TWO DOF SYSTEM: Identification of system parameters (K,C,M) using IVF method.

LSA/04/08 revised 11/09

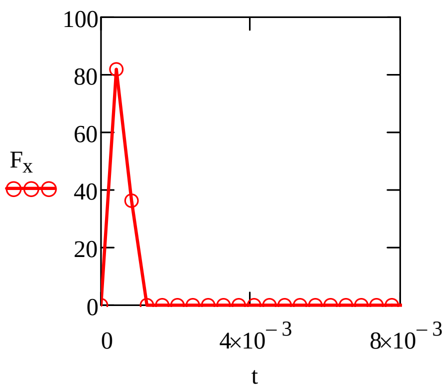
a. Read time domain data files:

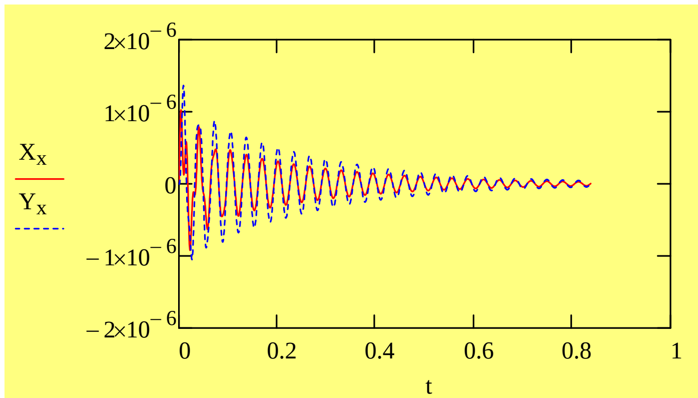
Response obtained from Impact_response.mcd

transfer data

Plot excitation force in X dir. and responses Xx, Yx versus time

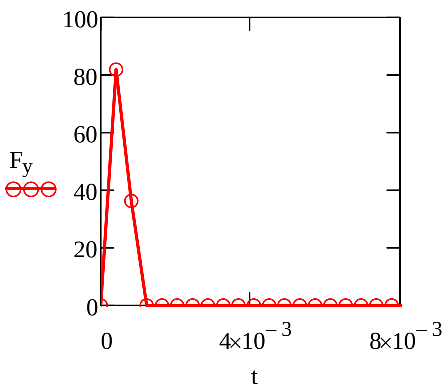
Note time scale differences in load and displacements gaphs

