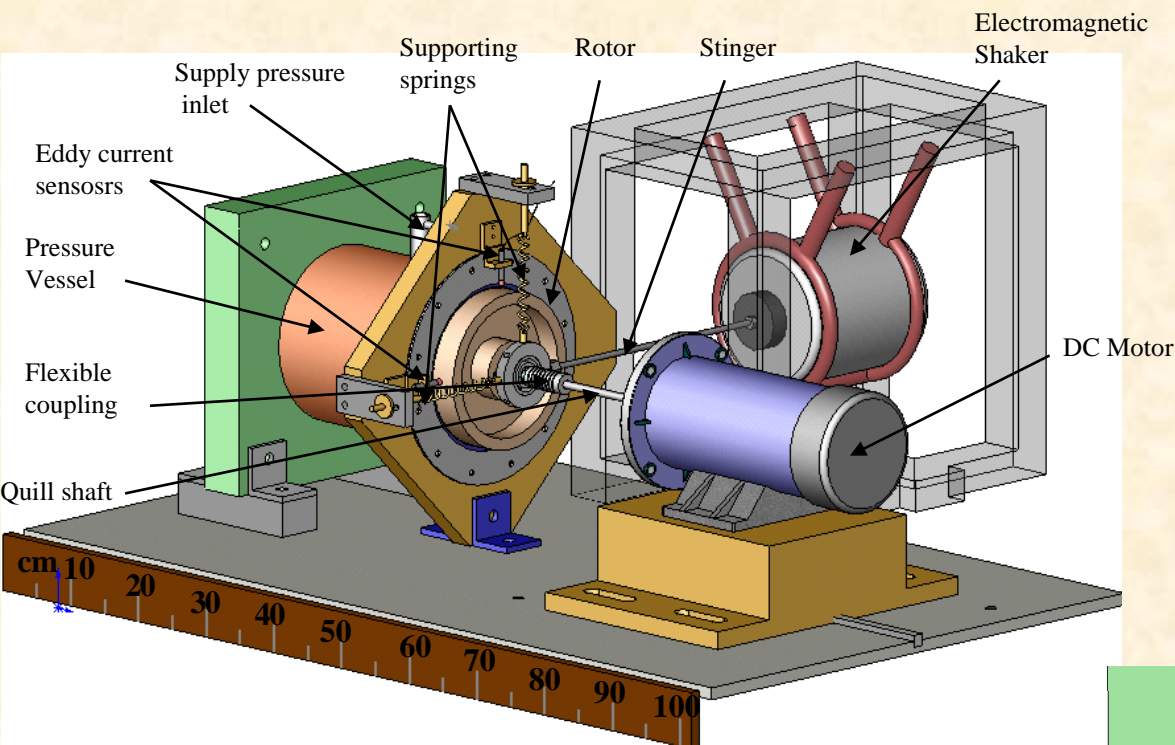


# **A 2<sup>nd</sup> example of system parameter identification (Hybrid Brush Seal)**

**Luis San Andrés (lecturer)**

**Thanks to Adolfo Delgado, José Baker & the  
support from Siemens Power Generation**

# Experimental Facility



**Test Rig: Rotordynamic Configuration**

## Structural parameters

$$K_{\text{shaft}} = 243 \text{ lbf/in (42.5 kN/m)}$$

$$M_{\text{s+d}} = 9.8 \text{ lb (4.45 kg)}$$

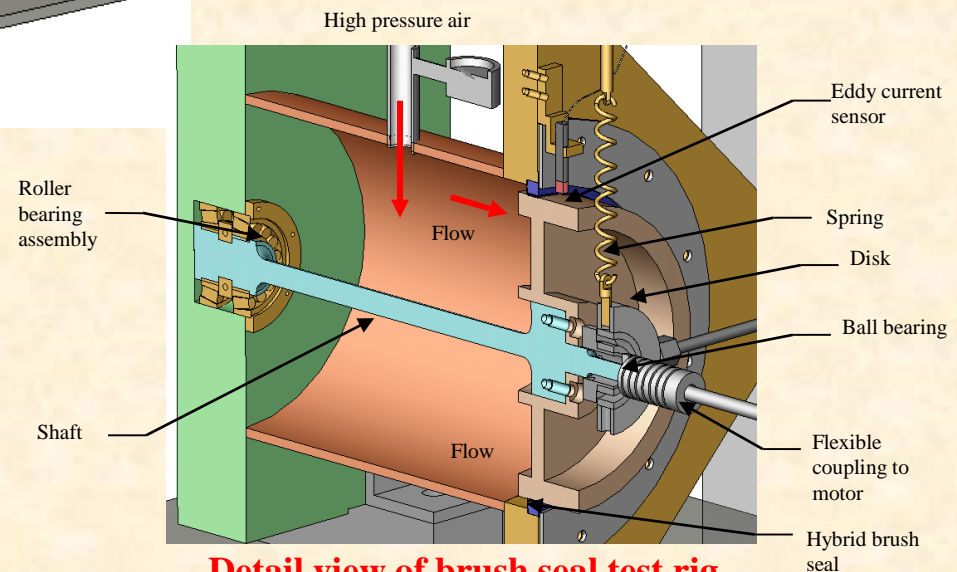
$$\zeta: 0.01 \% \text{ (damping ratio)}$$

## Installation:

6.550" diameter brush seal

Max. air Pressure: 60 psig

Shaker (20 lb max)



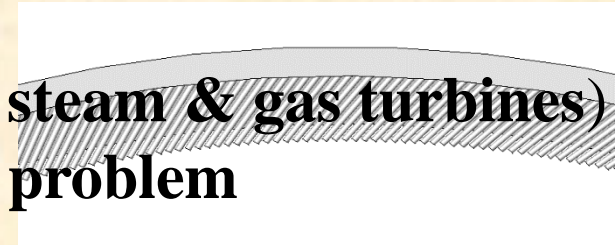
**Detail view of brush seal test rig**

# Brush Seals

Reduce secondary leakage in turbomachinery

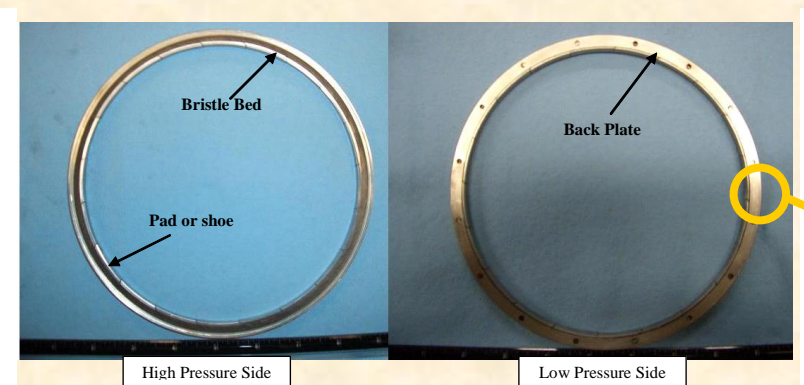
Replace labyrinth seals in HP TM (hot side of steam & gas turbines)

Wear and thermal distortions are a reliability problem



## Hybrid Brush Seals

Novel improvement over BS. Reduce more leakage and do not introduce wear or thermal distortion. Allow bi-directional rotation



