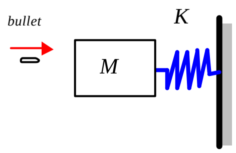
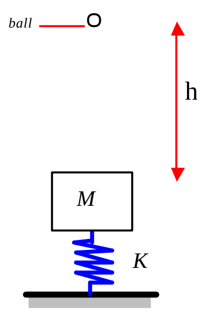
A bullet of mass *(m)* equal to 10 grams traveling at a speed of 100 m/s hits and embeds into a stationary massive flexible support (M=10 kg, K=100 kN/m).

a) Explain the physics of the problem and note the major assumptions and simplifications used
b) Determine the maximum velocity [m/s] the massive support will ever achieve.
c) Find the maximum deflection [mm] of the spring element.



A 1 kg steel ball is dropped from a height (h=1 m) and impacts (an initially stationary) massive and elastic table (M=10 kg, K=100 kN/m). If the collision is perfectly elastic, and the ball rebounds to a height of 0.50 m,

d) Explain the physics of the problem and note the major assumptions and simplifications used
e) Determine the maximum velocity [m/s] the massive support will ever achieve .
f) Find the maximum deflection [mm] of the spring element..



A block of mass M is connected to ground by a spring and subjected to a **nonlinear magnetic force**. This force varies with current $i_{(t)}$ and position $X_{(t)}$ as $f_M = \left[a\, i^2 / (g - X)^2\right]$, where a is a physical constant and g is the gap between M and the magnet when $X=0$. The static position is given when $i=0$.

a) derive the nonlinear equation of motion for the system,
b) linearize the equations of motion about the reference value $i=i_o$ and $X=X_o$,
c) determine the natural frequency of the system for the reference values of current and displacement. Does this natural frequency show a peculiar behavior? Explain your answer.

A flexible cable of spring constant *K* is used to lift a vehicle of mass *M* using a helicopter. The helicopter and vehicle are at rest and the cable is taut at time $t=0$ so the vehicle is suspended. The helicopter moves vertically upwards according to $y_h=t$ *for* $t>0$.

a) Derive the equation of motion for the vehicle vertical motion y_v, and
b) Determine the vehicle motion $y_v(t)$ *for* $t>0$ as a function of the parameters *K*,*M*.

A machine, mass M=500 kg, is known to cause unacceptable vibrations in nearby equipment and is to be mounted on available **vibration isolators** as shown in the figure. The isolators collectively have an effective stiffness K = 200,000 N/m and viscous damping D=2,000 N.sec/m. If the excitation force **on** the machine is periodic, i.e. $F=F_o \cos(\omega t)$, where $F_o=3{,}000\ N$ and $\omega=60\ rad/sec$ is the operating frequency of the machine, determine

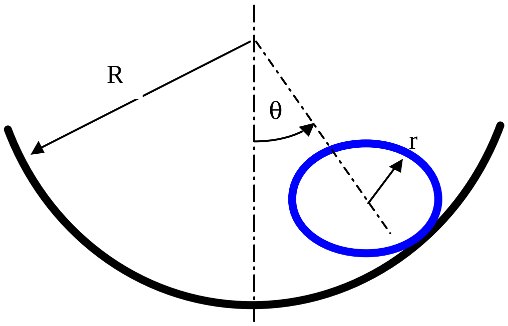
a) The amplitude of the transmitted force [N] to the foundation at the frequency of excitation (ω=60 rad/s).
b) The maximum transmitted force if (**for some unfortunate reason**) a force with a frequency equal to the system natural frequency, i.e. $F=F_o \cos(\omega_n t)$, is exerted **on** the machine.
c) The amount by which the stiffness of the vibration isolators can be decreased in order to reduce the transmitted force by 50% at the operating frequency ω=60 rad/sec, i.e. reduce (a).

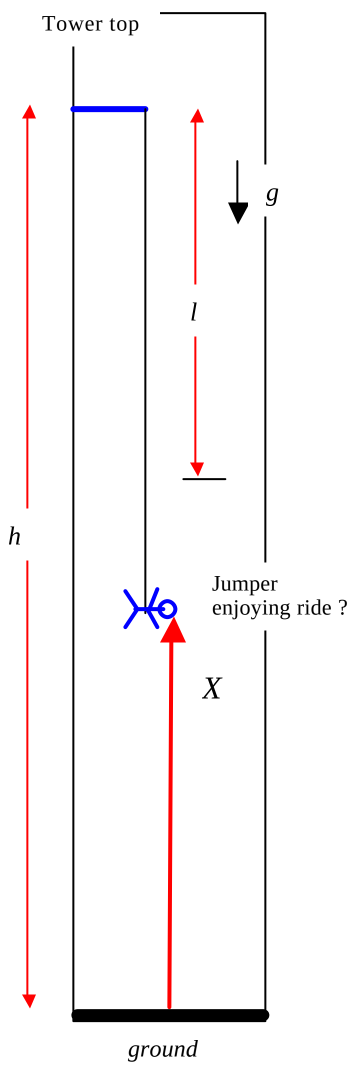
An electric motor operates mechanical equipment at a speed of 1,800 rpm. The system is supported on rubber pads which show a static deflection of 4 mm under the motor weight. Determine the transmissibility (ratio of force to the base foundation) if the damping ratio of the rubber pads is ξ=0.20