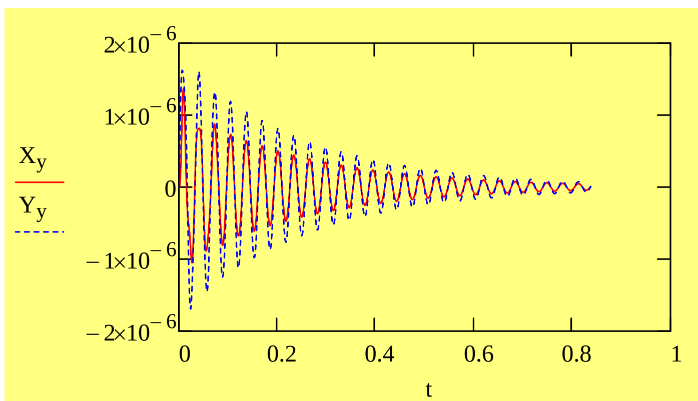




Plot excitation force in Y dir. and responses Xy, Yy versus time

$N_t = 2.05\times10^3$ number of points





$\frac{1}{\Delta T} = 2.44\times10^3$

SAMPLING RATE

$\max(t) = 0.84$

MAX time

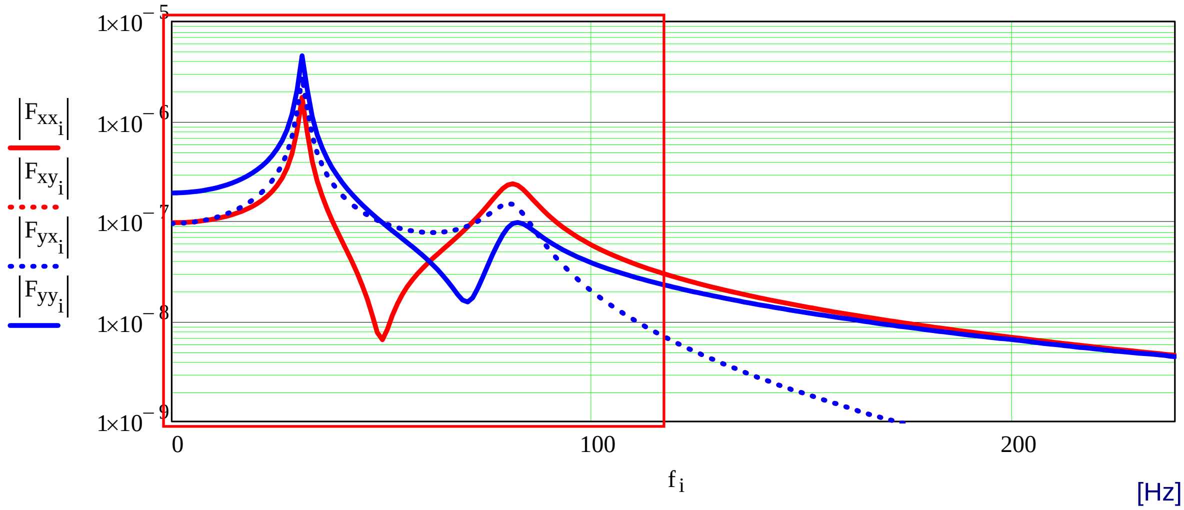
b. Transform forces and displacements to the frequency domain:

Select max index for identification: $Max := 200$ < $\frac{N_t}{2} = 1.024\times10^3$

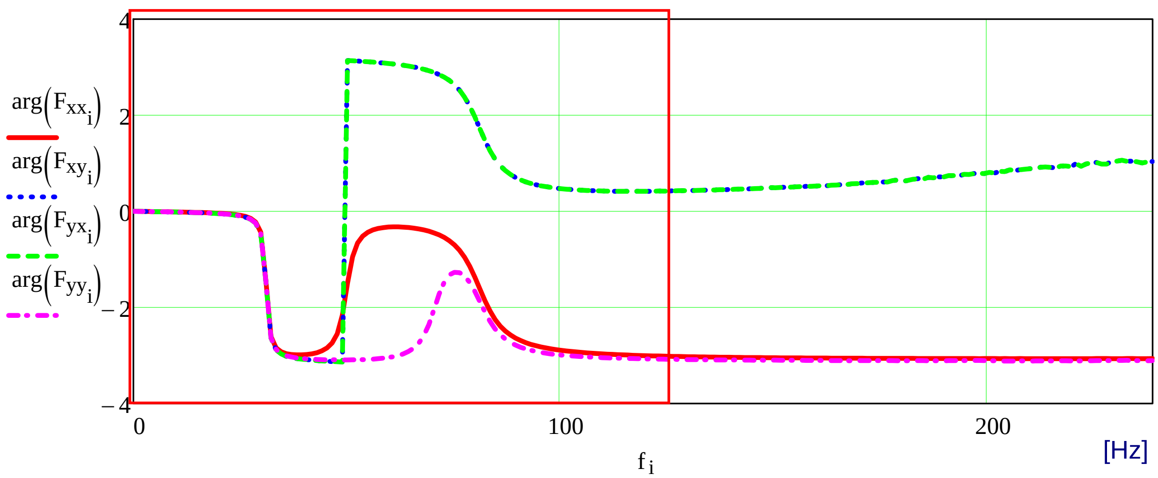
get DFTs

System Flexibilities (amplitude)

$f_{Max} = 238.59$ Hz for identification range



System Flexibilities (phase)



Select the frequency range for the identification of parameters:

initial point: $i_o := 5$ | final point: $i_f := \frac{Max}{2}$

Corresponding to: $f_{i_o} = 5.96$ Hz | $f_{i_f} = 119.3$ Hz

$Max = 200$

1. IMPEDANCE method:

$i := i_o .. i_f$ Sweep over frequencies $f_{io} := f_{i_o}$ $f_{if} := f_{i_f}$

$w_i := \omega_i$

$w2_i := (\omega_i)^2$

Extract Real and Imaginary parts (only those in frequency range selected):

$H_{Rxxm_i} := (Re(H_{xx_i}))$ $H_{Ryxm_i} := (Re(H_{yx_i}))$ $H_{Ixxm_i} := (Im(H_{xx_i}))$ $H_{Iyxm_i} := (Im(H_{yx_i}))$

