P3 Rolling of a cylinder inside of a hollow pipe

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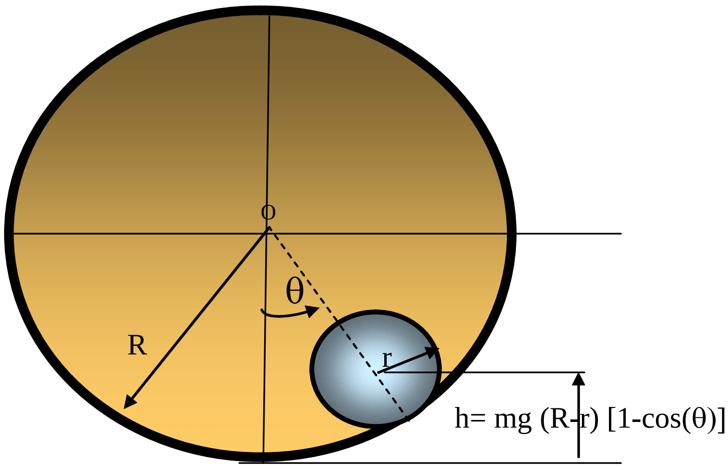


At time t=0 s, the cylinder is at the bottom of the hollow pipe and given an initial angular velocity ω_o about point O. The cylinder climbs the pipe wall as it rolls without slipping.

Tasks:

a) find equation of motion while cylinder rolls w/o slipping.
b) Find the maximum angle θ the cylinder climbs while rolling w/o slipping OR find maximum height reached?
c) Find the angle at which contact is lost, i.e. N=0.
d) Given the coefficient of friction μ between the cylinder and pipe surface, will the cylinder slip before reaching the maximum angle or EVEN before losing contact?

Given:

$\mu_s := 0.5$ coefficient of dry-friction

$R := 200 \cdot mm$

$r := 50 \cdot mm$

$V_o := 1.5 \cdot \frac{m}{sec}$ $\qquad \omega_o := \frac{V_o}{s}$

$s := R - r$ $\qquad I_G = \frac{1}{2} \cdot m \cdot r^2$

$\omega_o = 10\,s^{-1}$ $\qquad \omega_o = \left(\frac{d}{dt}\theta\right)$ at time t=0 s

$m := 2 \cdot kg$

Let: [' = d/dt]

a) Derive EOM for rolling without slipping condition

Assumed conservative system.

Let ϕ = angle of spinning of cylinder, θ = angle of cylinder rotation about center of pipe O

*** Kinematic constraint:** translational velocity of cylinder center of mass equals $V_t = \theta' \cdot s = r \cdot \phi'$ [0]

first term indicates rotation about center of hollow pipe, second term is due to rolling w/o slipping.

*** Kinetic energy (T):** (translation and rotation) of cylinder

$$T = \frac{1}{2} \cdot I_G \cdot \phi'^2 + \frac{1}{2} \cdot m \cdot V_t^2 = \frac{1}{2} \cdot \left(I_G \cdot \frac{s^2}{r^2} + m \cdot s^2\right) \cdot \theta'^2$$

since, $I_G := \frac{1}{2} \cdot m \cdot r^2$

$$T = \frac{1}{2} \cdot I_\theta \cdot \theta'^2 \quad ; \quad I_\theta = I_G \cdot \frac{s^2}{r^2} + m \cdot s^2 \quad\quad I_\theta = \left(\frac{1}{2} + 1\right) \cdot m \cdot s^2 = \frac{3}{2} \cdot m \cdot s^2 \qquad (1)$$

*** Potential Energy:** **gain** in potential gravitational energy. Consider Max Pot energy at θ=0

$$V = m \cdot g \cdot h = m \cdot g \cdot s \cdot (1 - \cos(\theta)) \qquad (2)$$

*** Derive EOM from PCME:** $\frac{d}{dt}(T + V) = P_{drive} - Pdis = 0$ **There is no dissipated power or input power (conservative system)**

$$I_\theta \cdot \theta'' \cdot \theta' + m \cdot g \cdot s \cdot \sin(\theta) \cdot \theta' = 0$$

$$I_\theta \cdot \theta'' + m \cdot g \cdot s \cdot \sin(\theta) = 0 \qquad (3)$$

substituting the equivalent inertia into Eqn (3)

$$\theta'' = \frac{-g}{s} \cdot \frac{2}{3} \cdot \sin(\theta) \qquad (4)$$

Note: $(R - r) = s$

$$\theta'' = c \cdot \sin(\theta)$$

where: $c := \frac{-g}{s} \cdot \frac{2}{3}$ is a constant

$c = -43.585\, s^{-2}$ **Since c<0, the cylinder angular velocity about O steadily decreases (constant deceleration)**