The figure below shows a solid cylinder of mass *(M)* and radius *(r)* rolling without slipping on a concave surface of radius of curvature (R). The cylinder mass moment of inertia about its cg is $I=0.5\ M\ r^2$. Determine

a) the appropriate kinematic constraint(s) between the cylinder rotation and translation.
b) The equation of motion for small amplitudes angular motions about the equilibrium position.
c) The natural frequency of the system.

You may use Newton's laws or the principle of conservation of mechanical energy to derive the EOM.





Consider the motion of a bungee jumper **as seen from ground** (see Figure). The jumper falls from a structure at a height *(h)* 50 m above ground. The bungee cord natural length (*l*) is 15 m, with stiffness *K*=73.6 N/m and viscous damping *D*=15 N.s/m. neglect the effect of air drag.

a) Determine the jumper speed at the instant in time when the bungie cord starts to stretch.

b) Derive the equation of motion of the jumper (average weight *W*=75 kg) **as seen from a coordinate system *X(t)* with its origin at ground level**. Give the initial conditions for the motion. (**DO NOT use any other reference frame but the one requested**).

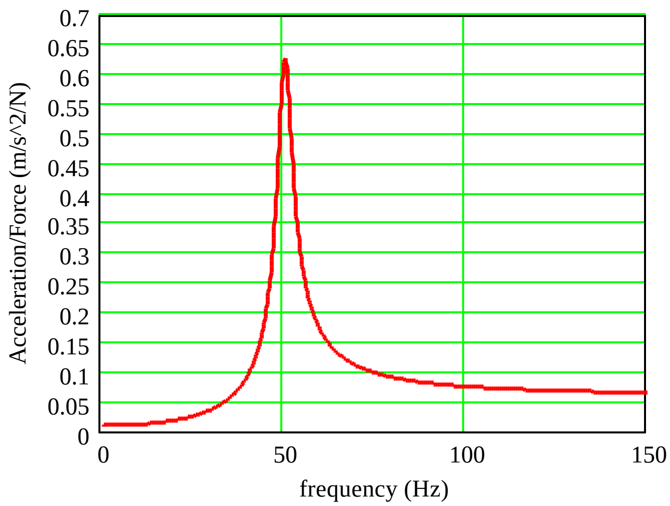
c) Calculate the natural frequency (w_n [rad/s]) and viscous damping ratio (*x*) of the cord-jumper system.

d) Calculate the final (steady state) position of the jumper. How far from ground would the jump er "hung"?

e) Consider the response without any damping and determine (1) How close will the jumper ever be from ground? (2) The largest peak acceleration he/she feels?
DO NOT solve the EOM, Use physical knowledge.

-

Dynamic measurements were conducted on a mechanical system to determine its FRF. Forcing functions with multiple frequencies were exerted on the system and a digital signal analyzer (FFT) recorded the magnitude of the ACCELERATION/FORCE ([m/s^2]/N) Frequency Response Function, as shown below. From the recorded data determine the system parameters, i.e. natural frequency (ω_n:rad/s) and damping ratio (ζ), and system stiffness (K:N/m), mass (M:kg), and viscous damping coefficient (D:N.s/m). Explain procedure of ANALYSIS/INTERPRETATION of test data for full credit.