$H_{Rxym_i} := (Re(H_{xy_i}))$ $H_{Ryym_i} := (Re(H_{yy_i}))$ $H_{Ixym_i} := (Im(H_{xy_i}))$ $H_{Iyym_i} := (Im(H_{yy_i}))$

Determine PARAMETERS from curve fits to impedances over frequency range selected

$kxx := intercept(w2, H_{Rxxm})$ $kxy := intercept(w2, H_{Rxym})$

$kyx := intercept(w2, H_{Ryxm})$ $kyy := intercept(w2, H_{Ryym})$

$mxx := -slope(w2, H_{Rxxm})$ $mxy := -slope(w2, H_{Rxym})$

$myx := -slope(w2, H_{Ryxm})$ $myy := -slope(w2, H_{Ryym})$

$Re(H) = K - M \cdot \omega^2$

$Ima(H) = \omega \cdot C$

$cxx := slope(w, H_{Ixxm})$ $cxy := slope(w, H_{Ixym})$

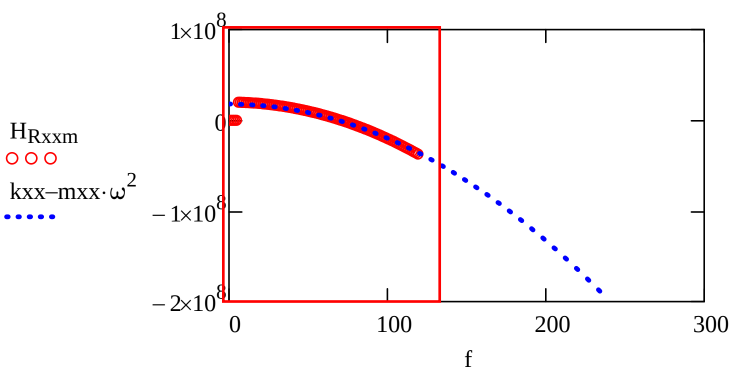
$cyx := slope(w, H_{Iyxm})$ $cyy := slope(w, H_{Iyym})$

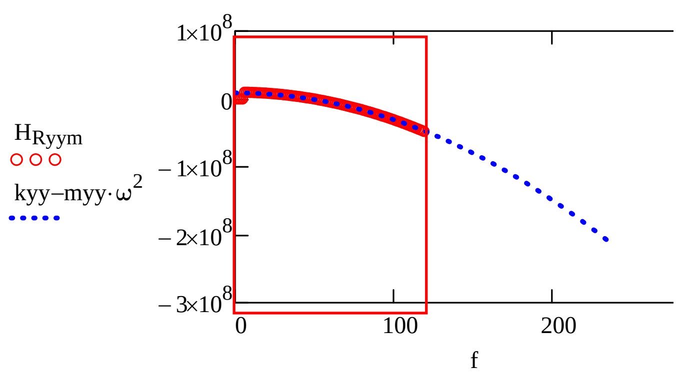
Make matrices of parameters $K_I := \begin{pmatrix} kxx & kxy \\ kyx & kyy \end{pmatrix}$ $C_I := \begin{pmatrix} cxx & cxy \\ cyx & cyy \end{pmatrix}$ $M_I := \begin{pmatrix} mxx & mxy \\ myx & myy \end{pmatrix}$