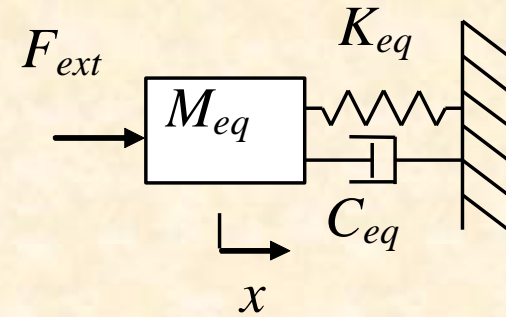
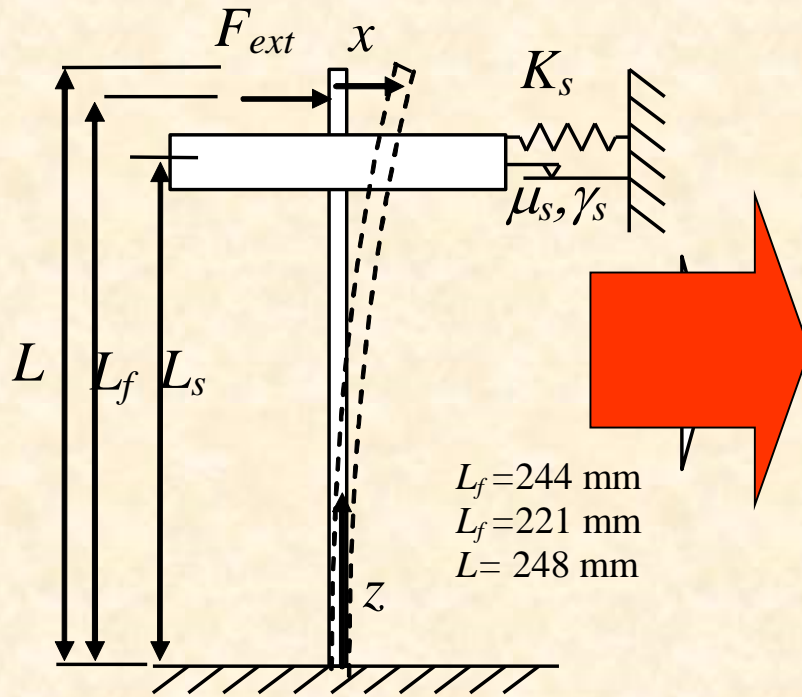
Courtesy of Advanced Turbomachinery Group®



\* Close-up courtesy of Advanced Technologies Group, Inc.