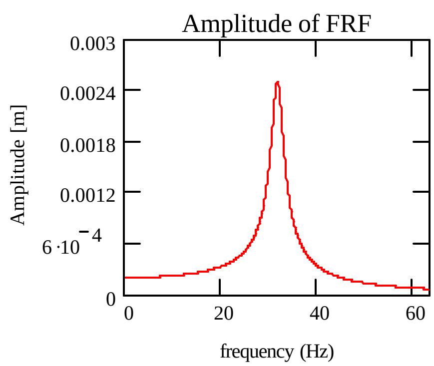


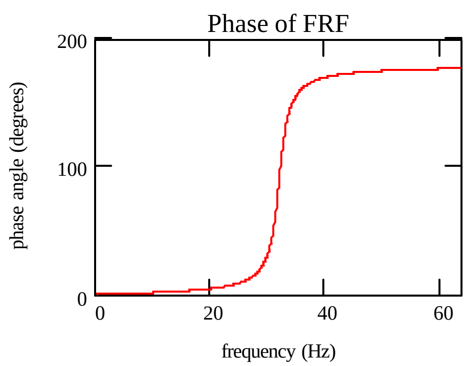
Magnitude of FRF for mechanical system

*

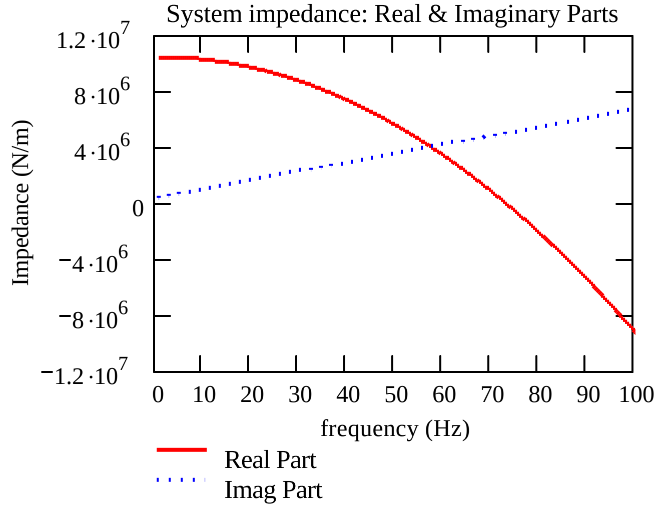
The figures below show the recorded amplitude and phase of the FRF of a simple mechanical system. A periodic force of constant magnitude (2000 N) and varying frequency was applied into the system to obtain the dynamic displacement measurements. From the test data shown

a) Indicate the major characteristics of the system (1st or 2nd order, under or overdamped, etc.)
b) Estimate the time constant (s) or natural frequency of the system (rad/s), whichever is appropriate.
c) Estimate the stiffness of the system in [N/m].
d) Estimate the damping coefficient of the system in [N s/m].
e) Sketch on the Figure, the FRF (amplitude and phase) for a value of damping equal to two times the magnitude determined in (d). Give a value of the maximum amplitude of motion in [m].



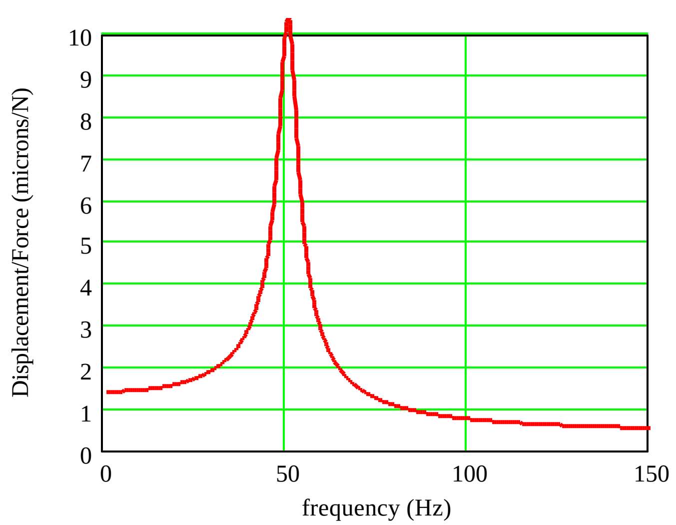


The estimation of physical parameters in a mechanical system is of importance for their appropriate design and performance. Some experiments where performed in a simple system where an external swept-sine load of constant magnitude and frequency ranging from 1 to 100 Hz excited the system. Measurements of the applied load and system displacement response were recorded with a FFT analyzer. The graph below shows the real and imaginary parts of the system impedance, i.e. the ratio of applied force to displacement [N/m], versus frequency [Hz]. From the data shown, determine the system parameters (stiffness, mass and damping = K, C, M) and provide values for the system natural frequency and damping ratio.



*

Dynamic measurements were conducted on a mechanical system to determine its FRF. Forcing functions with multiple frequencies were exerted on the system and a digital signal analyzer (FFT) recorded the <u>magnitude</u> of the DISPLACEMENT/FORCE (microns/N) Frequency Response Function, as shown below. From the recorded data determine the system parameters, i.e. natural frequency (ω_n:rad/s) and damping ratio (ζ), and system stiffness (K:N/m), mass (M:kg), and viscous damping coefficient (D:N.s/m). Explain procedure of ANALYSIS/INTERPRETATION of test data for full credit. NOTE that 1 micron= 10^{-6} m