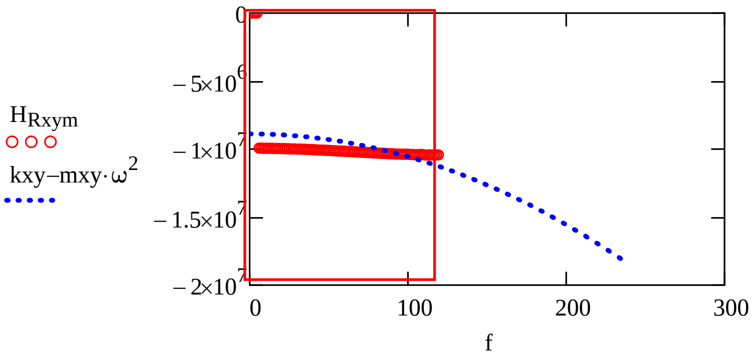
REAL PARTS

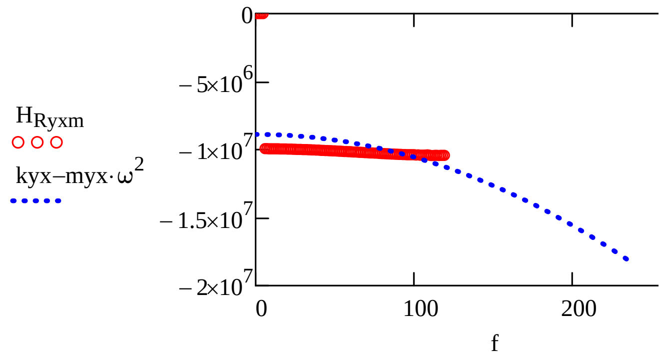
$f_{i_o} = 5.96$ Hz $f_{i_f} = 119.3$ Hz

SYMBOLS: data, LINE: CURVE FIT

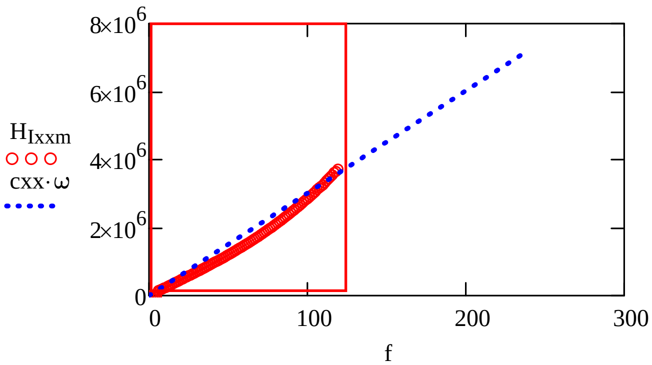


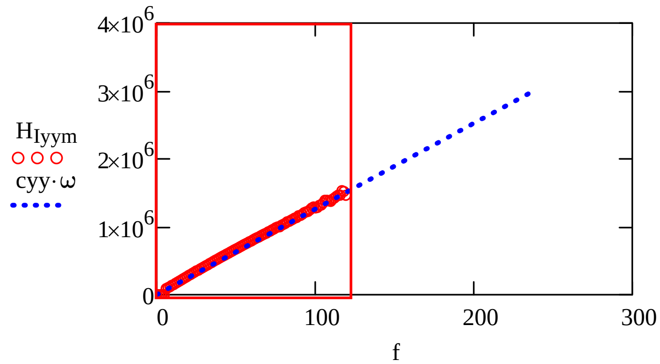


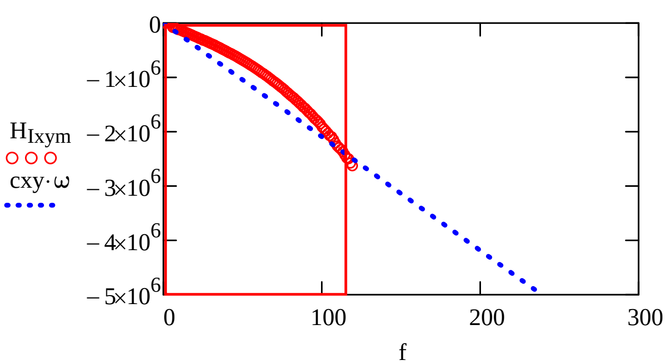


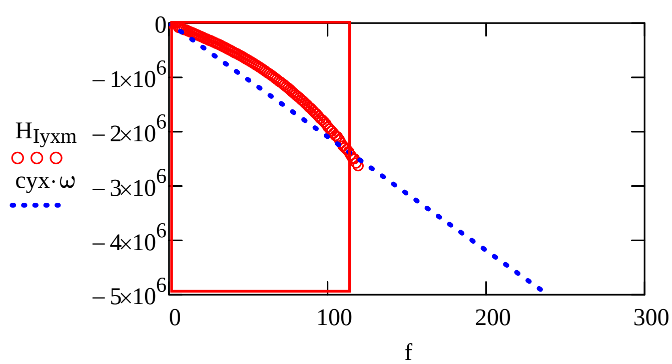


IMAGINARY parts









Identified force coefficients over freq range:

