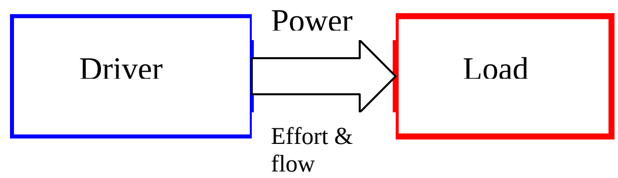
Lecture content intended for use in MEEN 363. However, I never find the time to actually deliver this important material. I hope you agree to its importance. LSA 04/2011

Introduction to Impedance matching

The following discussion pertains to a driver connected to a load operating at steady-state conditions.[1]

The driver delivers an effort (e), a function of the flow (f). The simplest relationship (**performance curve at steady state**) is of the form



$$e = e_s - s\,f,\quad s = e_s / f_i \qquad (1)$$

where (e_s) is the effort at zero flow, i.e. that require to stall (stop) the driver; while (f_i) is the flow at idle conditions (maximum flow). The slope of the effort vs. flow curve is ($-s$) <0, see Figure 1a. The slope has units of effort over flow, and thus the "s" parameter is known as the driver impedance.

The power delivered by the driver is simply $$P = e\,f = e_s\left(1 - \frac{f}{f_i}\right) f \qquad (2)$$

And is as a quadratic relationship in flow, increasing from zero to towards a maximum value at a certain flow and the decreasing towards null power at f_i, see Figure 1b.

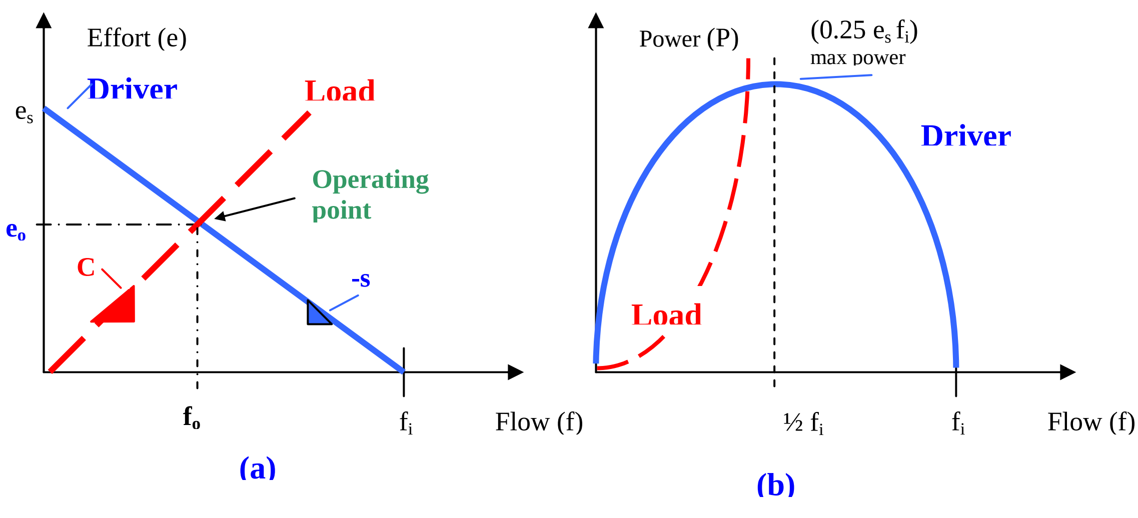


Figure 1. (a) Graphical relationship of effort vs. flow for simple driver and load. (b) Power = effort x flow. *C* and *s* correspond to load and driver impedances (slopes)

[1] A few examples of drivers are power supplies and batteries, motors, turbines, IC engines, etc. A few loads are electrical appliances (ovens, lights), pumps, compressors, fans, generators, vehicles, etc. Efforts include forces, torque, voltages and pressure; while flows encompass translation and rotational speeds, currents, flow rates. The list is clearly not comprehensive and probably inexhaustible.

The maximum power available from the driver is obtained from (*dP/df=0*) and equals

$$P_{max} = \frac{e_s\ f_i}{4} = \frac{e_s^2}{4\,s} \qquad \text{at } f=½f_i \qquad (3)$$

i.e. maximum power occurs at a flow equal to 50% of the maximum or idle flow.

Now, let's assume the (selected) driver connects to a load to perform some useful transference of power. Furthermore, let's assume for simplicity that the load effort is proportional to the flow, i.e. (see Figure 1a)

$$e_{load} = C\ f \qquad (4)$$

Nota that the "*C*" parameter is known as the **load impedance**. When the driver is connected to the load, some "equilibrium position" or **operating point** is achieved at steady-state operation, and which requires the balance of efforts, i.e.

$$e_{load} = e_{drive} \qquad (5).$$

From this equation, it follows that the "operating point" (flow & effort) and transmitted power from driver to load equal

$$f_o = \frac{e_s}{s+C};\ e_o = C\ f_o;\ P_o = \frac{C\,e_s^2}{(s+C)^2} \qquad (6)$$

See Figure 1 for the graphical representation of the operating point.

