Combined rotational-translation system with viscous damping

The figure shows a schematic view of a drum-hoist system. Pinion A is keyed on the shaft of a DC motor and drives gear B (teeth not shown). Drum C is keyed on the same shaft as gear B. An inextensible cable connects the drum to a box weighing $W=Mg$. The electric motor delivers Torque $T_{M(t)}$ to lift a heavy box that is inside a mining shaft. The shaft walls react with a drag force whose viscous (damping) coefficient is C_s. The support bearings B_A and B_B have rotational damping coefficients $C_{\theta A}$ and $C_{\theta B}$, respectively. The figure includes the mass moments of inertia *(I)* as well as the radius for each rotational component.

a) Define variables, establish kinematics constraints, and **derive** the EOM:
$I_{eq}\,\dot{\omega}_M + C_{\theta eq}\,\omega_M = T_{M(t)} - W\,s$, where ω_M is the rotor speed and s is a function of the gears and drum radii. Specify **symbolic formulae** and physical values for the system equivalent inertia I_{eq} and viscous damping $C_{\theta eq}$

b) What is the minimum T_M to hold the box away from the floor?

Start-up response: The box is resting on the floor. Then at time t=0 s, the motor is turned ON with torque T_M=450 Nm

c) Calculate the operating (steady state) speed of the motor (Hz) and the speed of box (m/s), and the power lost & drive of the system. What is its mechanical efficiency?

d) Solve EOM and find the time response of the motor ω_M(t). Sketch the motor speed ω_M [Hz] versus time? What is the system time constant? How long (time) does it take the motor to reach its steady state response?

Mass moments of inertia

$I_A := 0.1\cdot kg\cdot m^2$ motor & pinion A;