$f_{io} = 5.96$ to $f_{if} = 119.3$ Hz

Compare to ACTUAL parameters

$K_I = \begin{pmatrix} 1.79 \times 10^7 & -8.88 \times 10^6 \\ -8.88 \times 10^6 & 8.99 \times 10^6 \end{pmatrix}$ N/m

$M_I = \begin{pmatrix} 95.38 & 4.23 \\ 4.23 & 99.67 \end{pmatrix}$ kg

$C_I = \begin{pmatrix} 4794.2 & -3330.4 \\ -3330.4 & 2008.6 \end{pmatrix}$ Ns/m

$$\begin{pmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \\ m_{xx} & m_{xy} \\ m_{yx} & m_{yy} \\ c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{pmatrix} := \begin{pmatrix} 2\cdot 10^7 & -1\cdot 10^7 \\ -1\cdot 10^7 & 1\cdot 10^7 \\ 100 & 0 \\ 0 & 100 \\ 4500 & -2500 \\ -2500 & 2500 \end{pmatrix}$$

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2) LEAST SQUARES METHOD

Construct a vector whose elements are the flexibility matrices at each frequency (each element of the vector is a matrix):

$i := i_o .. i_f$

$$F_i := \begin{pmatrix} H_{xx_i} & H_{xy_i} \\ H_{yx_i} & H_{yy_i} \end{pmatrix}^{-1}$$

Now, the problem to solve is: $F \cdot H = I + E$

where F are the measured flexibilities, H the approximated impedances, I the identity matrix, and E the error to be minimized.

The left hand side can be rearranged as: $A \cdot \begin{pmatrix} k \\ m \\ c \end{pmatrix} = I + E'$

Now, the equations of each frequency decomposed into real and imaginary part. Stacking all the equations renders an undetermined system of equations (more equations than unknowns) of the same form, where:

A (as a function of the flexibilities) is given by:

$$a(F) := \left| \begin{array}{l} A \leftarrow \mathrm{Re}\left[F_{i_o}\cdot\begin{bmatrix}1 & 0 & -\left(\omega_{i_o}\right)^2 & 0 & i\cdot\omega_{i_o} & 0\\ 0 & 1 & 0 & -\left(\omega_{i_o}\right)^2 & 0 & i\cdot\omega_{i_o}\end{bmatrix}\right] \\ A \leftarrow \mathrm{stack}\left[A, \mathrm{Im}\left[F_{i_o}\cdot\begin{bmatrix}1 & 0 & -\left(\omega_{i_o}\right)^2 & 0 & i\cdot\omega_{i_o} & 0\\ 0 & 1 & 0 & -\left(\omega_{i_o}\right)^2 & 0 & i\cdot\omega_{i_o}\end{bmatrix}\right]\right] \\ \text{for } i \in i_o + 1 .. i_f \\ \quad \left| \begin{array}{l} A \leftarrow \mathrm{stack}\left[A, \mathrm{Re}\left[F_i\cdot\begin{bmatrix}1 & 0 & -\left(\omega_i\right)^2 & 0 & i\cdot\omega_i & 0\\ 0 & 1 & 0 & -\left(\omega_i\right)^2 & 0 & i\cdot\omega_i\end{bmatrix}\right]\right] \\ A \leftarrow \mathrm{stack}\left[A, \mathrm{Im}\left[F_i\cdot\begin{bmatrix}1 & 0 & -\left(\omega_i\right)^2 & 0 & i\cdot\omega_i & 0\\ 0 & 1 & 0 & -\left(\omega_i\right)^2 & 0 & i\cdot\omega_i\end{bmatrix}\right]\right] \end{array}\right. \\ A \end{array}\right.$$

<= A = real part of the first frequency

<= stacks what was on A with the imaginary part of the first frequency

<= for loop from the second to the last frequencies.