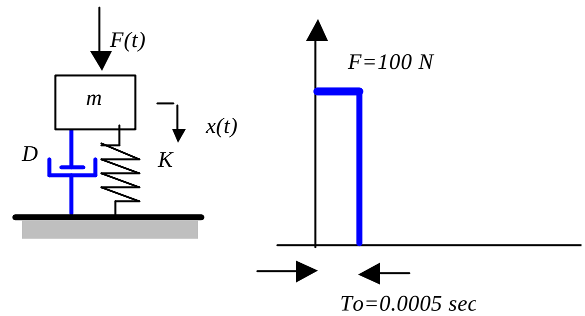


Magnitude of FRF for mechanical system

*

A spring-mass-damper system is subjected to a **pulse** force $F(t)$ as shown. If k=1000 N/m, m=2.5 kg, and C=1 N-sec/m,

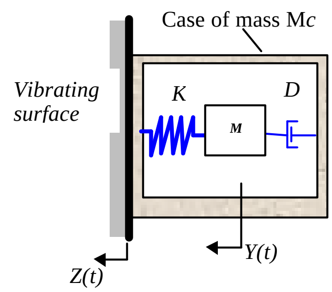
a) Calculate the system natural frequency (rad/sec) and damping ratio.
b) Provide an **engineering** estimation for the maximum velocity of the system.
c) What would be the maximum displacement response of the system? Use the knowledge found in (a).
d) Draw sketches of the system displacement and velocity responses vs. time.



<u>Detail all used assumptions or simplifications for full credit</u>

The figure shows a seismic instrument with its case rigidly attached to a vibrating surface. The instrument displays the motion $Y(t)$ **<u>RELATIVE</u>** to the case motion $Z(t)$.

a) Derive the equation of motion for the instrument using the **relative motion *Y(t)*** as the output variable.
b) Determine the instrument natural frequency w_n [rad/s] and critical damping D_c [lb.s/in] for M=0.02 lb and K=13 lb/in.
c) When will the instrument show $Y=0$, i.e. no motion? .
d) If the vibrating surface moves with a constant acceleration of **3 g,** what will the instrument display at s-s? Give value in inches.
e) What is the instrument response (magnitude and phase) if the surface vibrates with an acceleration of amplitude **3 g** at a frequency of 200 Hz ? Assume a damping ratio of 5 %.



Note: the sensor works under all attachment configurations, i.e. vertical, horizontal, top, bottom, etc.

The mechanism shown consists of two <u>massless</u>, <u>rigid</u> links (*AB* and *BC*) and a slider of mass *M*. Both links have the same length (*l*). Connecting pins *B* and *C* are frictionless, while pin *A* has a rotational viscous drag coefficient, D_q. The slider's velocity is resisted by a viscous like damping action (D_v), one on <u>each</u> side of the guide channel. A motor applies an external torque $T_m(t)$ to link *AB* which makes $q(t)$ oscillate about its reference position, $q=0$. Determine (give or write)

a) The kinematic constraint equation relating the motion of the slider (X) to the rotation $q(t)$.

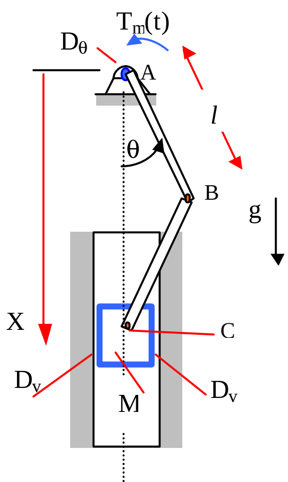
b) An expression for the system kinetic energy (E_k) in terms of $q(t)$ (and its time derivatives) and appropriate system constants.

c) An expression for the system potential energy (E_p) in terms of $q(t)$ and appropriate system constants.

d) Expressions for the dissipated power (P_v) and drive (input) power (P_{in}) in terms of $q(t)$.

e) Using the PCME, derive the equation of motion for the system in terms of $q(t)$ and its time derivatives.

Note: the final nonlinear equation is of the form, $f_{1(q)}\ddot{q} + f_{2(q)}\dot{q}^2 + f_{3(q)}\dot{q} + f_{4(q)} = T_m$

