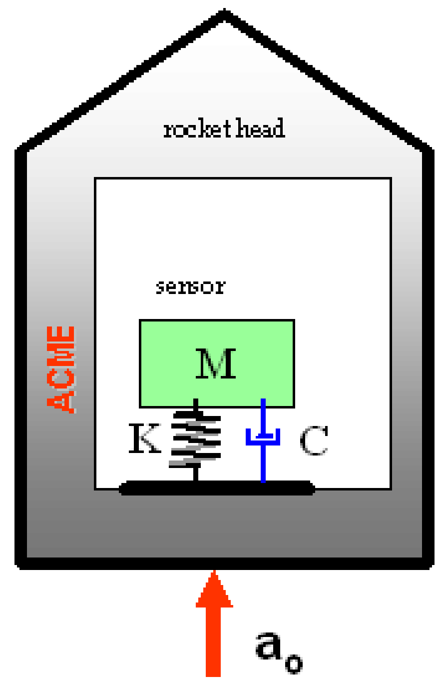
MEEN 363 QUIZ 3 (FA10) Names:

35 min.



An instrument (sensor) installed in the nose of a rocket is cushioned against vibration with a soft spring-damper. The rocket, fired vertically from rest, has a constant acceleration a_o. The instrument mass is *M*, and the support stiffness is *K* with damping *C*. The instrument-support system is underdamped. The **motion** of the instrument **relative to the rocket** is of importance.

a) Define motion coordinates and derive the equation of relative motion for the instrument [25]

b) Give or find an analytical expression for the relative displacement of the instrument as a function of time. Express your answer with well defined physical parameters and variables. [25]

c) Given M=1 kg, K=1 N/mm, and damping ratio ζ=0.10. Find the natural frequency & damping coefficient *C* of the system [25]

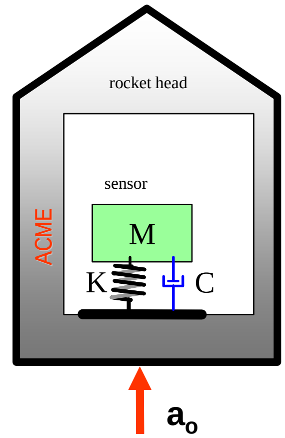
d) For a_o=3g, find the steady-state displacement of the instrument, relative to rocket **and** absolute (with respect to ground). [25]

Note: Please remember that Newton's EOMs are valid in an inertial reference frame ONLY.

Q3: Description of motion in a moving reference frame

An instrument package installed in the nose of a rocket is cushioned against vibration with a soft spring-damper. The rocket, fired vertically from rest, has a constant acceleration *ao*. The instrument mass is *M*, and the support stiffness is *K* with damping *C*. The instrument-support system is underdamped. The **relative motion** of the instrument **with respect to the rocket** is of importance.

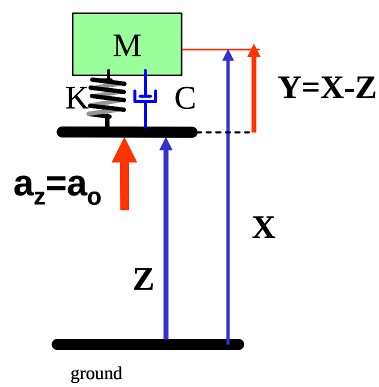
a) Derive the equation of relative motion for the instrument
b) Give or find an analytical expression for the relative displacement of the instrument vs. time. Express your answer with well defined parameters and variables.
c) Given *M*=1 kg, *K*=1 N/mm, and damping ratio ζ=0.10. Find the natural frequency & damping coefficient of the system
d) For ao=3g, find the steady-state displacement of the instrument, relative to rocket and absolute.



Definitions: coordinate systems

$X(t)$ Absolute displacement of instrument recorded from ground

$Z(t)$ Absolute displacement of rocket from ground.



$Y = X - Z$ displacement of instrument relative to rocket

$a_Z = \frac{d^2}{dt^2}Z = a_o$ acceleration of rocket fired from REST

sensor parametes: $M := 1 \cdot kg \quad K := 1000 \cdot \frac{N}{m}$

$a_o := 3 \cdot g \qquad \zeta := 0.10$

Equation of motion for instrument

Newton's Laws are applicable to inertial CS

from **free body diagram**, let Y=(X-Z)>0

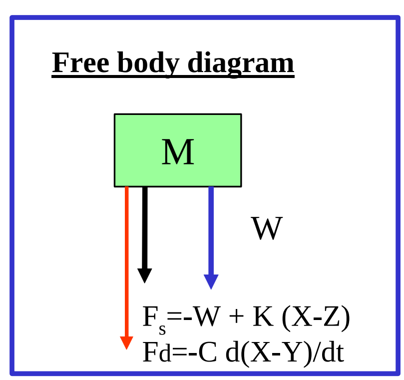
$$M \cdot \frac{d^2}{dt^2}X = -W - F_s - F_d \quad (1)$$

where

$$F_s = -W + K \cdot (X - Z) = -W + K \cdot Y \quad (2a)$$

is the spring force supporting instrument. The dashpot force is

$$F_d = C \cdot \frac{d}{dt}(X - Z) \quad (2b)$$



Substitute Eqs. (2) into Eq.(1):

$$M\cdot\frac{d^2}{dt^2}X = -W + W - K\cdot Y - C\cdot\frac{d}{dt}Y$$

$$M\cdot\frac{d^2}{dt^2}X = -K\cdot Y - C\cdot\frac{d}{dt}Y$$

But interest is in the **relative motion Y;** hence, substitute X=Y+Z

$$M\cdot\frac{d^2}{dt^2}(Y+Z) + K\cdot Y + C\cdot\frac{d}{dt}Y = 0$$

$$M\cdot\left(\frac{d^2}{dt^2}Y + C\cdot\frac{d}{dt}Y\right) + K\cdot Y = -M\cdot a_Z = -M\cdot a_o$$

(3) is the desired EOM.