It is of interest to determine the condition at which the power transmission maximizes given a certain load. From (6), determine ($dP_o/dC=0$), i.e.

$$\frac{d\,P_o}{d\,C} = 0 = \frac{e_s^2\,(s+2C-C)}{(s+C)^2} = 0 \ \rightarrow\ s = C \qquad (7)$$

and the maximum transmitted power equals $P_{max} = \frac{C\,e_s^2}{(s+C)^2} = \frac{e_s^2}{4\,s}$ (3)

That is, maximum power transmission occurs when the load impedance (*C)* equals the driver impedance (*s).*

The analysis outlined is known as **IMPEDANCE MATCHING.** It is very useful to ensure maximum power transmission (and efficiency) in the operation of systems. The procedure demonstrates the **NEED** to appropriately select drivers to accommodate the desired loads.

It also indicates that if a driver is selected to operate a load with optimum transmission; then, variations in the load (change in $C \neq s$) will cause an **IMPEDANCE MISMATCHING** and inefficient operation; i.e. away from optimum or maximum power transmission.

Actual drivers do not show the "ideal" steady-state performance curve shown in Figure 1. Most notably compressors and pumps show effort vs. flow relationships like the one shown in Figure 2. Note that in actual hardware, the driver impedance (*s)* varies with the flow *(f)* in a complicated form. One should never allow operation of this type of driver in a flow region where the slope is positive (–s>0), i.e. negative impedance. Attempts to operate at this, typically low flow condition, will cause damage to the equipment since severe instabilities will occur. This is the case of compressors undergoing surge and stall, for example.

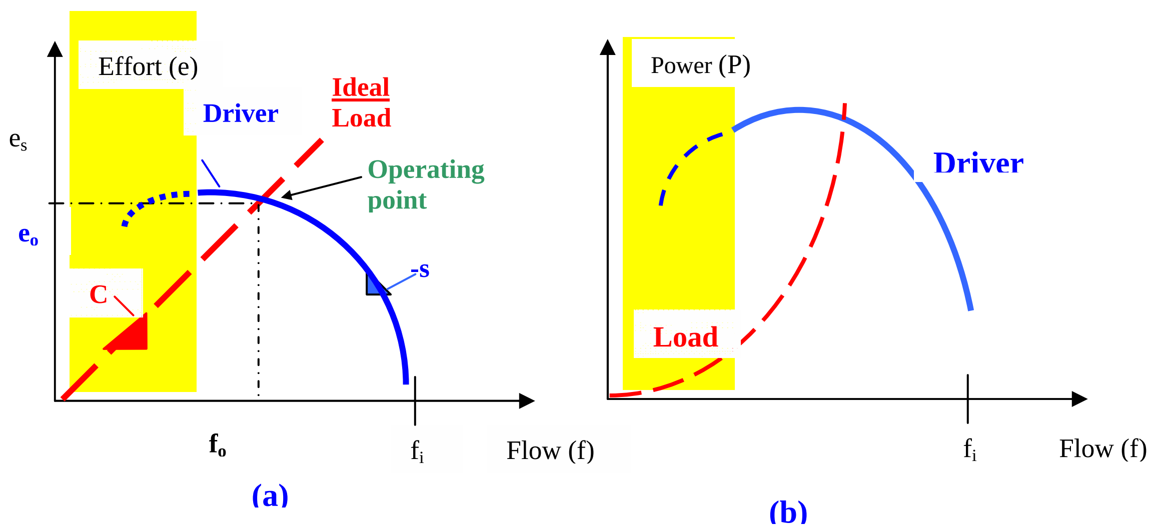


Figure 2. (a) Effort vs. flow for actual driver and simple load. (b) Power = effort x flow. ***C*** **and** ***s*** **correspond to load and driver impedances (slopes). Yellow zone indicates region of instability – forbidden (NO-NO) operation.**

Note that in the discussion above, the loads considered are ideal. In reality, actual loads may also show complicated relationships between effort and flow, for example of the form $e_{load}=e_{DRY}+C\,f + D\,f^2$, etc.

The knowledge gained will allow you to properly select the best pair of audio speakers that match an audio amplifier, for example. However, the most enduring concepts for you to ponder are those of driver and load impedances and the importance of matching impedances in an actual engineering application. Whenever designing or specifying components for a system, do no forget to exercise these important concepts.