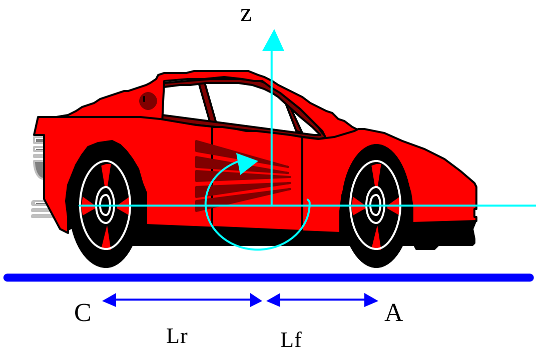


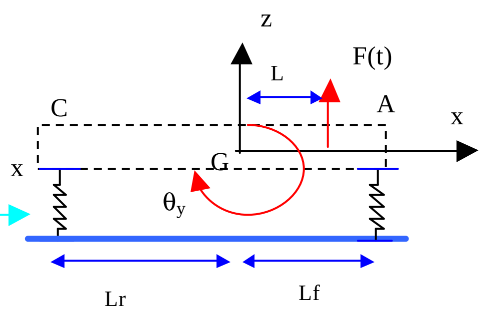
The diagram below shows the simplest <u>planar</u> model of an automobile suspension <u>without any damping</u>. It accounts for the vertical displacement of the center of mass z_G and pitching rotations q_y. The rigid chassis has mass (m [kg]) and centroidal radii of gyration, r_y [m]. The stiffness and viscous dashpots of the front end and rear end supports are K_A and K_C [N/m], and C_A and C_C [N.s/m], respectively.

a) Determine expressions for the motion of the chassis corners (A, C) as a function of the translation (z_G) and small rotations (q_y) of the chassis.

b) Derive the equations of motion for the system and express them in matrix form.

c) Why is not the car weight a part of your final EOMs? Explain in words.





Model of simple suspension system

The equations of motion of the suspension system (above) including viscous damping are given in matrix form as:

$$\begin{bmatrix} m & 0 \\ 0 & I_y \end{bmatrix} \begin{Bmatrix} \ddot{z}_G \\ \ddot{q}_y \end{Bmatrix} + \begin{bmatrix} C_{zz} & C_{zy} \\ C_{yz} & C_{yy} \end{bmatrix} \begin{Bmatrix} \dot{z}_G \\ \dot{q}_y \end{Bmatrix} + \begin{bmatrix} K_{zz} & K_{zy} \\ K_{yz} & K_{yy} \end{bmatrix} \begin{Bmatrix} z_G \\ q_y \end{Bmatrix} = \begin{Bmatrix} F_z \\ M_y \end{Bmatrix} \quad \text{or} \quad M\,\ddot{q} + C\,\dot{q} + K\,q = Q \qquad (1)$$

where $q=(z_G\ ,\ q_y\ )^T$ and $Q=(\ F_z\ ,\ M_y)^T$ are the vectors of generalized displacements and forces; and M, K and C are $(2x2)$ matrices containing the generalized system inertia, stiffness and viscous damping coefficients, respectively. **Write equation (1) in the <u>state space</u> form**, i.e. as

$$\dot{W} = AW + b\,, \qquad \text{where} \quad W = \{\dot{q} \quad q\}^T = \{\dot{z}_G \quad \dot{q}_y \quad z_G \quad q_y\}^T .$$

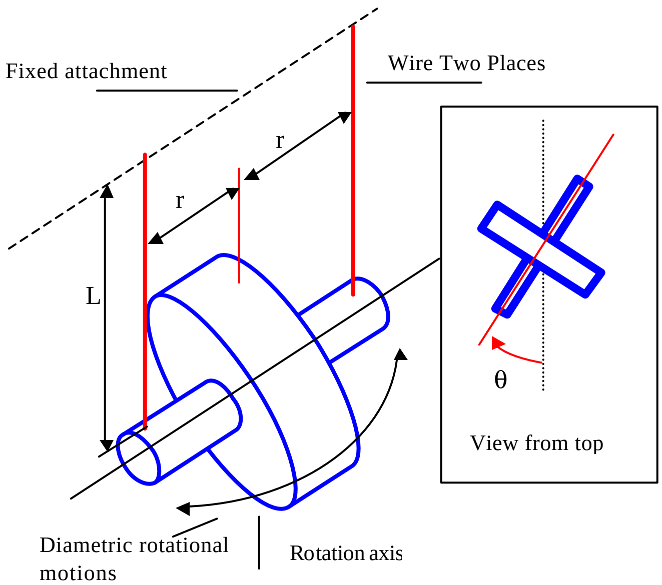
Specify the elements of the transfer matrix A and the forcing vector b.

The figure shows a typical rotational pendulum arrangement for measuring the radius of gyration (*k*) (i.e. mass moment of inertia $I_D = m\,k^2$) of a rotor. The rotor is hanged on wires that have negligible mass. Each wire of length *L* is located a distance *r* from the rotor center of mass. From small amplitude motions *q(t)*, the procedure requires to record the natural period of motion and then extract the radius of gyration from a simple engineering formula.

a) Show that the (linearized) equation of motion for the arrangement shown is $\ddot{q} + \frac{r^2 g}{k^2 L} q = 0$ (1), where *k* is the radius of gyration.

b) Some measurements were conducted with a rotor for a small water pump application. The rotor weighed 10 lb, and *L*=30 inch, *r*= *5* inch. The rotor performed 12 oscillations in 10 seconds. Determine, the natural frequency of the system [rad/s], the radius of gyration *k* [inch] and the rotor diametrical mass moment of inertia I_D [lb-inch-s^2].