Find natural frequency and damping coefficient

natural frequency of sensor is:

$$\omega_n := \left(\frac{K}{M}\right)^{.5} \qquad \omega_n = 31.623\,\frac{rad}{s}$$

Natural period: $T_n := \frac{2\cdot\pi}{\omega_n}$

damping coefficient.

$$C := \zeta\cdot 2\cdot(K\cdot M)^{0.5} \qquad C = 6.325\,N\cdot\frac{s}{m}$$

$$T_n = 0.199\,s$$

damped natural frequency

$$\omega_d := \omega_n\cdot\left(1-\zeta^2\right)^{0.5} \qquad \omega_d = 31.464\,\frac{rad}{s}$$

Solution of ODE - prediction of relative motion

The solution of ODE Eq. (3) with null initial conditions since motion starts from rest is (Use cheat sheet)

Motion starts from rest

$$Y_o := 0\cdot m \qquad V_o := 0\cdot\frac{m}{s}$$

$$Y_s := \frac{-M\cdot a_o}{K}$$

$$Y = Y_s + e^{-\zeta\cdot\omega_n\cdot t}\cdot\left(C_1\cdot\cos\left(\omega_d\cdot t\right) + C_2\cdot\sin\left(\omega_d\cdot t\right)\right)$$

$$C_1 := Y_o - Y_s \qquad C_2 := \frac{\left(V_o + \zeta\cdot\omega_n\cdot C_1\right)}{\omega_d}$$

is the formula describing the motion of instrument relative to rocket

$$C_1 = 0.029\,m \qquad C_2 = 2.957\times 10^{-3}\,m$$

and, after long time Y approaches: $Y_s = -0.029\,m$

To find the instrument absolute displacement, first determine the absolute motion of the rocket, i.e.

velocity $V_z(t) := a_o\cdot t$ and displacement $Z(t) := a_o\cdot\frac{t^2}{2}$

The absolute displacement of the sensor is $X = Y + Z$

after very-long tines, times removes the homogenous (transient) response; and the senso reaches its steady state motion

$$X_{SS}(t) := Z(t) + Y_s$$

not for Quiz Lets graph the relative and absolute displacements of the sensor

without damping

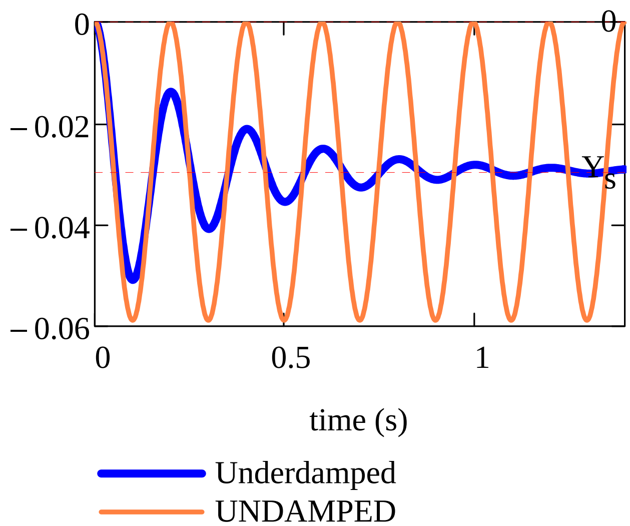
$$Y(t) := Y_s + e^{-\zeta\cdot\omega_n\cdot t}\cdot\left(C_1\cdot\cos\left(\omega_d\cdot t\right) + C_2\cdot\sin\left(\omega_d\cdot t\right)\right)$$

$$Y\_(t) := Y_s\cdot\left(1 - \cos\left(\omega_n\cdot t\right)\right)$$

for plots, set $T_{max} := 7\cdot T_n$

Relative displacement of sensor w/r to rocket

Y [m]



$Y\left(5\cdot T_n\right) = -0.028\,m$

note the effect of damping

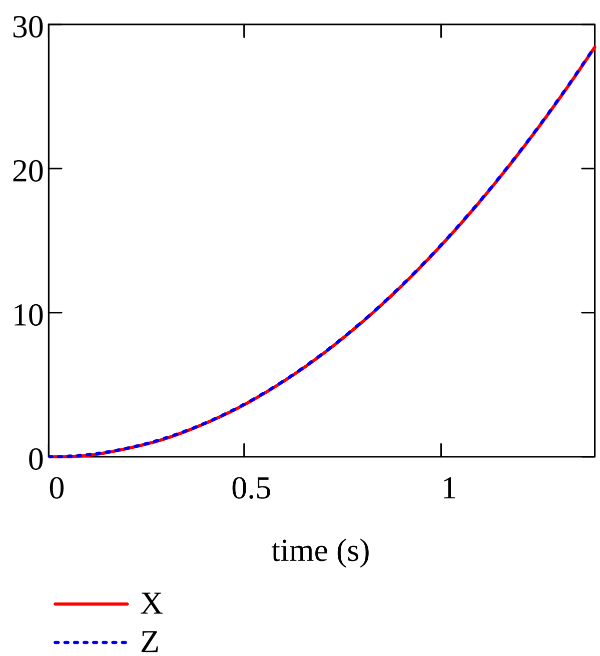
$Y_s = -0.029\,m$

The absolute displacement of the sensor is $X = Y + Z$ where

Displacements - Sensor (X) and Rocket (Z)

[m]

X, Z



$$kmh := \frac{1}{3.6}\cdot\frac{m}{s}$$