Spring Lever Mechanism

# Dynamic Load Tests (no shaft rotation)



**Equivalent Test System**

$$M_{eq} \ddot{x} + K_{eq} x + C_{eq} \dot{x} = F_{ext}$$

# Parameter Identification (no shaft rotation)

ASME DETC2005-84159

$$\bar{x} = x e^{i\omega t}$$

$$\bar{F} = F_{ext} e^{i\omega t}$$



**Harmonic force  
& displacements**

$$Z = \frac{\bar{F}}{\bar{x}} = (K_{eq} - \omega^2 M_{eq}) + i \omega C_{eq}$$



**Impedance Function**

$$W = \oint F_{ext} \dot{x} dt$$



**Work External**

$$E_{dis} = \pi \omega C_{eq} |\bar{x}|^2$$



**Viscous Dissipation**

$$E_{dis} = \gamma_{eq} \pi K_{eq} |\bar{x}|^2 + 4 \mu |\bar{F}| |\bar{x}|$$

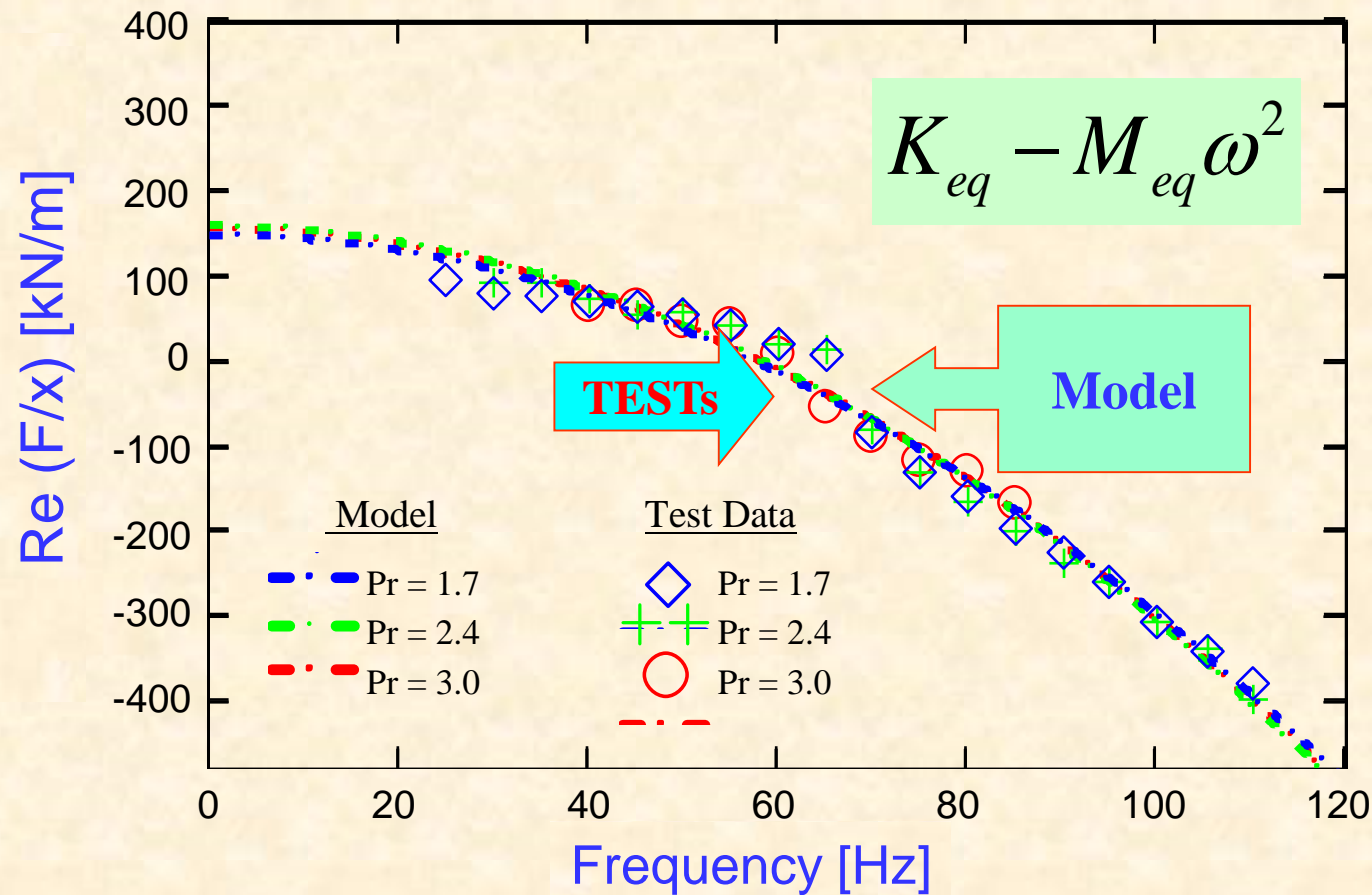


**DRY  
FRICTION &  
STRUCTURAL  
DAMPING**

# HBS Dynamic Stiffness vs. Frequency (no shaft rotation)

Load = 63 N, frequency: 20-100Hz

Pressure ratio ( $P_r = P_s/P_d$ ) = Discharge/Supply



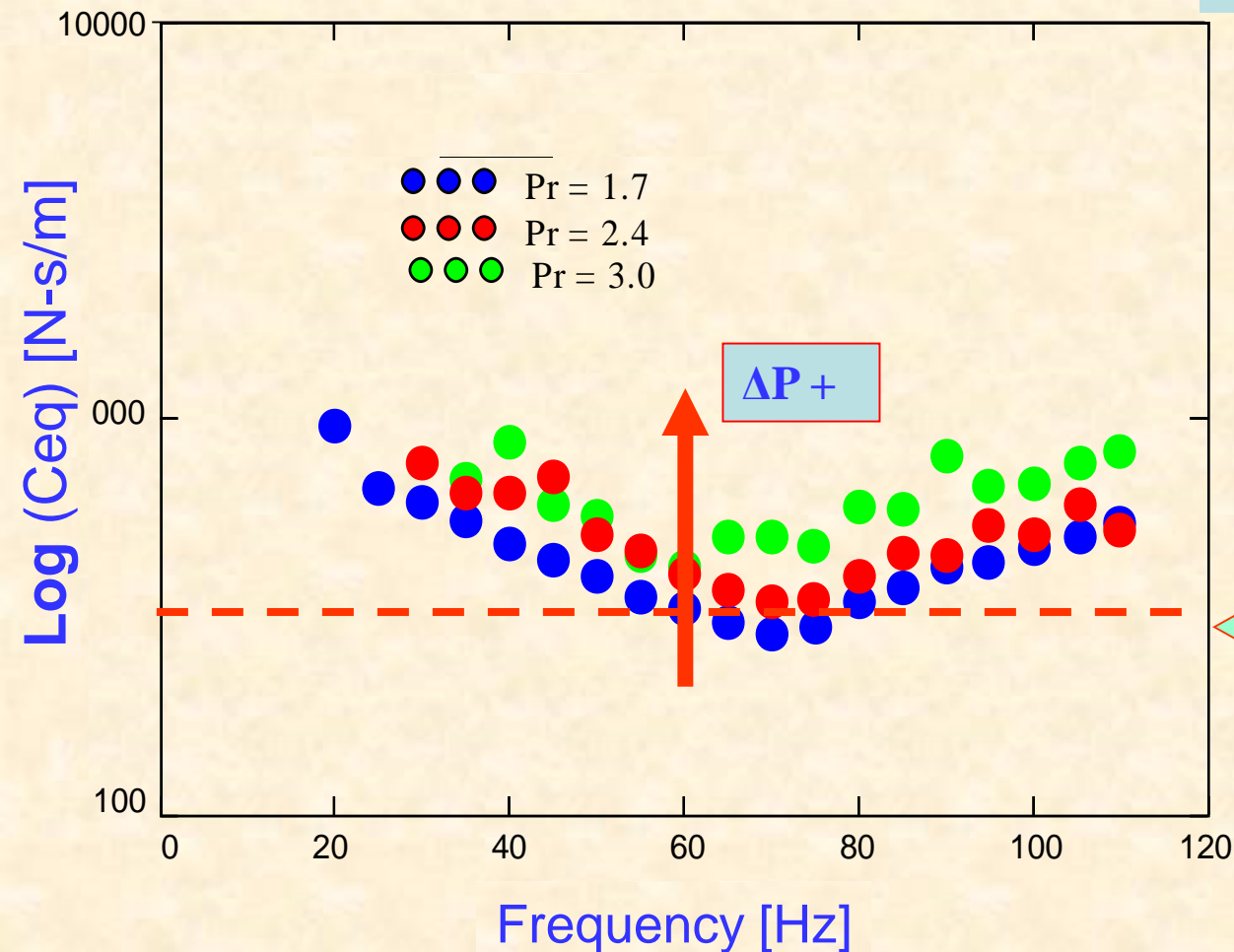
**Model reproduces  
real part of the  
impedance under  
the given supply  
pressure  
conditions.**

# HBS Equivalent Viscous Damping vs. Frequency (no shaft rotation)

Load = 63 N, frequency 20-110Hz

Pressure ratio ( $P_r = P_s/P_d$ ) = Discharge/Supply

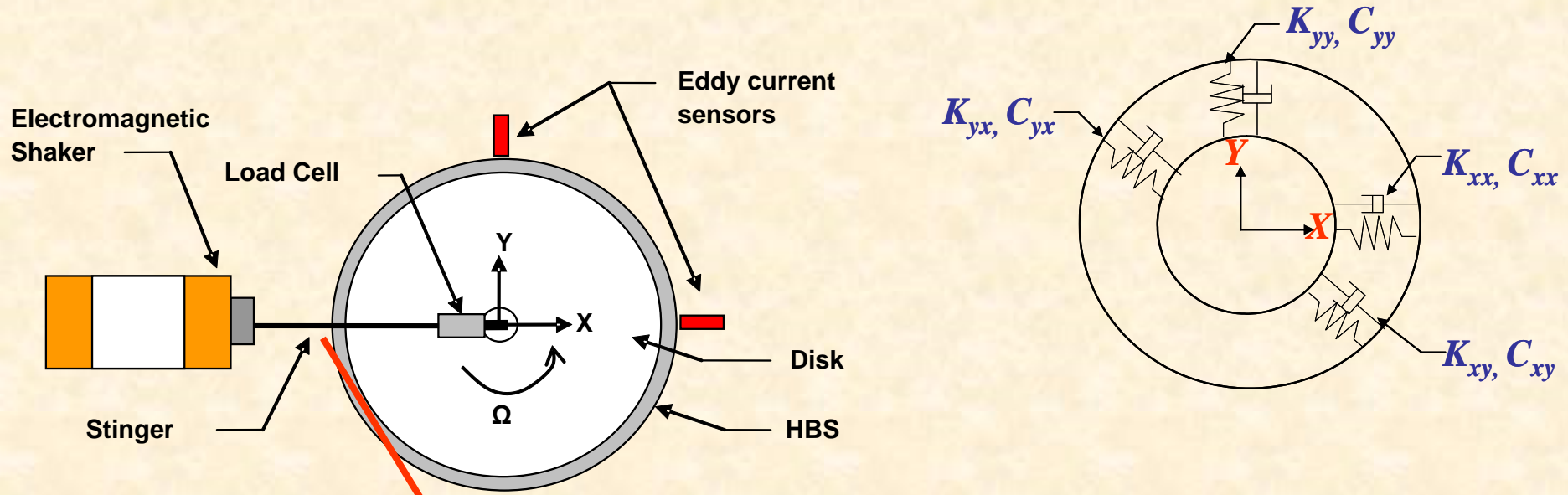
$$C_{eq} = \frac{\gamma_{eq} K_{eq}}{\omega} + \frac{4\mu|\bar{F}|}{\pi\omega|\bar{x}|}$$



Equivalent damping increases slightly with pressure differential. Results typical of a system with dry-friction & material damping energy dissipation

