Derive EOMs for Mechanical System

The figure shows a heavy block weighing $W=Mg$ that is connected to springs with stiffness K_1 and K_2. A frictionless pulley and (inextensible) cable connect spring 1 to the block. The cable does NOT slip on the pulley. Upon assembly of the system, both springs are pre-stretched (in tension or preloaded). The system is initially at rest at its **static equilibrium position** (SEP=assembled configuration).

At time $t>0$ s, an external force **F**(t) acts on the block and **drives** the system into motion. **There is viscous drag along the inclined wall and represented by damping coefficient *C*.**

a) Find the **equations of static equilibrium**, i.e. forces necessary to keep system together.

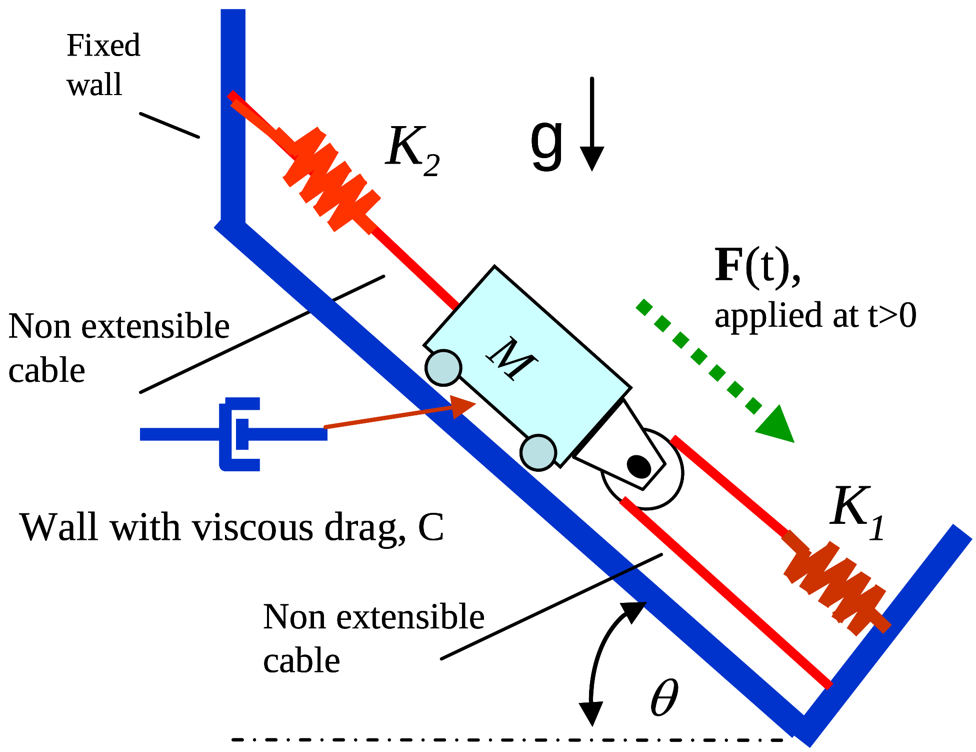
b) Specify coordinates for motion of the block and springs and **with origins at the SEP**. Draw coordinates on figure.

c) Identify the kinematic constraint relating motions of the block and spring 1.

d) Draw **complete** free body diagrams applicable for time $t>0$ s. Label (name) all forces and relate them to the motion coordinates.

e) Using Netwon's Laws, derive the EOM for the block ($t>0$ s).

Explain your solution procedure, detail assumptions for full credit. Work that only shows formulas without descriptions will be counted as 50% of your grade.



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P3 SP09 - Derive EOM for simple mechanical system

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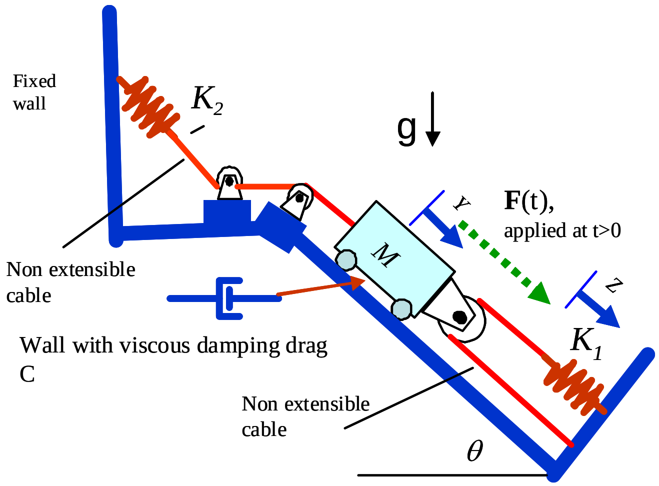
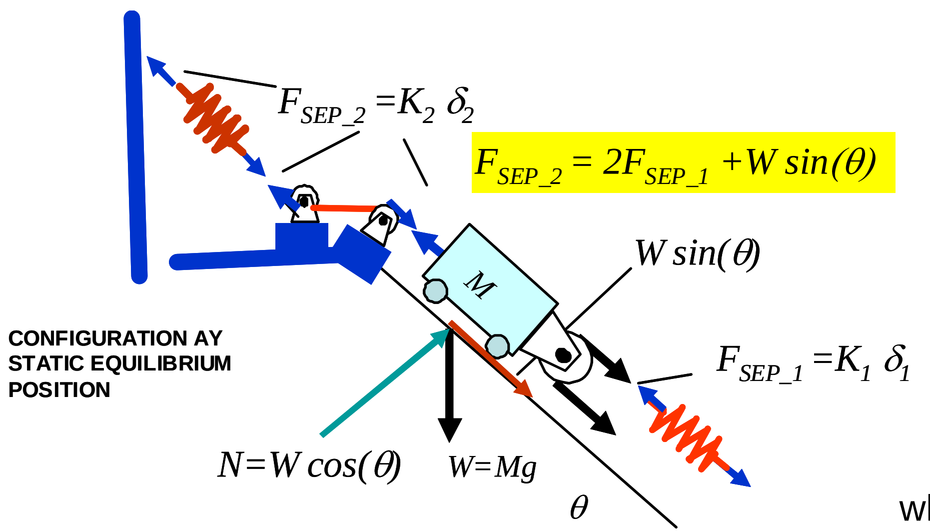


