SWINGING GONDOLA - FUN RIDE!

The figure shows a SWINGING GONDOLA (SG) at an amusement park. The gondola's mass and centroidal radius of gyration equal M and r_k. The total mass of the passengers does not affect significantly the magnitudes given above or CG location of SG. Two bars of length L and mass M_b, $I_b=(1/12)\, M_b L^2$, attach the SG CG to a shaft connected to a drive motor. The shaft is supported on frictionless bearings. The motor applies torque T_m turning the bars and lifting the gondola to a release angle θ_o. At this time, the motor is disengaged and the gondola starts to swing, giving a good thrill to its riders. Determine:

a) System EOM after the motor is disengaged. Express the angular acceleration $\ddot{\theta}$ in terms of the system parameters & θ. [10]

b) Given the (large) initial angle θ_o, derive an analytical expression (symbolic) for the SG angular velocity $\dot{\theta}$ in terms of the system parameters, and angles θ and θ_o. [10]

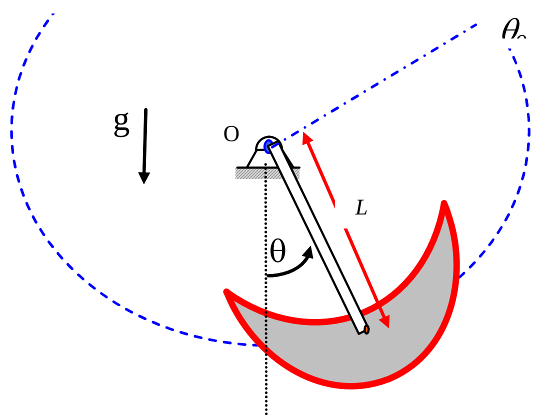
c) Given, M=4000 kg, r_k=4 m, L=10 m, M_b=0.05 M, and θ_o = 125°, calculate, when gondola crosses θ=0°, its angular speed [Hz], and SG CG tangential speed and radial acceleration What is the type of "thrill" or sensation the riders feel when the SG passes through θ=0°? [2.5 x 4]

Data:

$\theta_{o\_} := 125$ degree release angle

$M := 4000 \cdot kg$ $\quad L := 10 \cdot m$ $\quad r_k := 4 \cdot m$

$M_b := 0.05 \cdot M$



From Tables: (mass moment of inertia)

$$I_{b\_cg} = M_b \cdot \frac{L^2}{12}$$

bar

$$I_{SG} := M \cdot r_k^2$$

Gondola inertia about its cg

Moments of inertia about pivot O:

$$I_{b\_o} := M_b \cdot \frac{L^2}{12} + M_b \cdot \left(\frac{L}{2}\right)^2 \qquad I_{SG\_o} := I_{SG} + M \cdot L^2 \qquad \frac{I_{b\_o}}{I_{SG\_o}} = 0.014$$

Total Mass moment of inertia about pivot O: $I_o := 2 \cdot I_{b\_o} + I_{SG\_o}$

Equation of motion:

From summation of moments about pivot:

$$I_o \cdot \frac{d^2}{dt^2}\theta = -M \cdot g \cdot L \cdot \sin(\theta) - 2 \cdot M_b \cdot g \cdot \frac{L}{2} \cdot \sin(\theta)$$

assume: no friction

$$I_o \cdot \frac{d^2}{dt^2}\theta + g \cdot L \cdot \sin(\theta) \cdot (M + M_b) = 0$$

[2] $$\frac{d^2}{dt^2}\theta = \frac{-g \cdot L \cdot (M + M_b)}{I_o} \cdot \sin(\theta)$$

For **small amplitude motions**, Eqn(2) reduces to: $\sin(\theta) \equiv \theta$

$$I_o \cdot \frac{d^2}{dt^2}\theta + g \cdot L \cdot (M + M_b) \cdot \theta = 0 \qquad [3]$$

natural frequency, for small amplitude motions:

Disregard bar masses

$$\omega_n := \left[\frac{g \cdot L \cdot (M + M_b)}{I_o}\right]^{0.5} \qquad \omega_{na} := \left[\frac{g}{L} \cdot \frac{1}{1 + \left(\frac{r_k}{L}\right)^2}\right]^{.5}$$

Natural period

$$T_n := \frac{2 \cdot \pi}{\omega_n} \qquad T_{na} := 2 \cdot \pi \cdot \left[\frac{L}{g} \cdot \left[1 + \left(\frac{r_k}{L}\right)^2\right]\right]^{0.5}$$

$$T_n = 6.764\,s \qquad T_{na} = 6.834\,s$$

Not much difference if bars are omitted from analysis

SOLUTION for SMALL ANGLES θ - LINEARIZED EOM:

The solution fo the linear EOM (3) is $\theta(t) = \theta_o \cdot \cos(\omega_n \cdot t) + \omega_o \cdot \sin(\omega_n \cdot t)$

where $\theta_o := \theta_{o\_} \cdot \frac{\pi}{180}$ $\omega_o = \frac{d}{dt}\theta$ are initial conditions

since $\omega_o = 0$ starting from rest. Then

$\theta(t) = \theta_o \cdot \cos(\omega_n \cdot t)$ and $\omega(t) := -\theta_o \cdot \omega_n \cdot \sin(\omega_n \cdot t)$

from these 2 equations, square both sides $\frac{\theta^2}{\theta_o^{\,2}} = \cos(a)^2$ $\frac{\omega^2}{\theta_o^{\,2} \cdot \omega_n^{\,2}} = \sin(a)^2$ $a = \omega_n \cdot t$

and add both equations to give $\frac{\omega^2}{\omega_n^{\,2}} = \theta_o^{\,2} - \theta^2$

Thus, the **linearized solution is** $\omega_{Lin}(\theta) := \omega_n \cdot \left(\theta_o^{\,2} - \theta^2\right)^{0.5}$

Find angular speed of gondola for large release angles

From Conservation of Energy, **T+V = Vo**; since gondola is released from initial angle θo with null speed

Kinetic energy $T = \frac{1}{2} \cdot I_o \cdot \left(\frac{d}{dt}\theta\right)$ **Potential Energy** $V = (M + M_b) \cdot g \cdot L \cdot (1 - \cos(\theta))$

$$V_o = (M + M_b) \cdot g \cdot L \cdot (1 - \cos(\theta_o))$$

obtain:

$$\frac{1}{2} \cdot \left(\frac{d}{dt}\theta\right)^2 = \frac{(M + M_b) \cdot g \cdot L \cdot (\cos(\theta) - \cos(\theta_o))}{I_o} = \omega_n{}^2 \cdot (\cos(\theta) - \cos(\theta_o))$$

where

$$\omega_n := \left[\frac{g \cdot L \cdot (M + M_b)}{I_o}\right]^{0.5}$$

Then, the gondola angular speed dθ/dt=ω:

$$\frac{d}{dt}\theta = \omega(\theta) := \omega_n \cdot \left[2 \cdot (\cos(\theta) - \cos(\theta_o))\right]^{0.5}$$

$$f(\theta) := \frac{\omega(\theta)}{2 \cdot \pi}$$ in Hz

$$f_n := \frac{\omega_n}{2 \cdot \pi}$$

Tangential speed and normal acceleration at θ=0 $f(0) = 0.262\,Hz$

$$f_n = 0.148\,Hz$$

$$V_t(\theta) := L \cdot \omega(\theta)$$

$$a_n(\theta) := -L \cdot \omega(\theta)^2$$

$$V_t(0) = 16.479\,\frac{m}{s}$$

$$\frac{\omega(0)}{\omega_n} = 1.774$$

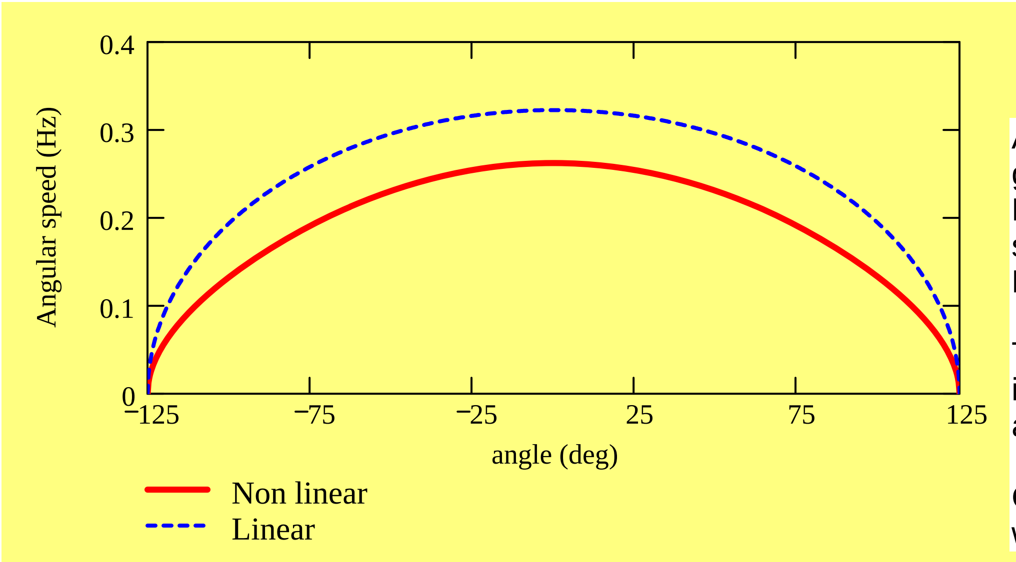
$$a_n(0) = -27.156\,\frac{m}{s^2}$$

$$\frac{a_n(0)}{g} = -2.769$$

rider feels something in his/her tummy!
(a weighless sensation)

Graphs: v**ariation of gondola angular speed and radial acceleration for entire ride:**

$\theta_{o\_} = 125$ degrees



Angular speed of gondola derived from NONLINEAR solution is smaller than using LINEAR solution.

The differences increase as the release angle increases

Greatest diference when θo= 180 deg