Viscous Model

# Identification of Rotordynamic Force Coefficients



Excitation force  
(frequency  $\omega$ )

$$\begin{bmatrix} M_{xx} & M_{xy} \\ M_{yx} & M_{yy} \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} + \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} = \begin{pmatrix} F_x \\ 0 \end{pmatrix} + \begin{pmatrix} F_{ix} \\ F_{iy} \end{pmatrix}$$

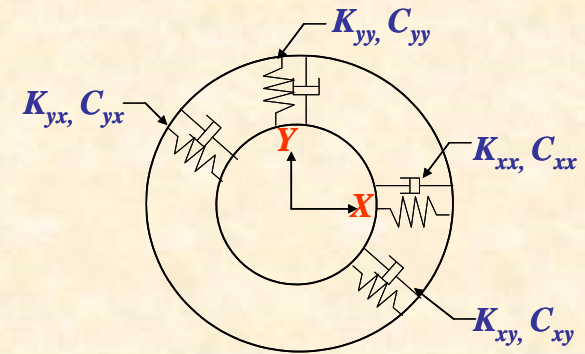
Imbalance forces ( $1X=\Omega$ )



# Identification of Rotordynamic Force Coefficients

For periodic force excitation:

$$\bar{F}_x = F_x e^{i\omega t} \quad \rightarrow \quad \bar{y} = ye^{i\omega t}$$
$$\bar{x} = xe^{i\omega t}$$



EOMS reduce to:

$$Z_{xx} \cdot \bar{x} + Z_{xy} \cdot \bar{y} = \bar{F}_x$$

$$Z_{yx} \cdot \bar{x} + Z_{yy} \cdot \bar{y} = 0$$

With impedances:

$$Z_{\alpha\beta} = \left\{ K_{\alpha\beta} - M_{\alpha\beta} \omega^2 + iC_{\alpha\beta} \omega \right\}, \alpha\beta=x,y$$

For centered operation  
(axi-symmetry)

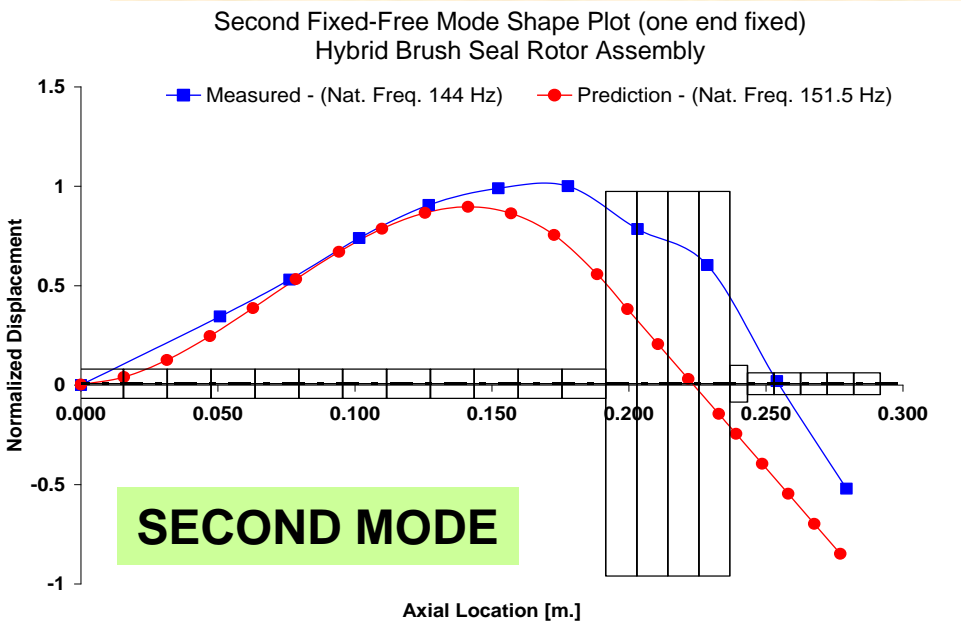
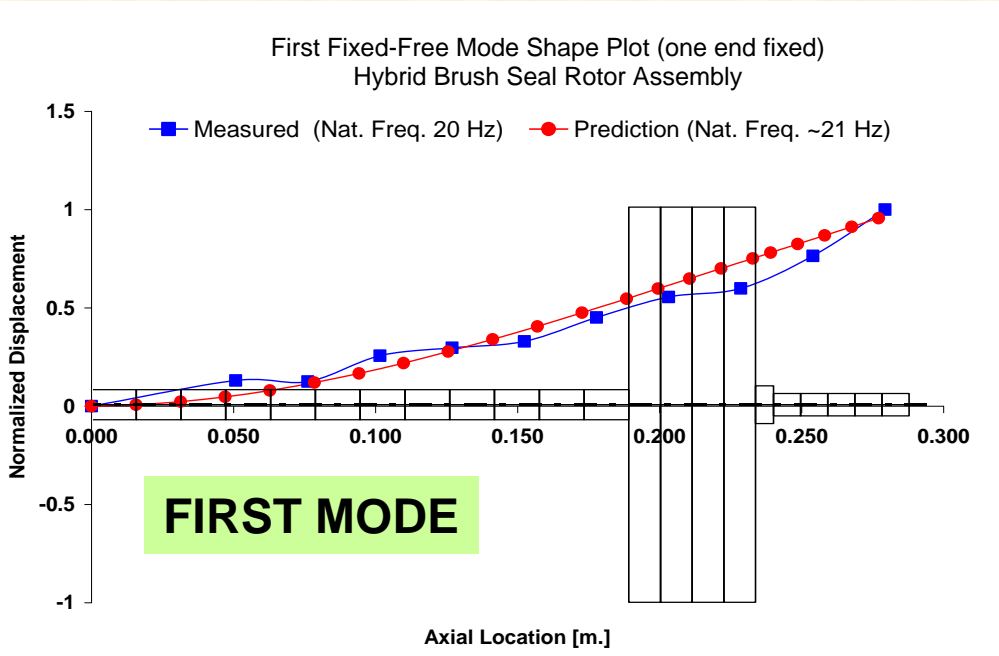
$$Z_{xx} = Z_{yy}$$

$$Z_{xy} = -Z_{yx}$$

$$\begin{bmatrix} M_{xx} & 0 \\ 0 & M_{xx} \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} K_{xx} & K_{xy} \\ -K_{xy} & K_{xx} \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} + \begin{bmatrix} C_{xx} & 0 \\ 0 & C_{xx} \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} = \begin{pmatrix} F_x \\ 0 \end{pmatrix}$$

