MEEN FA11 – Exam 3 – Problem 3: Derive EOM for simple 1DOF system

In the figure, a towing truck (not shown) is about to pull a trailer. The trailer transports a cargo of mass M that is held in place with two elastomeric cables of stiffness **K**, each initially stretched with force $\mathbf{F_{asy}}$. The cables also offer viscous damping (energy dissipation) as denoted by the damping coefficient **C**. At t>0 s, the truck pulls the trailer with known displacement $\mathbf{Z}_{(t)}>0$.

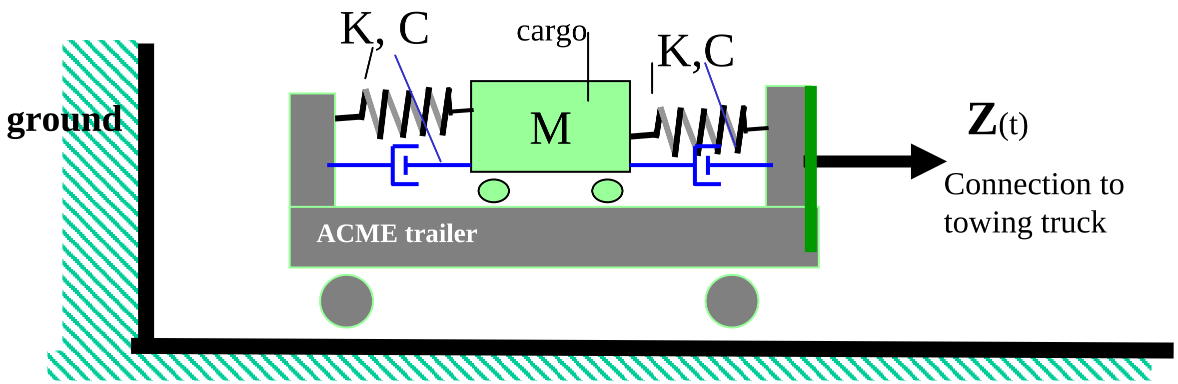
a) Define a coordinate system for the motion of the cargo. Explain your choice [5]

b) Assume a state of motion and draw a complete free body diagram for the system. [10]

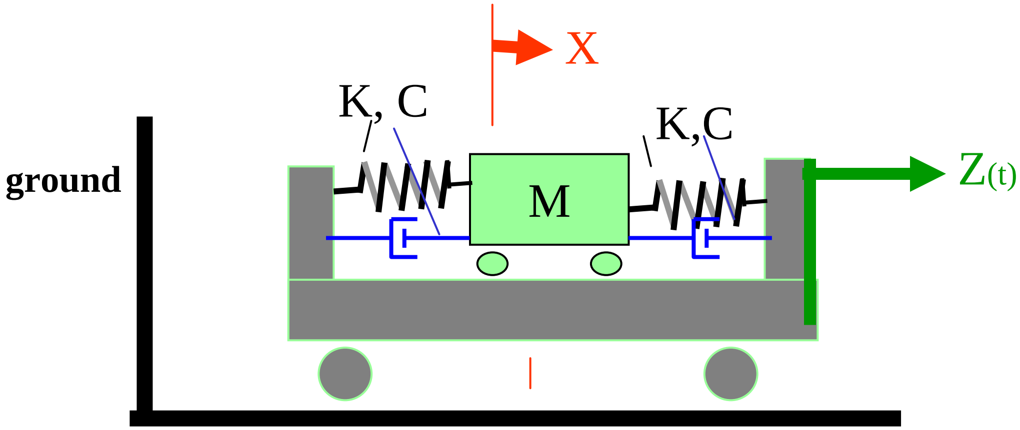
c) State an EOM for the cargo [5]. Using definitions for spring and dashpot forces in terms of selected motion coordinates, **derive** the equation governing the motion of the cargo for t>0 s. [10].

d)The motion is better observed in terms of a motion coordinate relative to the trailer displacement **Z**, say $\mathbf{Y=X_{cargo}-Z}$. Express the equation of motion with **Y**(t) as the independent variable. [5]

Trailer: A large transport conveyance designed to be pulled by a truck or tractor

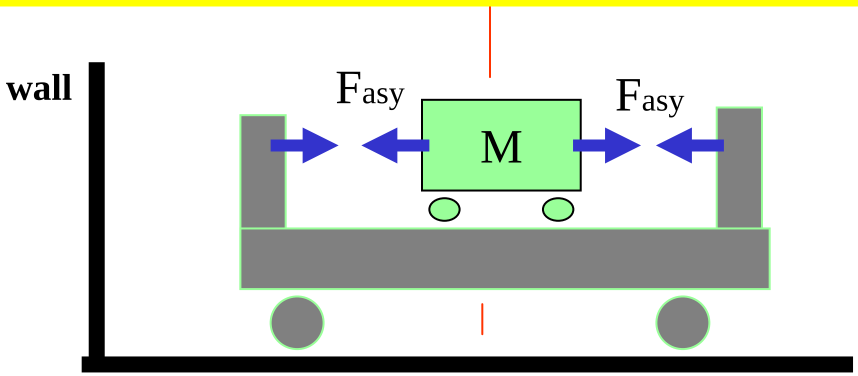


Assume: no friction between wheels and surfaces



Assumed: no friction between wheels and surfaces

DIAGRAM of forces at STATIC EQUILIBRIUM POSITION



SEP: X =Z = 0

Notes:
Towing truck not acting, *Z(t)*=0
Trailer motion starts at *t>0*
Weights and Normal forces omitted from Free Body Diagram