(b) Max angle θ traveled (climbed) OR maximum heigh reached

From Conservation of mechanical energy (T+V) = [To at θ=0] since motion starts at bottom θ=0 with ang speed ω_o

$$\frac{1}{2} \cdot I_\theta \cdot \theta'^2 + m \cdot g \cdot s \cdot (1 - \cos(\theta)) = \frac{1}{2} \cdot I_\theta \cdot \omega_o^2$$

since equiv inertia=

hence

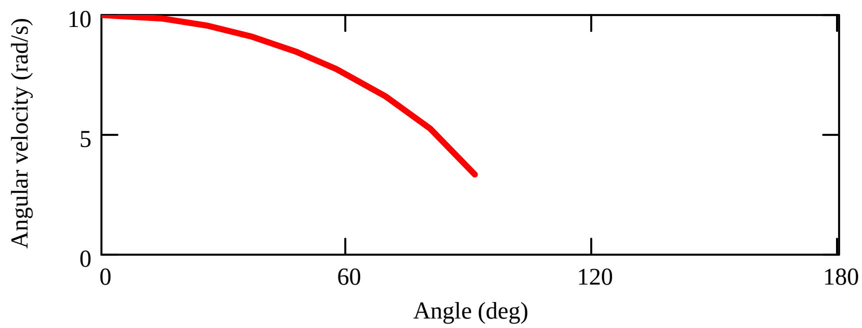
$$\theta'^2 - \omega_o^2 = \frac{-m \cdot g \cdot s \cdot (1 - \cos(\theta))}{\frac{1}{2} \cdot I_\theta}$$

$$I_\theta := \frac{3}{2} \cdot m \cdot s^2$$

$\omega_o = 10\, s^{-1}$

$\theta' = \omega$

$$\omega(\theta) := \left[\omega_o^2 - \frac{g \cdot (1 - \cos(\theta))}{s} \cdot \frac{4}{3}\right]^{\frac{1}{2}} \quad (5)$$



Eq. (5) gives the cylinder angular velocity ω versus angle θ & as a function of the initial speed. The cylinder climbs along the wall of the hollow cylinder. The maximum angle corresponds to the position with zero angular velocity (maximum potential energy). From Eq. (5):

$$0 = \omega_o^2 - \frac{g \cdot (1 - \cos(\theta_{max}))}{s} \cdot \frac{4}{3}$$

Hence:

$$\frac{\omega_o^2 \cdot s}{g} \cdot \frac{3}{4} = 1 - \cos(\theta_{max})$$

Let:

$$k := 1 - \frac{\omega_o^2 \cdot s}{g} \cdot \frac{3}{4} \quad (6)$$

$$\theta_{max} := \operatorname{acos}(k) \cdot \frac{180}{\pi} \quad (7)$$

$\theta_{max} = 98.464$ degrees

$k = -0.147$

max height

$$h := s \cdot \left(1 - \cos\left(\theta_{max} \cdot \frac{\pi}{180}\right)\right)$$

$\frac{h}{R} = 0.86$

$\frac{r}{R} = 0.25$

$h = 0.172\, m$

NOTE: This result can be obtained very seasily since Energy is conserved:
Max Potential energy = Max Kinetic energy; i.e.

$$h_{max} := \frac{\frac{1}{2} \cdot I_\theta \cdot \omega_o^2}{m \cdot g}$$

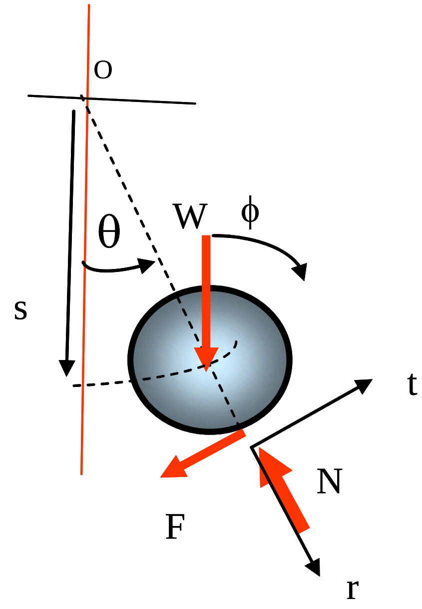
$h_{max} = 0.172\, m$

Plot Energies: T &V

(c) Angle at which contact is lost

Contact is lost when normal force = N= 0

Draw a free body diagram, and set SUM(Forces) radial and tangential directions, Σ(moments)



FREE BODY DIAGRAM

Since s is constant

$$\Sigma F_r = m \cdot a_r = -N + W \cdot \cos(\theta) \qquad a_r = -s \cdot \theta'^2 \qquad (8a)$$

$$\Sigma F_t = m \cdot a_t = -W \cdot \sin(\theta) - F \qquad a_t = s \cdot \theta'' \qquad (8b)$$

$$\Sigma M_G = I_G \cdot \phi'' = F \cdot r \qquad (8c)$$

from equations above:

Normal force $N = -m \cdot a_r + W \cdot \cos(\theta) = m \cdot s \cdot \theta'^2 + m \cdot g \cdot \cos(\theta)$ (9a)