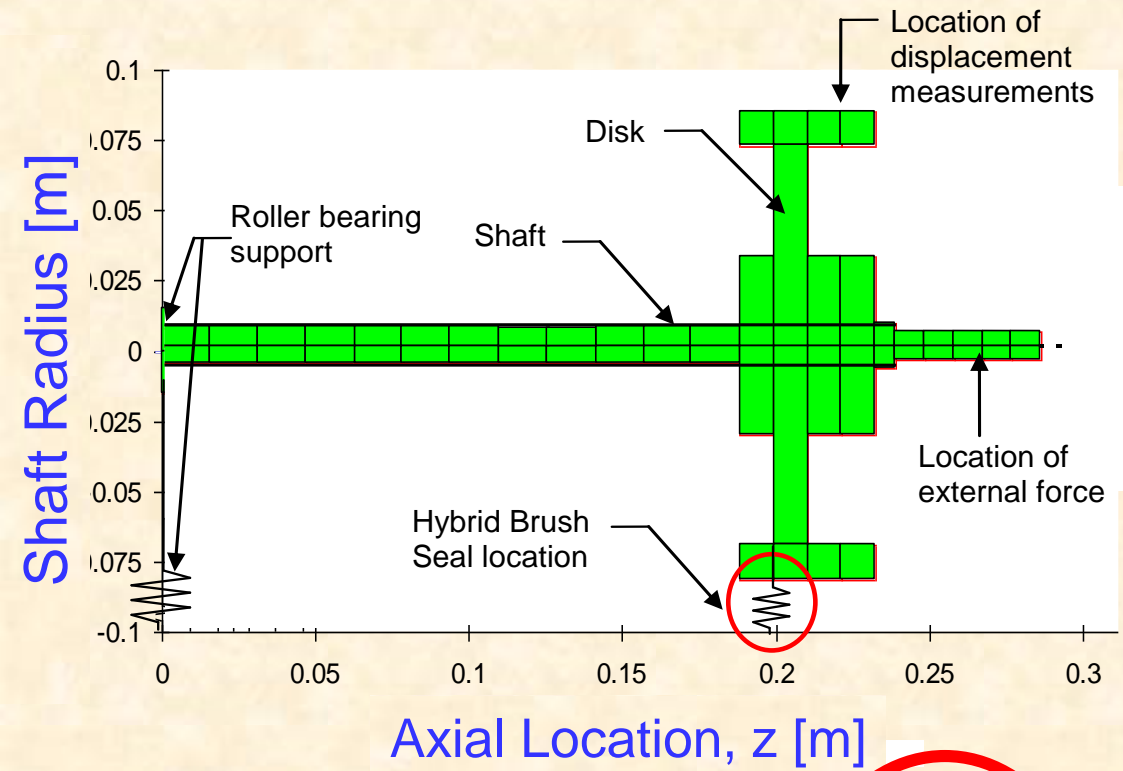
# Rotor mode shapes

Only first mode excited  
in rotor speed range (0-  
1200 rpm)



# Effect of rotor speed on rotor-HBS natural frequency

Gyroscopic effects negligible for test rotor speeds (600 and 1,200 rpm [20 Hz])

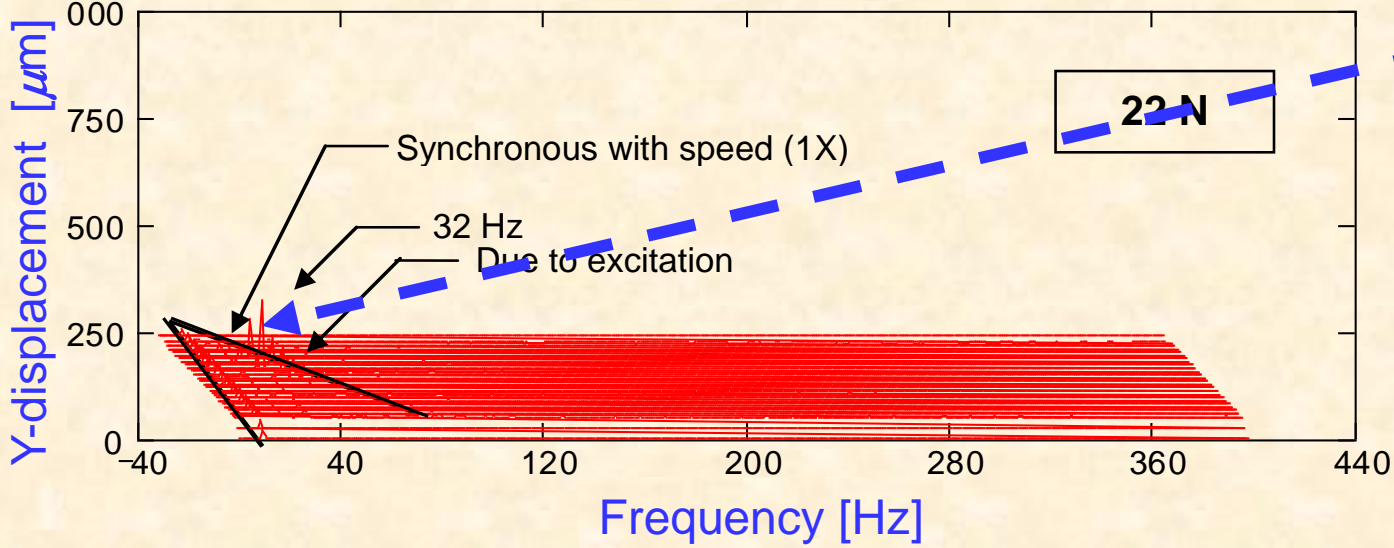
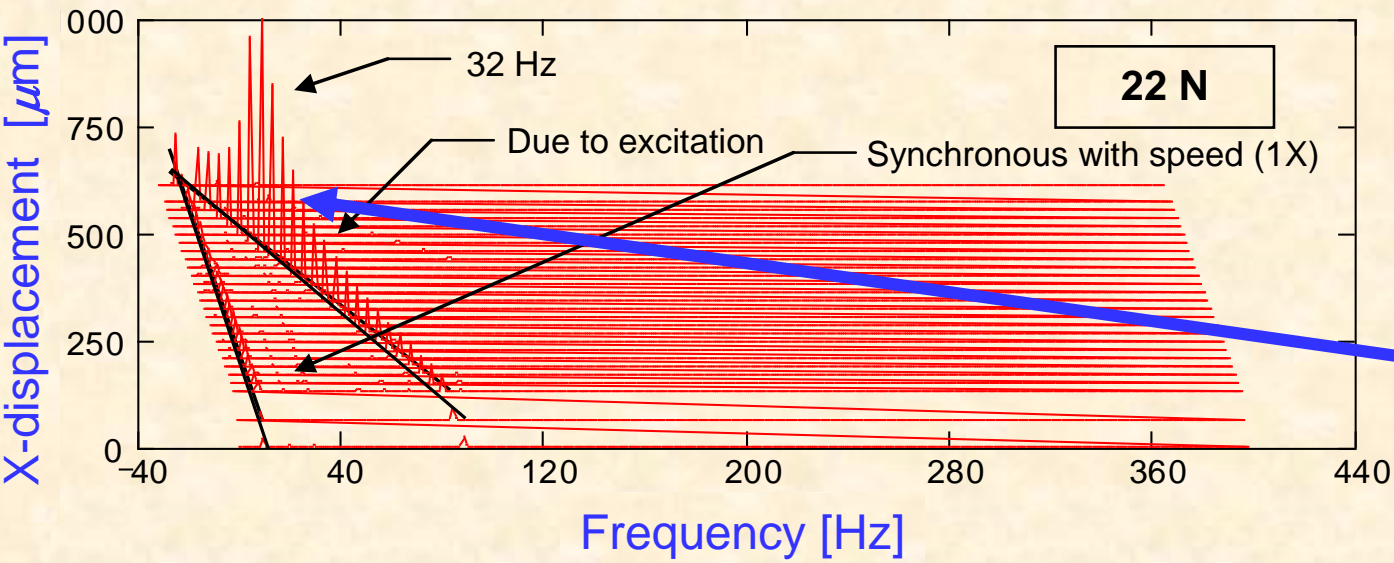