(*) Hint: Use PCME and small amplitude motions to derive the EOM.



The figure shows a simple mechanical system. The flexible cable, represented by a stiffness K_c, pulls a massive block (M) which is attached to a ground spring (K_d). The rough walls of the channel introduce drag, of the dry-friction (μ) and viscous damping (D) types. You pull with motion $Z(t)$ and are interested on finding out the motion of the block $X(t)$.

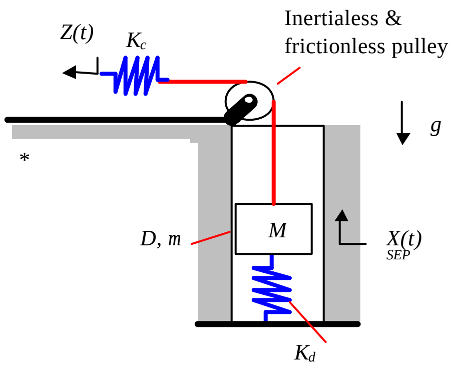
f) Draw a free body diagram of the block and note <u>all</u> forces of importance.

g) Establish the appropriate constitutive relationships for the mechanical elements.

h) Derive the equation of motion for the block.

i) Determine the system natural frequency (w_n [rad/s]) if M=10 lb, K_c=1000 lb/in, K_d=100 lb/in.

j) If the dry-friction force equals 20 lbs, what is the minimum displacement Z_* one must pull to initiate the block motion ?



The mechanism shown consists of a cylinder of mass (M_c) and mass moment of inertia (I_c) connected with a frictionless pin to a uniform bar of mass (m_b) and centroidal mass moment of inertia (I_b). The other end of the bar is also pinned at A and which has a rotational viscous drag coefficient, D_q. A motor applies an external torque $T_m(t)$ to the bar and which makes the bar and cylinder oscillate with large motions $q(t)$. Note that the cylinder rolls without slipping on the cylindrical surface shown. Determine (give or write)

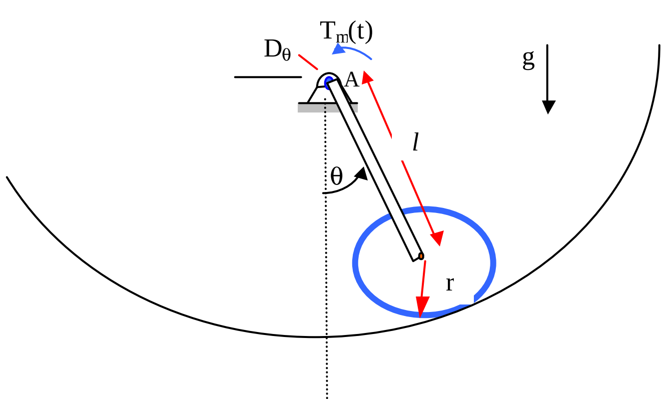
f) The kinematic constraint relating the bar rotation $q(t)$ to the rolling motion of the cylinder.

g) An expression for the system kinetic energy (E_k) in terms of $q(t)$ (and its time derivatives) and appropriate system constants.

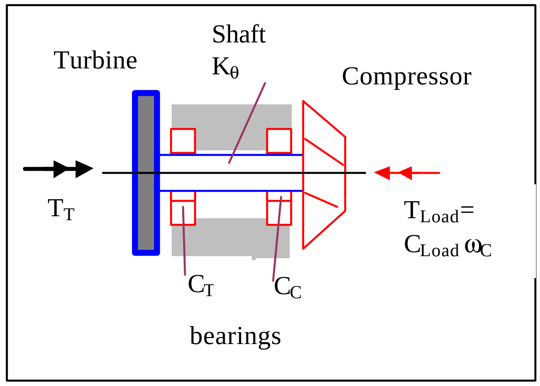
h) An expression for the system potential energy (E_p) in terms of $q(t)$ and appropriate system constants.

i) Expressions for the dissipated power (P_v) and drive (input) power (P_{in}) in terms of $q(t)$.

j) Using the PCME, derive the equation of motion for the system in terms of $q(t)$ and its time derivatives.