$I_B := 0.02\cdot kg\cdot m^2$ gear B; $I_C := 0.5\cdot kg\cdot m$

box: $W := 10000\cdot N$ $M := \frac{W}{g}$

$R_A := 1\cdot in$ $R_B := 2.5\cdot in$

$R_C := 4\cdot in$

Bearing & shaft drag coefficients

$C_{\theta A} := C_{\theta B}$ $C_s := 561.876\cdot N\cdot\frac{s}{m}$

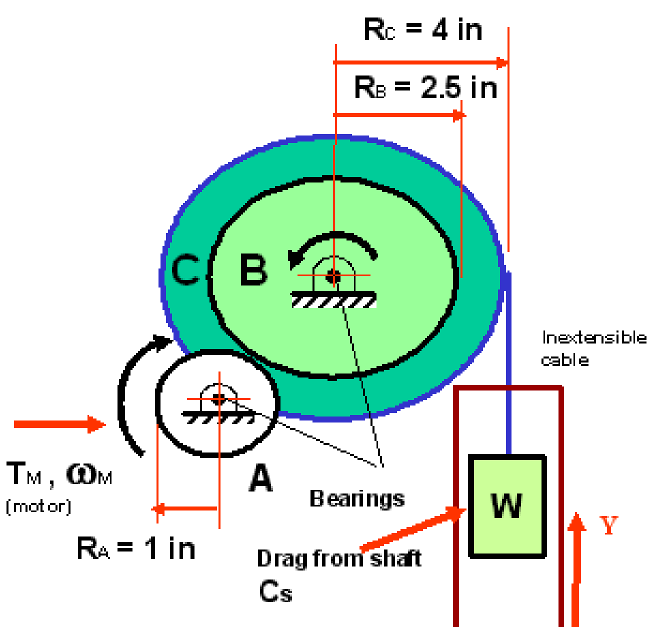


Figure NOT to scale

MECHANICAL SYSTEM: Modeling and response

Mass moments of inertia

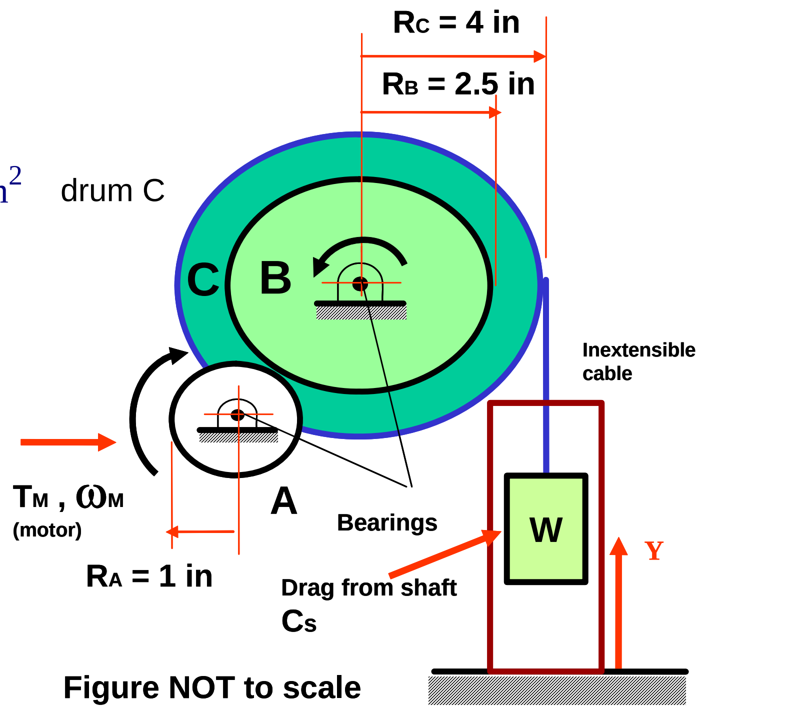


Figure NOT to scale

$I_A := 0.1 \cdot kg \cdot m^2$ motor & pinion A;

$I_B := 0.02 \cdot kg \cdot m^2$ gear B; $I_C := 0.5 \cdot kg \cdot m^2$ drum C

box: $W := 10000 \cdot N$ $\quad M := \dfrac{W}{g}$

$R_A := 1 \cdot in$ $\quad R_B := 2.5 \cdot in$

$R_C := 4 \cdot in$

Bearing & shaft drag coefficients

$C_{\theta B} := 0.8 \cdot \dfrac{N \cdot s \cdot m}{rad}$

$C_{\theta A} := C_{\theta B}$ $\quad C_s := 561.876 \cdot N \cdot \dfrac{s}{m}$

DEFINITION OF VARIABLES

T_M Motor - torque

ω_M Motor - pinion A angular speed

ω_B Angular speed of gear B

ω_C Angular speed of drum C

$V = \dfrac{d}{dt} Y$ Speed of box

Let: $r_G := \dfrac{R_A}{R_B}$ gear ratio

$ss := R_C \cdot r_G$

Kinematic CONSTRAINTS

$\omega_B \cdot R_B = \omega_M \cdot R_A$

$\omega_C = \omega_B = \omega_M \cdot r_G$

$V = \omega_C \cdot R_C$

$V = ss \cdot \omega_M$

KINETIC ENERGY of SYSTEM

= Σ individual kinetic energies: motor, gears and box

$$T = \left(\frac{1}{2} \cdot I_A \cdot \omega_M^2\right) + \left(\frac{1}{2} \cdot I_B \cdot \omega_B^2\right) + \left(\frac{1}{2} \cdot I_C \cdot \omega_C^2\right) + \left(\frac{1}{2} \cdot M \cdot V^2\right)$$

Substitution of constraints in rotational speeds and translational speed V gives:

$$T = \left(\frac{1}{2} \cdot I_A \cdot \omega_M^2\right) + \left[\frac{1}{2} \cdot (I_B + I_C) \cdot \omega_M^2 \cdot r_G^2\right] + \left(\frac{1}{2} \cdot M \cdot R_C^2 \cdot r_G^2 \cdot \omega_M^2\right)$$

$$T = \frac{1}{2} \cdot I_{eq} \cdot \omega_M^2$$

$$I_{eq} := I_A + \left(I_B + I_C + M \cdot R_C^2\right) \cdot r_G^2$$

$$I_{eq} = 1.867\, kg\, m^2$$

POTENTIAL ENERGY in SYSTEM: increase in gravitational potential energy since box is lifted. Use Y=0 as datum

$V = W \cdot Y$

Power Dissipated (LOST) by SYSTEM

ONE bearing set on motor side,
ONE bearing set on drum sidee

= Σ individual drag losses from each bearing

$$\left[P_D = \left(C_{\theta B} \cdot \omega_B^2\right) + \left(C_{\theta A} \cdot \omega_M^2\right)\right] + C_s \cdot V_s^2$$

$$C_{\theta eq} := C_{\theta A} \cdot \left(1 + r_G^2\right) + C_s \cdot ss^2$$

$$P_D = C_{\theta e} \cdot \omega_M^2$$

$$C_{\theta eq} = 1.856 \, N \cdot m \cdot s$$

Drive Power from MOTOR

$$P_{drive} = T_M \cdot \omega_M$$

Principle of conservation of mechanical energy

$$\frac{d}{dt}(T + V) + P_D = P_{drive}$$

==>

$$I_{eq} \cdot \left(\frac{d}{dt}\omega_M\right) \cdot \omega_M + C_{\theta e} \cdot \omega_M^2 + W \cdot V = T_M \cdot \omega_M$$

canceling similar terms, the **final EOM is**

$$I_{eq} \cdot \left(\frac{d}{dt}\omega_M\right) + C_{\theta eq} \cdot \omega_M = T_M - W \cdot ss$$

where

$$ss = 0.041 \, m$$

Minimum motor torque needed to lift box (W) is

$$T_{Mmin} := W \cdot ss$$

$$T_{Mmin} = 406.4 \, N \cdot m$$

(b) Operating point

At the operating point, the motor speed is constant and the box moves with constant velocity.