Rotor Speed [RPM]	1 <sup>st</sup> Backward Nat. Frequency, [Hz]	1 <sup>st</sup> Forward Nat. Frequency, [Hz]	2 <sup>nd</sup> Forward Nat. Frequency, [Hz]	3 <sup>rd</sup> Forward Nat. Frequency, [Hz]
0	30.5	30.5	146	1351
600	29.7	31.4	154	1351
1200	28.8	32.2	163	1351

T=32 Hz

# CROSS-Coupling Effects under rotation

Load=22 N, 600 rpm



For load along X direction, rotor principal (X) motions

>>>

cross (Y) motions

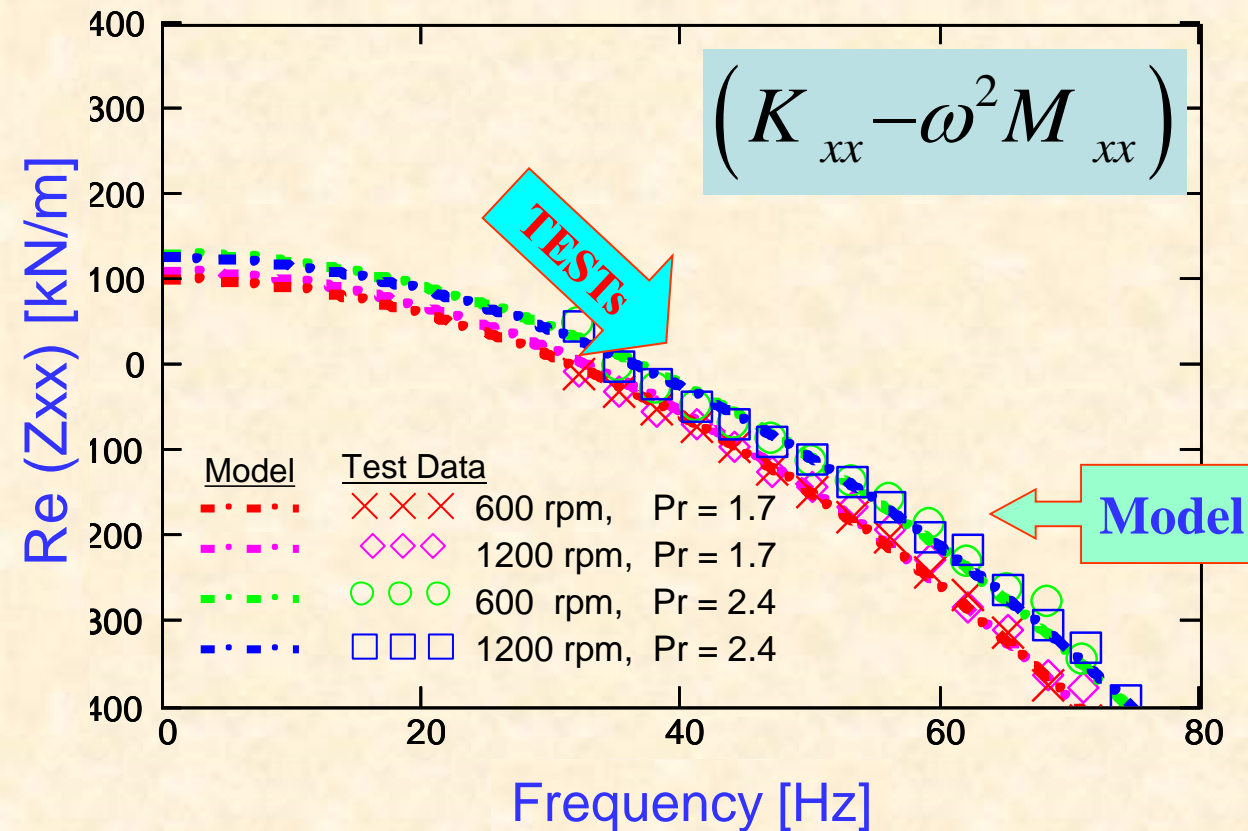
3X motions always small

# Test dynamic stiffness vs. frequency

Load = 22 N, frequency 25-80Hz

Pressure ratio ( $P_r = P_s/P_d$ ) = Discharge/Supply

$$Z_{xx} = \frac{\bar{F}_x \cdot \bar{x}}{(\bar{x}^2 + \bar{y}^2)}$$



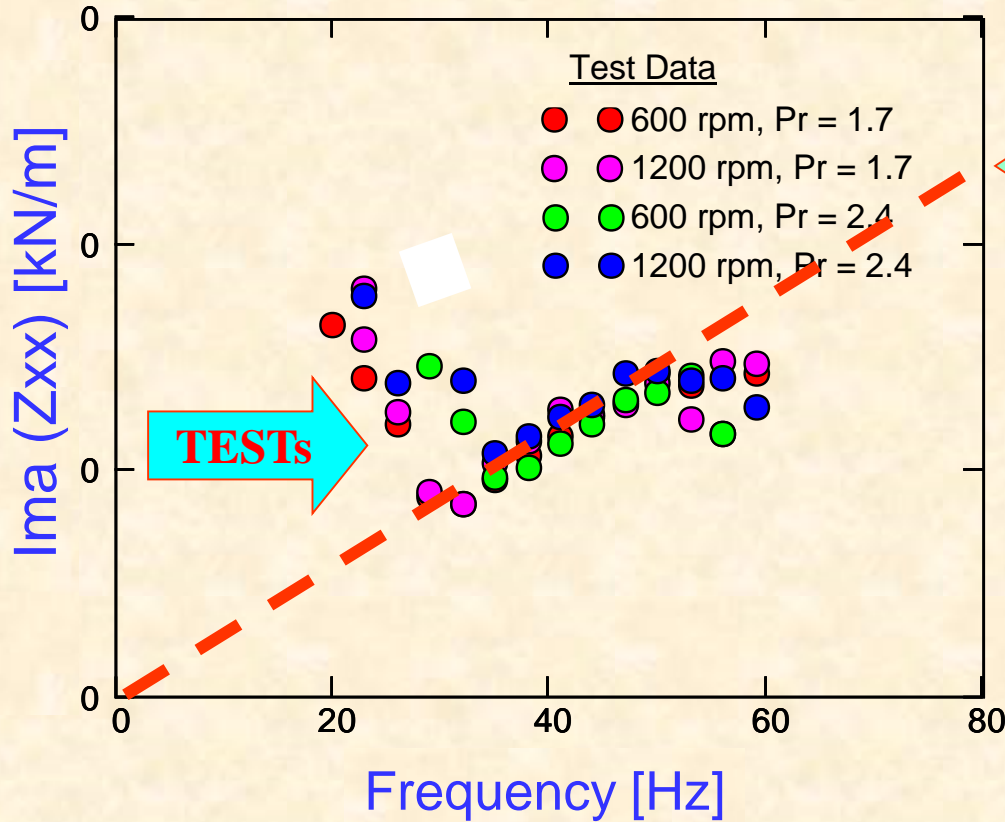
**Model reproduces the measured real part of impedance.**  
**Little effect of pressurization**