<= stacks to A the real part of the i^{th} frequency

<= stacks to A the imaginary part of the i^{th} frequency

<= returns the matrix A

Auxiliary matrix of zeros [2x2]: $zero_{1,1} := 0$

The right hand side of the equation is given by:

$$I := \left| \begin{array}{l} I \leftarrow \mathrm{identity}(2) \\ I \leftarrow \mathrm{stack}(I, zero) \\ \text{for } i \in i_o + 1 .. i_f \\ \quad \left| \begin{array}{l} I \leftarrow \mathrm{stack}(I, \mathrm{identity}(2)) \\ I \leftarrow \mathrm{stack}(I, zero) \end{array}\right. \\ I \end{array}\right.$$

The least squares solution of the problem (minimum E) is:

$$q_1 := \left(A^T\cdot A\right)^{-1}\cdot A^T\cdot I$$

$A := a(F)$ <= Matrix A for the measured flexibilities

$$\begin{pmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \\ m_{xx} & m_{xy} \\ m_{yx} & m_{yy} \\ c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{pmatrix} := q_1$$

$$q_1 = \begin{pmatrix} 2.01 \times 10^7 & -10 \times 10^6 \\ -9.99 \times 10^6 & 1.01 \times 10^7 \\ 101.92 & 1.16 \\ 1.15 & 103.13 \\ 4365.36 & -2434.94 \\ -2426.08 & 2447.98 \end{pmatrix}$$

Estimated system parameters based on least squares fit to flexibilities

$$\begin{pmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \\ m_{xx} & m_{xy} \\ m_{yx} & m_{yy} \\ c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{pmatrix} := \begin{pmatrix} 2\cdot 10^7 & -1\cdot 10^7 \\ -1\cdot 10^7 & 1\cdot 10^7 \\ 100 & 0 \\ 0 & 100 \\ 4500 & -2500 \\ -2500 & 2500 \end{pmatrix}$$

Actual values:

3. Instrumental variable method

Actual values:

$$q = \begin{pmatrix} 2 \times 10^7 & -10 \times 10^6 \\ -10 \times 10^6 & 1 \times 10^7 \\ 101.9 & 1.1 \\ 1.1 & 103.1 \\ 4366.4 & -2435.6 \\ -2426.8 & 2448.5 \end{pmatrix} \leftarrow \begin{pmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \\ m_{xx} & m_{xy} \\ m_{yx} & m_{yy} \\ c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{pmatrix} := \begin{pmatrix} 2\cdot 10^7 & -1\cdot 10^7 \\ -1\cdot 10^7 & 1\cdot 10^7 \\ 100 & 0 \\ 0 & 100 \\ 4500 & -2500 \\ -2500 & 2500 \end{pmatrix}$$

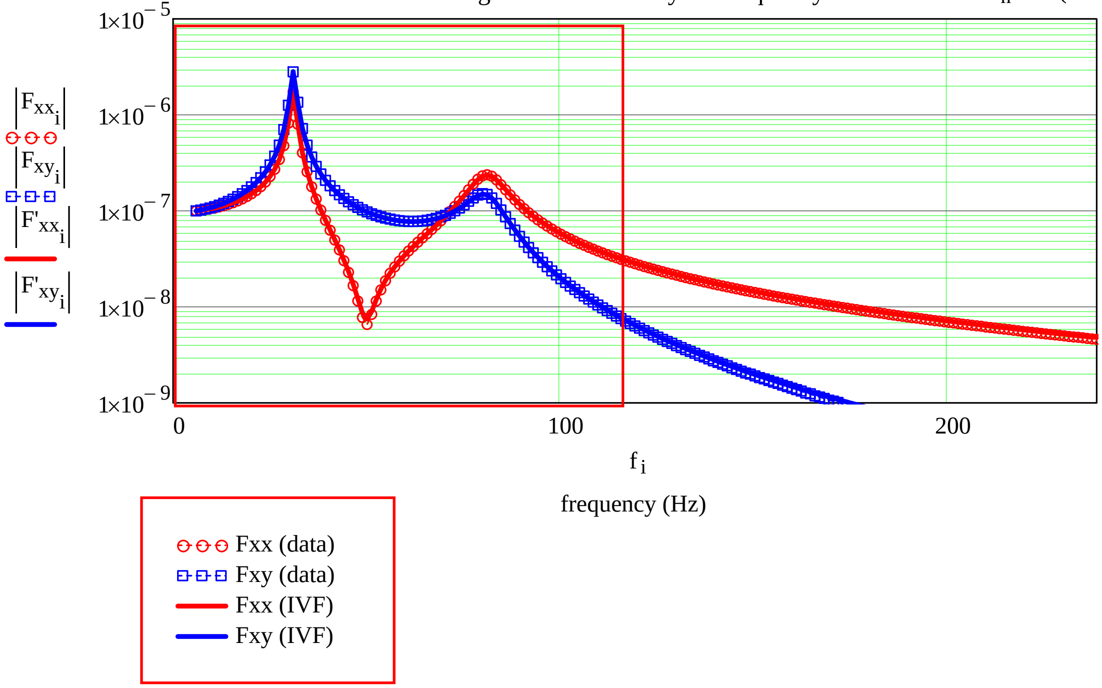
4. Build output/input (transfer functions)