The figure shows a turbine (T) driving a compressor (C) through an elastic shaft. The turbine delivers a constant torque T_T=6500 Nm, and the compressor reacts with a torque load proportional to the compressor speed, i.e. $T_{Load}=C_{Load}\,\omega_C$, where C_{Load}=20 Nm.s/rad. The bearings, located close to the T and C show viscous drag with coefficients C_T=0.5 Nm.s/rad and C_C=0.25 Nm.s/rad, respectively. The T and C mass moments of inertia are I_T =10 kg.m^2 and I_C=5kg.m^2. The (massless) transmission shaft stiffness is K_θ=0.8 x 10^5 Nm/rad.



a) Derive the equations of motion for the turbomachinery. Establish these equations in state-space form with $q^T=\{\Delta\theta, \omega_T, \omega_C\}$, i.e turbine and compressor speeds and shaft twist angle.
b) Determine the steady-state values for the rotating system, i.e. the operating T and C speeds and shaft twist angle. [Hint: Use Power Balance at s-s).
c) Calculate the power lost in the bearings, and if the cost of a kWh is 7 cents, estimate the ($) cost to overcome the bearing frictional drag in a day of operation (24 h)? Is this cost acceptable?

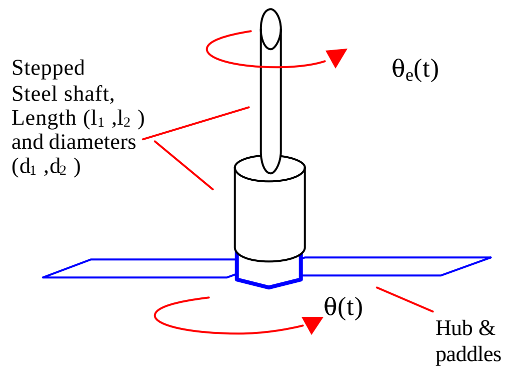
The T and C operate at its steady-state speed ($\omega_T=\omega_C$= 313.2 rad/s) and $\Delta\theta$=0.079 rad [4.54 °]). Suddenly, the shaft breaks causing a general plant alarm. Assuming that the shaft failure has not wrecked the whole system, determine:

a) dynamic response of the compressor as a function of time (i.e, provide the EOM, a formula for $\omega_C(t)$, and a graphical sketch of the response) . Will the compressor come to a stop, when will this happen (give a value in sec)? [
b) dynamic response of the turbine as a function of time (i.e, provide the EOM, a formula for $\omega_T(t)$, and a graphical sketch of the response) . Will the turbine over speed or decelerate? Determine a value of the system time constant and the terminal speed of the turbine? Is this speed safe?
c) Explain if the events in (b) are dangerous? What may have caused the shaft failure? As an engineer, what would you do to reduce the risks of severe damage to the turbine?.

A lawn mower is composed of the paddles and hub connected to a stepped shaft. The torsional spring constants (**k_q**) for the top and bottom shafts are equal to A and B, respectively. The mass moment of inertia (***I***) of the hub and blades is kg.m^2**.** The grass clips introduces a load modeled as viscous drag with a damping coefficient (**D_q**) equal to N.m/(rad/sec)**.**

Assume that the angular vibration of the engine $\theta_e(t)$ is known, i.e. an input to the shaft.

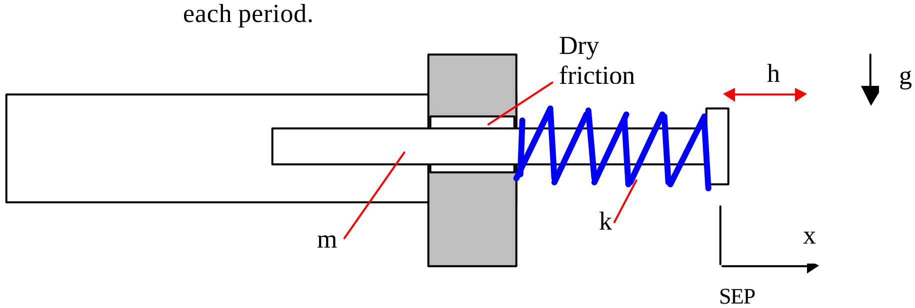
a) Give an expression for the torsional stiffness of the shaft, i.e. in terms of geometry and material properties.
b) Derive the equation of motion for the mower angular displacement $\theta(t)$.
c) What is the natural frequency (ω_n) and damping ratio (ζ) of the system.



Schematic view of paint mixer

The plunger mechanism of a pin ball machine is shown below. The plunger is displaced *h*=2 inch from its equilibrium position and then released. There is dry friction (μ=0.3) at the guiding cylinder. The plunger equivalent stiffness *(K)* and mass (*M*) are 1 lb/in and 0.25 lb, respectively.

a) Derive an equation describing the plunger motion.
b) Sketch the dynamic response of the plunger after its release.
c) For home many cycles of motion will the plunger move? How fast will the amplitude of motion decay each period.