# Test quadrature stiffness vs. frequency

Load = 22 N, frequency 25-80Hz

Pressure ratio ( $P_r = P_s/P_d$ ) = Discharge/Supply

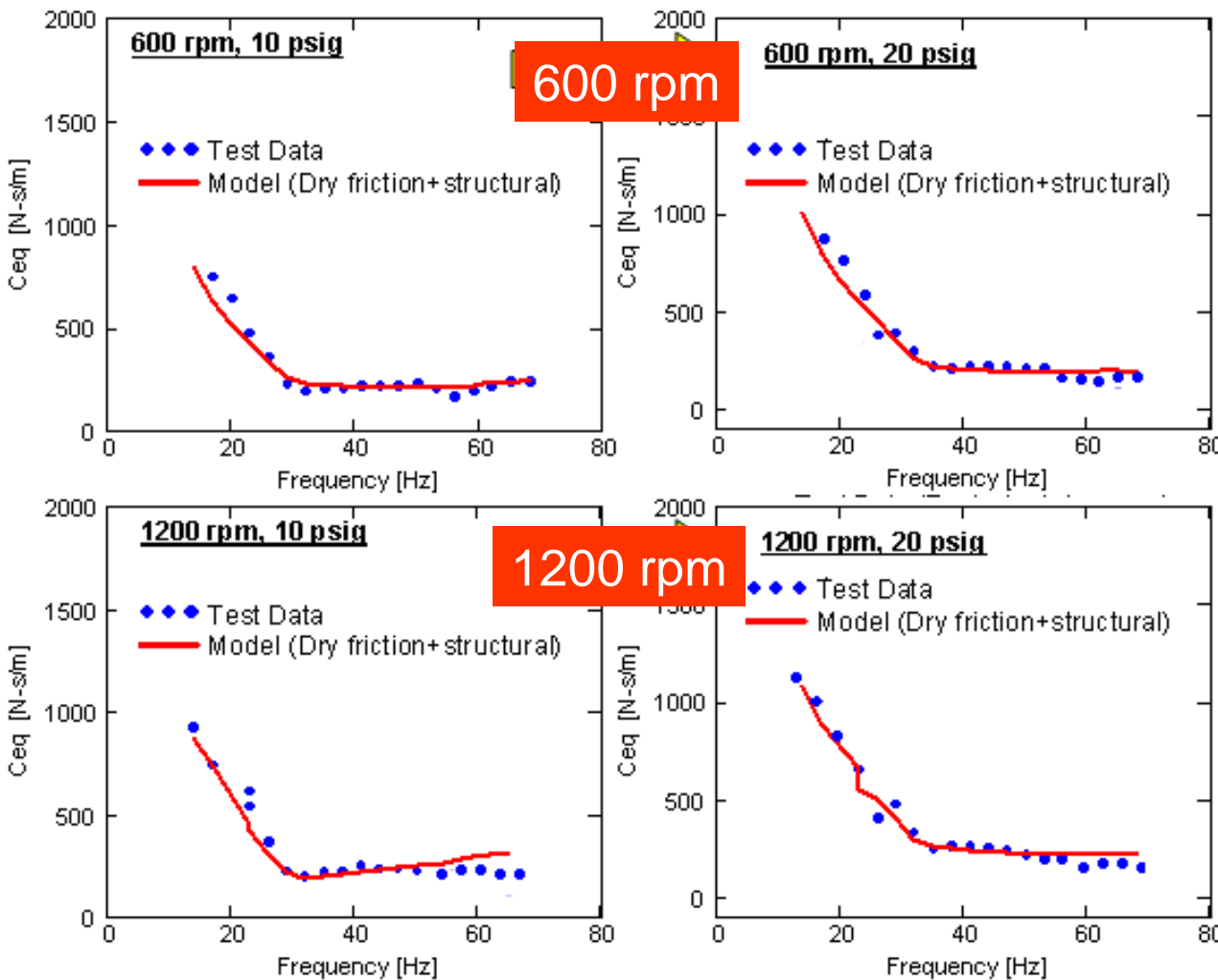
$$Z_{yx} = -Z_{yy} \frac{\bar{y}}{\bar{x}}$$



Viscous Model

Damping is NOT viscous

# Equivalent Viscous Damping ( $C_{xx} \sim C_{eq}$ ) vs. Frequency



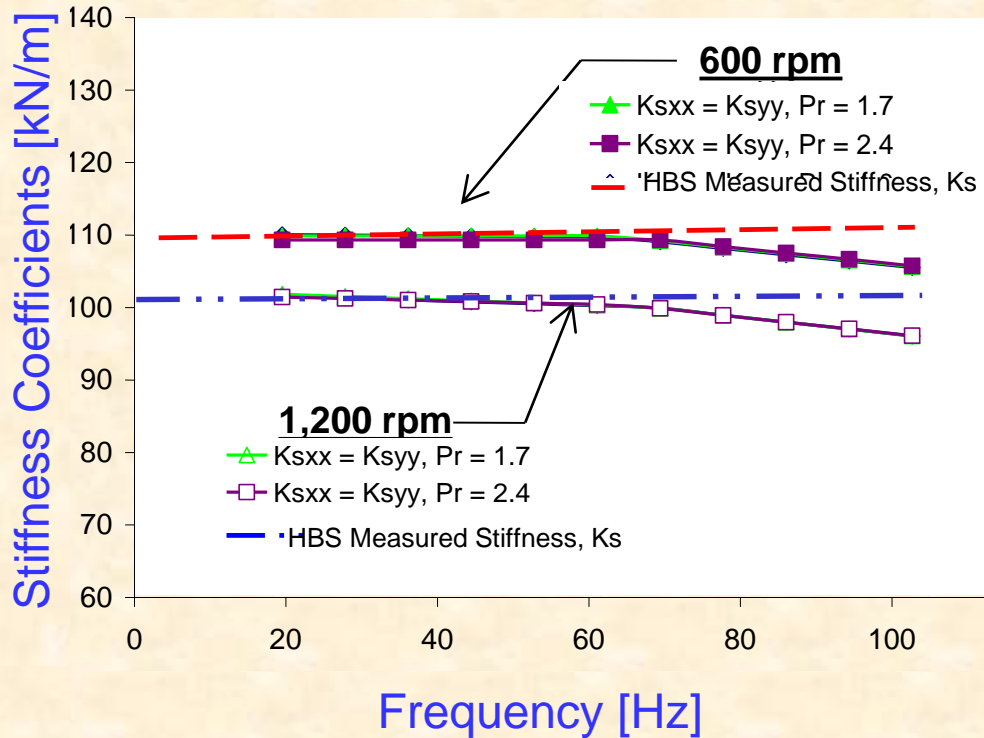
Damping decreases with frequency, with little effect of supply pressure. **Minimum value at test system natural frequency (~32 Hz)**

$Pr=1.7$

$Pr=2.4$

# HBS predicted & test direct stiffness vs. frequency

Frequency 25-100 Hz



Predicted HBS stiffness ( $K_{sxx}$ ) drops slightly in range from 20- 100 Hz.