FLEXIBILITY functions - data and IVF

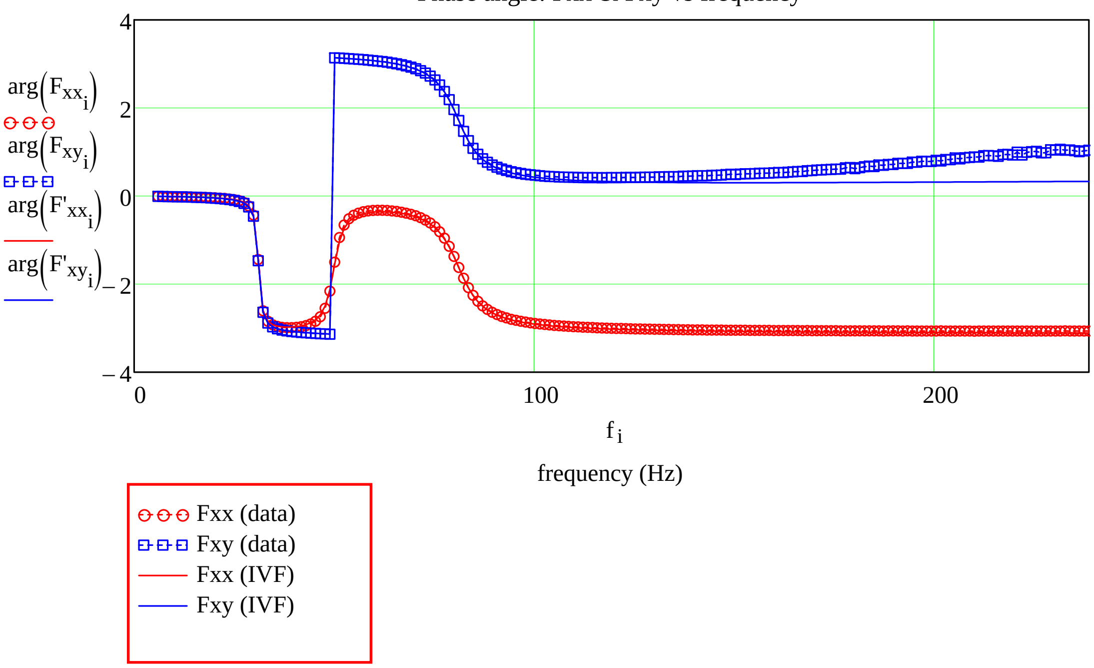
Magnitude: Fxx & Fxy vs. frequency

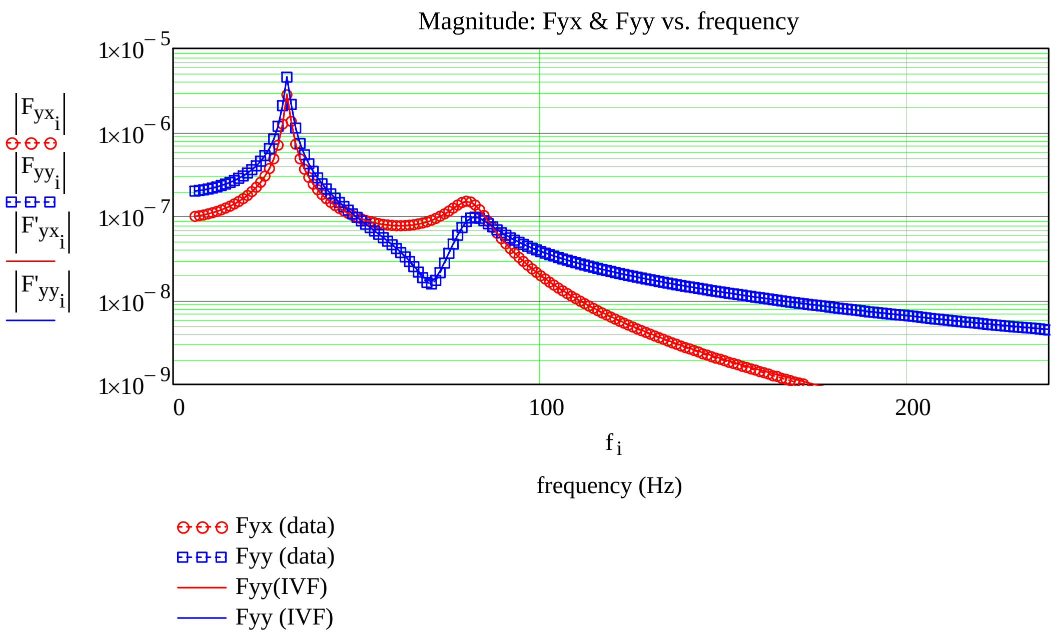
$f_n^T = (31.08 \quad 81.13)$ Hz

SYMBOLS DATA
LINES IVF results

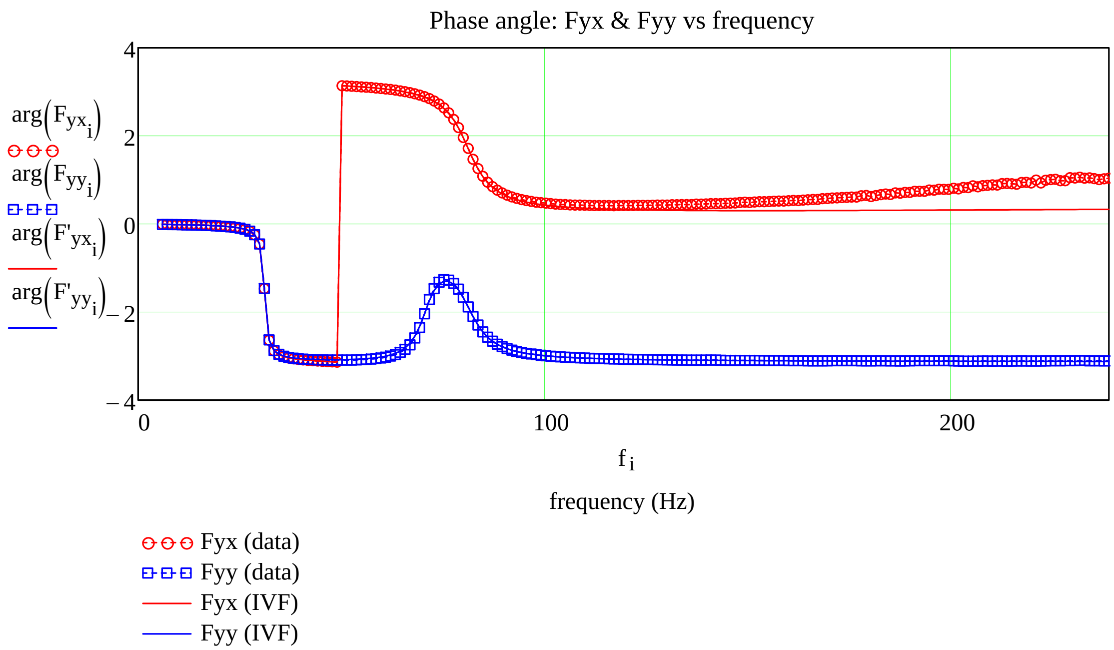


Phase angle: Fxx & Fxy vs frequency





Magnitude: Fyx & Fyy vs. frequency
$|F_{yx_i}|$
$|F_{yy_i}|$
$|F'_{yx_i}|$
$|F'_{yy_i}|$
1×10^{-5}
1×10^{-6}
1×10^{-7}
1×10^{-8}
1×10^{-9}
0
100
200
f_i
frequency (Hz)
Fyx (data)
Fyy (data)
Fyy(IVF)
Fyy (IVF)



Phase angle: Fyx & Fyy vs frequency
$\arg(F_{yx_i})$
$\arg(F_{yy_i})$
$\arg(F'_{yx_i})$
$\arg(F'_{yy_i})$
4
2
0
− 2
− 4
0
100
200
f_i
frequency (Hz)
Fyx (data)
Fyy (data)
Fyx (IVF)
Fyy (IVF)