Let:
Y(t) be coordinate describing displacements of block M from SEP, i.e. Y=0 means SEP
Z(t) be coordinate describing displacement of free end of spring 1 from SEP

Upon assembly both springs are pre-stretched (in tension)

(a) Static deflection of springs

By definition of SEP (Static equilibrium position), i.e. when Z=Y=0 and at rest (without motion):



where $F_{SEP} = T$ **is tension in a spring**

<u>For static equilibrium, the forces must be:</u>

$$F_{SEP\_2} = W \cdot \sin(\theta) + 2 \cdot F_{SEP\_1} \qquad \textbf{(1)}$$

$$F_{SEP\_2} = K_2 \cdot \delta_2 \qquad N = W \cdot \cos(\theta) \quad \text{normal force}$$

$$F_{SEP\_1} = K_1 \cdot \delta_1$$

where δ's are the spring deflections at the equilibrium condition, i.e. the assembly configuration

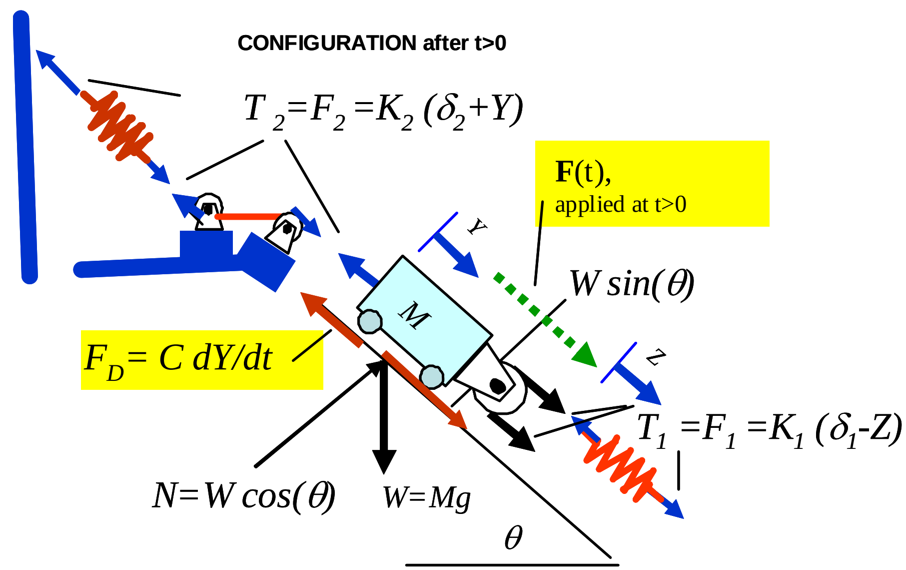
(b) coordinate systems and kinematic constraint in pulley- inextensible cable

The cable length is constant, thus

$l_c = l_c + Z - 2Y$ **and the kinematic constraint follows as** $Z = 2Y$ **(2)**

(c) Freee body diagram: assume a state of motion with Y=1/2 Z >0, F(t) > 0 for t>0

Assumed state of motion to draw FBD : *Y>0, Z>0*



Cable tensions = elastic forces from springs

$$F_2 = T_2 = F_{SEP\_2} + K_2 \cdot Y$$

$$F_1 = T_1 = F_{SEP\_1} - K_1 \cdot Z \qquad \textbf{(3)}$$

$$F_D = C \cdot \frac{d}{dt} Y$$ viscous drag force

Apply Newton's law to derive EOM:

$$M \cdot \frac{d^2}{dt^2} Y = 2 \cdot T_1 + W \cdot \sin(\theta) + F(t) - F_D - T_2 \qquad \textbf{(4)}$$

Note: EOM cannot contain internal forces (Tension for example). The tension is DETERMINED by the motion.

Substitute the constitutive forces, Eqn (3) into EOMs (4) to obtain

$$M \cdot \frac{d^2}{dt^2} Y = F(t) - C \cdot \frac{d}{dt} Y + W \cdot \sin(\theta) + 2 \cdot \left(F_{SEP\_1} - K_1 \cdot Z\right) - \left(F_{SEP\_2} + K_2 \cdot Y\right) \qquad \text{(5a)}$$

canceling the forces from assembly (static equilibrium position) $F_{SEP\_2} = W \cdot \sin(\theta) + 2 \cdot F_{SEP\_1}$ leads to

$$M \cdot \frac{d^2}{dt^2} Y = F(t) - C \cdot \frac{d}{dt} Y + 2 \cdot \left(-K_1 \cdot Z\right) - \left(K_2 \cdot Y\right) \qquad \text{(5b)}$$

Using the kinematic constraint of pulley $Z = 2Y$

gives:

$$M \cdot \frac{d^2}{dt^2} Y = F(t) - C \cdot \frac{d}{dt} Y - 2 \cdot 2 \cdot K_1 \cdot Y - K_2 \cdot Y$$

leads to the final EOM:

$$M \cdot \frac{d^2}{dt^2} Y + C \cdot \frac{d}{dt} Y + \left(4 \cdot K_1 + K_2\right) \cdot Y = F(t) \qquad \textbf{(6)}$$