Tests show nearly constant

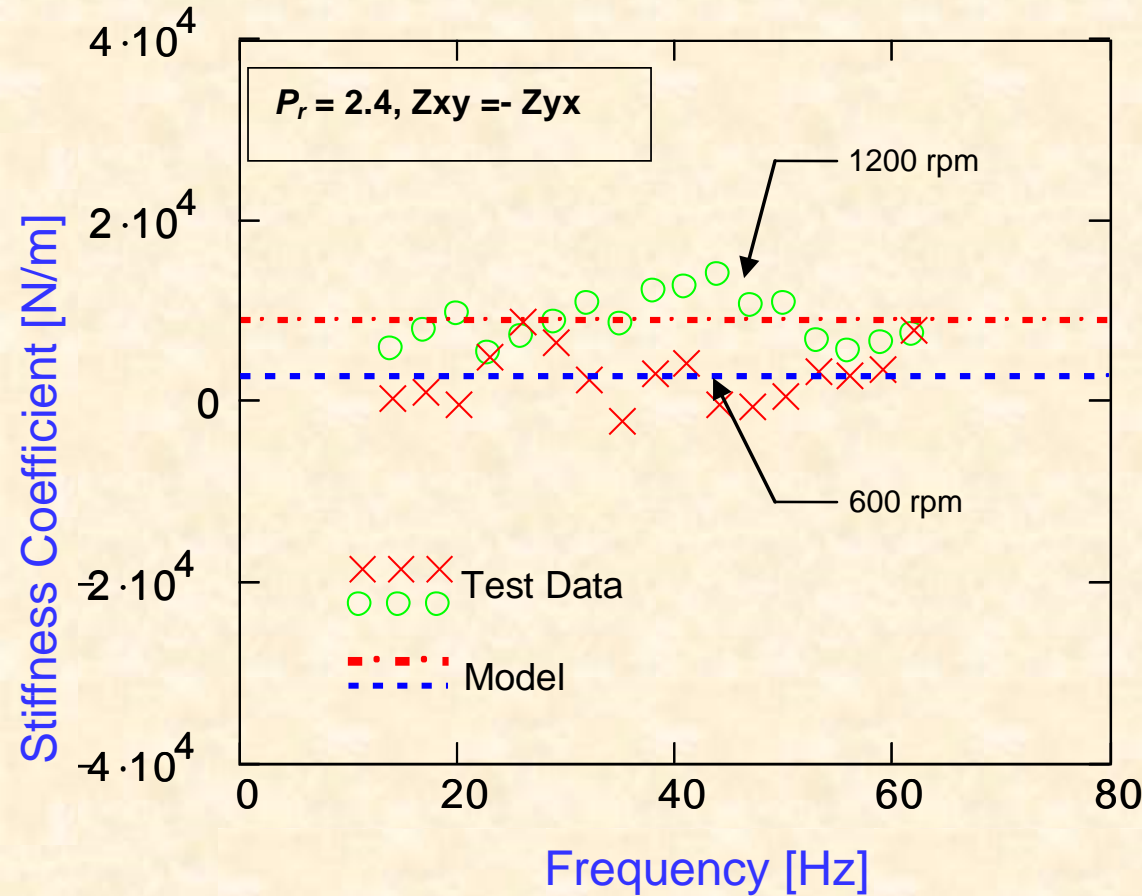
$K_{sxx}$

Pressure ( $P_r = P_s/P_d$ ) has negligible effect on seal direct stiffness,  $K_{sxx}$



# HBS predicted & test cross stiffness vs. frequency

Frequency 25-100 Hz



$$Z_{yx} = -Z_{xx} \frac{\bar{y}}{\bar{x}}$$

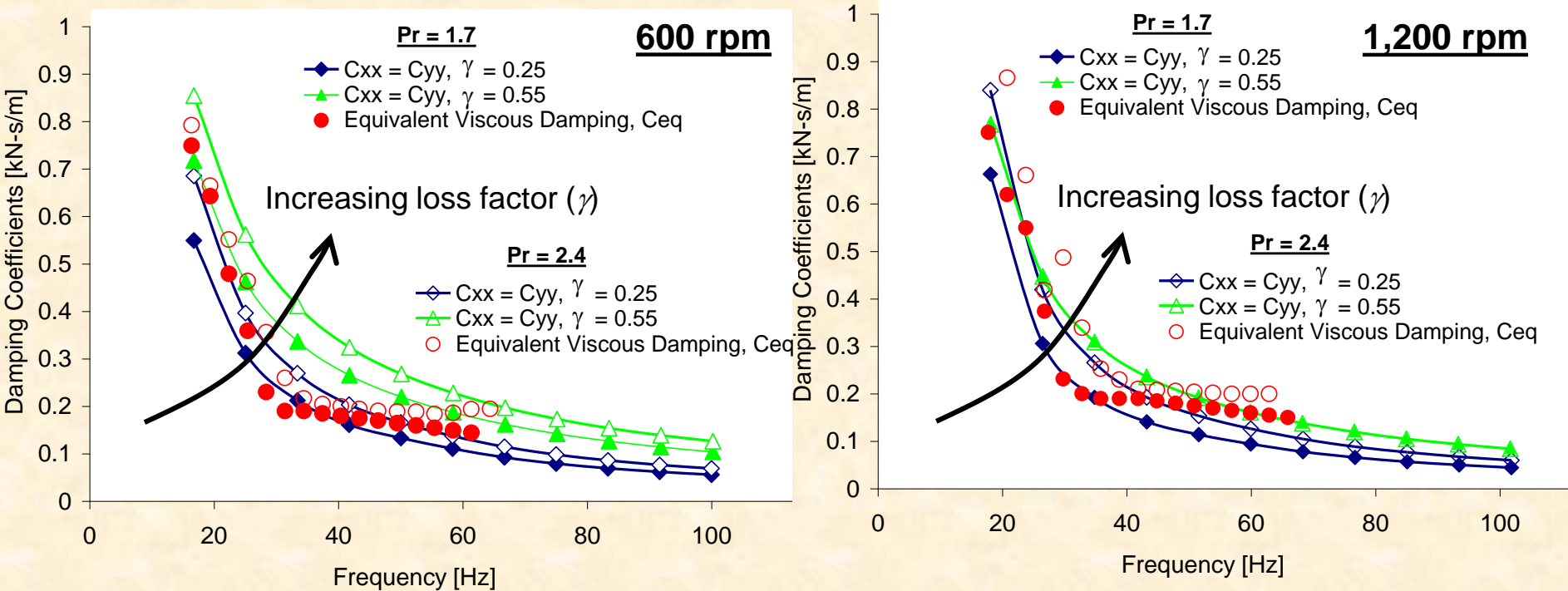
**HBS cross stiffness**  
 $(K_{sxy}) \ll$  **direct**  
**stiffness**  $(K_{sxx})$

**Pressure**  $(P_r = P_s/P_d)$  **has**  
**negligible effect on seal**  
**cross stiffness,  $K_{sxy}$**

# HBS predicted & test damping vs. frequency

Frequency 25-100 Hz

Pressure ratio ( $P_r = P_s/P_d$ ) = Discharge/Supply



HBS direct damping ( $C_{sxx}$ ) decreases with excitation frequency. Loss factor coefficient ( $\gamma$ ) models well seal structural (hysteresis) damping

# Conclusions

- A structural loss factor ( $\gamma$ ) and a dry friction coefficient ( $\mu$ ) effectively characterize the energy dissipation mechanism of a Hybrid Brush Seal (HBS).
- **HBS Direct stiffness ( $K_{sxx} = K_{syy}$ ) decreases minimally with rotor increasing rotor speed for  $P_r = 1.7$  and 2.4 HBS**  
Cross-coupled stiffness ( $K_{sxy} = -K_{syx}$ ) is much smaller than the direct stiffness.
- HBS Direct viscous damping coefficients decrease as a function of increasing excitation frequency.