A device to mix painting is composed of the paddles and rigid hub connected directly to a DC drive electric motor. The motor characteristic curve as a function of angular speed (ω) is shown in the figure. The mass moment of inertia (***I***) of the hub and blades is 2 kg.cm^2**,** and the painting introduces a viscous damping ($\mathbf{D}_q$) equal to $1x10^{-2}$ N.cm.sec/rad**.**

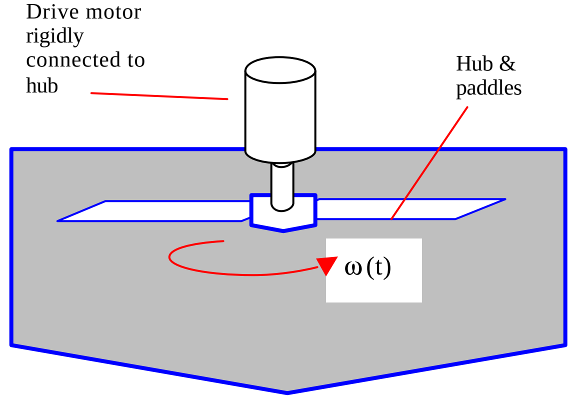
a) Derive the equation of motion for the angular velocity of the mixer $\omega(t)$.

d) Calculate the system time constant (τ). How does the motor affect it?

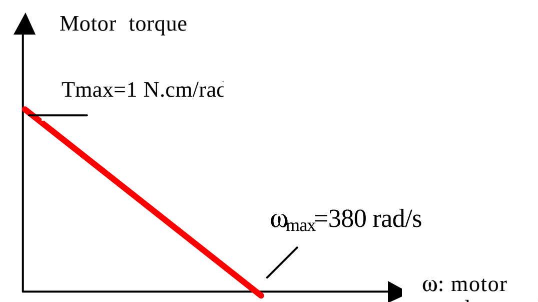
e) The mixer is stationary and the motor is turned on. Present a formula describing the mixer angular speed vs. time.

f) What is the steady state angular speed of the mixer? What would be this speed if the painting were twice as viscous?

g) If the mixer is suddenly removed from the paint bucket, how fast will the motor spin?



Schematic view of paint mixer



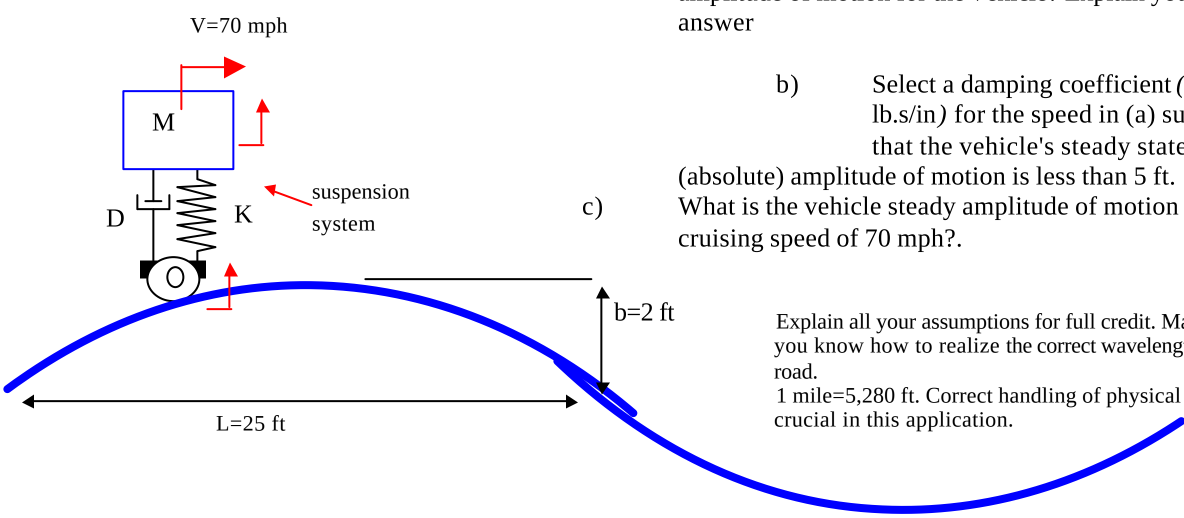
Characteristic performance curve of motor

The figure below shows a vehicle moving horizontally along a wavy path at a cruising speed (*V*) of 70 mph. Prior tests showed that the vehicle weighing 2000 lb has a natural frequency of 5 Hz. Neglect the influence of the tire's bouncing mode and determine,

a) What car speed in mph will cause the highest amplitude of motion for the vehicle? Explain your answer

b) Select a damping coefficient *(D,* lb.s/in*)* for the speed in (a) such that the vehicle's steady state (absolute) amplitude of motion is less than 5 ft.

c) What is the vehicle steady amplitude of motion at the cruising speed of 70 mph?.



Explain all your assumptions for full credit. Make sure you know how to realize the correct wavelength of the road.
1 mile=5,280 ft. Correct handling of physical units is crucial in this application.