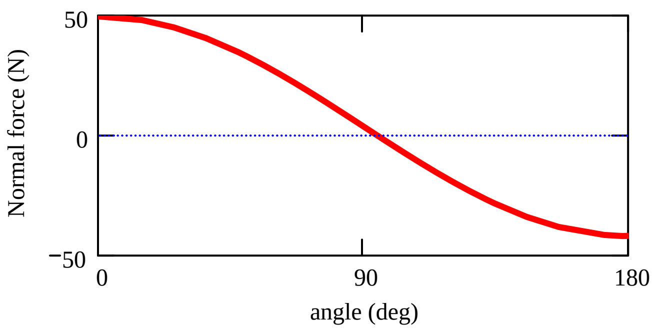
Tangential force $F = -m \cdot g \cdot \sin(\theta) - m \cdot s \cdot \theta''$ (9b)

re-derive EOM (3)

Substitution of Eq. (5) angular speed ω=θ' into Eqn (9a): $N = m \cdot s \cdot \left[\omega_o^2 - \frac{g \cdot (1 - \cos(\theta))}{s} \cdot \frac{4}{3}\right] + m \cdot g \cdot \cos(\theta)$

$$N(\theta) := m \cdot \left[s \cdot \omega_o^2 + \frac{g}{3} \cdot (7 \cdot \cos(\theta) - 4)\right] \qquad (10)$$

Note that if ωo=0, then at θ=0, N=mg; as it should



N=0 when contact is lost. This happens at angle θ=**θc**

From Eqn. (10)

$$s \cdot \omega_o^2 + \frac{g}{3} \cdot (7 \cdot \cos(\theta_c) - 4) = 0$$

$$-7 \cdot \cos(\theta_c) + 4 = \omega_o^2 \cdot \frac{3 \cdot s}{g}$$

Let: $kk := \left(4 - \omega_o^2 \cdot \frac{3 \cdot s}{g}\right) \cdot \frac{1}{7}$ kk must be betwwen -1 and +1 for real (physical) root $kk = -0.084$

$$\theta_c := \mathrm{acos}(kk) \cdot \frac{180}{\pi} \qquad (11)$$

$\theta_c = 94.824$ degrees **compare to** $\theta_{max} = 98.464$ $h := s \cdot \left(1 - \cos\left(\theta_c \cdot \frac{\pi}{180}\right)\right)$

Since θc < θmax, cylinder will lose contact before reaching its max height. $h = 0.163\ \mathrm{m}$

(d) Will cylinder SLIP before it loses contact or before reaching its max height?

To determine angle at which slip occurs. Find **tangential force**, from Eqn (9b): $F = -m \cdot g \cdot \sin(\theta) - m \cdot s \cdot \theta''$

$$F = -m \cdot g \cdot \sin(\theta) - m \cdot s \cdot \left(\frac{-g}{s} \cdot \frac{2}{3} \cdot \sin(\theta)\right) \quad \Rightarrow \quad F = -m \cdot g \cdot \sin(\theta) \cdot \left(1 - \frac{2}{3}\right) = -\frac{1}{3} \cdot m \cdot g \cdot \sin(\theta)$$

Note F<0,

(12)

Cylinder slips at angle θs, and at this angle $F = -\mu_s \cdot N$ (13)

Care here, since DRY-FRICTION FORCE force must oppose direction of motion, i.e. NEGATIVE in tangential direction. Hence the negative sign

SUBstitute F and N into Eqn. (13), i.e.:

$$F = -\frac{1}{3} \cdot m \cdot g \cdot \sin(\theta) \qquad N(\theta) := m \cdot \left[s \cdot \omega_o^{2} + \frac{g}{3} \cdot (7 \cdot \cos(\theta) - 4)\right]$$

given dry-friction coefficient $\mu_s = 0.5$

$$\frac{1}{3} \cdot m \cdot g \cdot \sin(\theta_s) = \mu_s \cdot m \cdot \left[s \cdot \omega_o^{2} + \frac{g}{3} \cdot (7 \cdot \cos(\theta_s) - 4)\right]$$

$$1 \cdot \sin(\theta_s) = \mu_s \cdot \left[\frac{3 \cdot s \cdot \omega_o^{2}}{g} + 1 \cdot (7 \cdot \cos(\theta_s) - 4)\right] \quad (14)$$

Solve nonlinear equation:

$$f(x) := 1 \cdot \sin(x) - \mu_s \cdot \left[3 \cdot \frac{s}{g} \cdot \omega_o^{2} + (7 \cdot \cos(x) - 4)\right]$$

$$guess\_upper\_\theta := 120 \cdot \frac{\pi}{180}$$

$$\theta_s := root(f(x), x, 0, guess\_upper\_\theta) \cdot \frac{180}{\pi}$$

$\theta_s = 78.693$ degrees

compare with $\theta_{max} = 98.464$, $\theta_c = 94.824$

since $\theta_s < \theta_c$, $\theta_s < \theta_{max}$

==> sphere will slip before losing contact with cylinder AND well before reaching its max height

▶ Graphs of forces, Normal & Tang

▶ preliminary work