DEFINITIONS:

Forces:
W: weight
N: normal force
Fs: force from elastomeric cable
F_{asy}: assembly force for springs (extension or stretched)

Parameters:
M: mass
K: stiffness coefficient
C: viscous damping coefficient

Variables:
Z: coordinate for motion of trailer - known
X: coordinate for motion of cargo
(Absolute frame of reference, with origin at state of rest of trailer and cargo)

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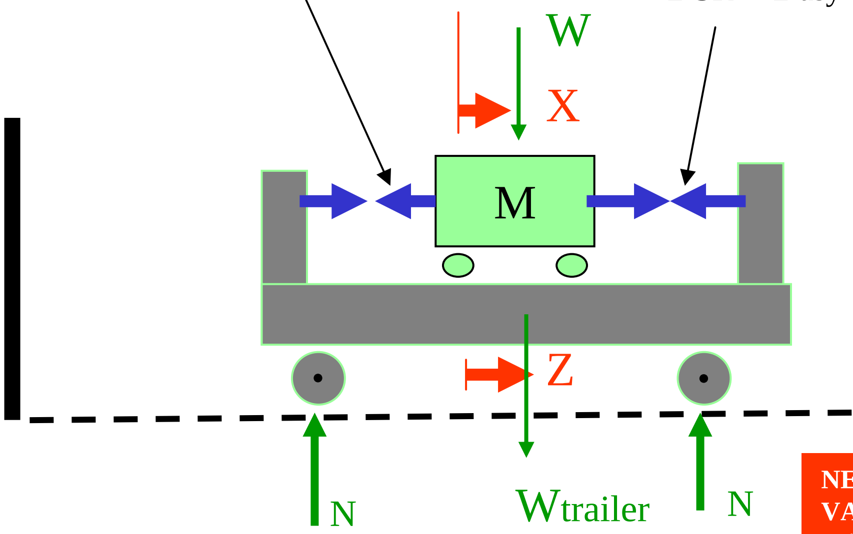
FREE BODY DIAGRAM

forces for SYSTEM UNDERGOING MOTIONS

Assume a state of motion to draw FBD: $Z > X > 0$

$F_{SL} = F_{asy} - K(Z - X) - C(\dot{Z} - \dot{X})$

$F_{SR} = F_{asy} + K(Z - X) + C(\dot{Z} - \dot{X})$



DEFINITIONS:

Forces:

W: weight

N: normal force

Fs: cable reaction force (adds elastic + damping effects), L: left, R: right side

F_{asy}: assembly force for cables (extension or stretched)

Parameters:

M: mass cargo

K: stiffness coefficient

C: viscous damping coefficient

Variables:

Z(t) : base motion (known)

X : coordinate for motion of cargo **(Absolute frame of reference)**

Derive Equations of Motion:

NEWTON's LAW is VALID ONLY With reference to an ABSOLUTE COORDINATE SYSTEM

STEP 1: State EOM for cargo

$$M\ddot{X} = F_{SR} - F_{SL} \quad (1)$$

STEP 2: Substitute spring and dashpot forces defined in terms of motion

$$M\ddot{X} = F_{asy} + K(Z - X) + C(\dot{Z} - \dot{X}) - F_{asy} + K(Z - X) + C(\dot{Z} - \dot{X})$$

$$M\ddot{X} = 2K(Z - X) + 2C(\dot{Z} - \dot{X}) \quad (2)$$

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Derive Equation of Motion:

STEP 3: Cancel common terms in Eqs. (2)

and move to LHS terms that depend on motion

$$M\ddot{X} + 2K\,X + 2C\,\dot{X} = 2K\,Z + 2C\dot{Z} \quad \text{(3a)}$$

$$M\ddot{X} + 2K\,X + 2C\,\dot{X} = 2K\,Z + 2C\dot{Z} \quad \text{(3b)}$$

STEP 4: Since the motion of the cargo relative to the trailer is of interest, rewrite EOM (3) with a relative displacement coordinate *Y(t)*.

Hence Eqn (2)

$$M\ddot{X} = 2K(Z - X) + 2C(\dot{Z} - \dot{X})$$

is recast as

Define a relative motion coordinate

$$Y = (X - Z), \text{ then } \ddot{Y} = (\ddot{X} - \ddot{Z}) \quad \text{(4)}$$

$$\ddot{X} = (\ddot{Z} + \ddot{Y}) \quad \text{(5)}$$

$$M\ddot{Y} + 2K\,Y + 2C\,\dot{Y} = -M\,\ddot{Z} \quad \text{(6)}$$

which is the desired EOM for the cargo system.

Note: It is (perhaps) evident that deflection *Y* is easier to record (measure) than absolute *X* or *Z*

Note that using the relative motion coordinate **Y**, the EOM in the moving coordinate system shows the "appearance" of an "inertial-like" force **–M d²Zdt²**.