Let: $T_M := 450 N \cdot m$

$$\omega_{Mss} := \frac{T_M - W \cdot ss}{C_{\theta eq}}$$

$$\frac{\omega_{Mss}}{2 \cdot \pi} = 3.739 \, Hz$$

$$V_{ss} := \omega_{Mss} \cdot R_C \cdot \frac{R_A}{R_B}$$

$$V_{ss} = 0.955 \, \frac{m}{s}$$

$$V_{ss} = 3.437 \, kph$$

power

(b) Power & efficiency

$P_{drive} := T_M \cdot \omega_{Mss}$ $P_{drive} = 1.057 \times 10^4 W$ Total power

$P_{load} := W \cdot V_{ss}$ $P_{load} = 9.547 \times 10^3 W$ Power taken by load

$P_{lost} := C_{\theta eq} \cdot \omega_{Mss}^{\ 2}$ $P_{lost} = 1.024 \times 10^3 W$ Power lost in bearings

Efficiency of system $\eta := \frac{P_{load}}{P_{drive}}$ $\eta = 0.903$

Efficiency decreases with applied torque

power

start up

(c) Start UP response

Solve EQN of motion, with system at rest

$$I_{eq} \cdot \left(\frac{d}{dt}\omega_M\right) + C_{\theta eq} \cdot \omega_M = T_{eq}$$

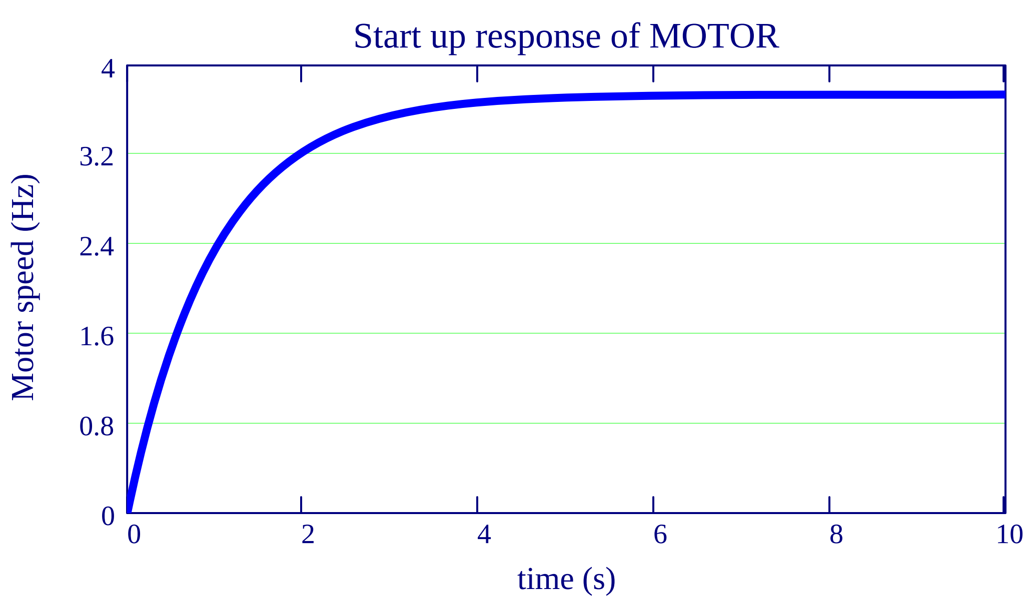
where

$T_{eq} := T_M - W \cdot ss$

LET: $\omega_M(0) = 0$ $\frac{\omega_{Mss}}{2 \cdot \pi} = 3.739\,Hz$ $T_{eq} = 43.6\,N \cdot m$

define time constant $\tau := \frac{I_{eq}}{C_{\theta eq}}$ $\tau = 1.006\,s$

and motor response as a function of tme is: $\omega_M(t) := \omega_{Mss} \cdot \left(1 - e^{\frac{-t}{\tau}}\right)$ [SOLN]



The system takes about 4-5*τ to reach its steady-state (operating) speed

$4 \cdot \tau = 4.025\,s$

$5 \cdot \tau = 5.031\,s$

start up

Find time t_ for motor to reach of its steady state speed

$$\omega_{t\_} := 0.99 \cdot \omega_{Mss}$$

from SOLN:

$$\frac{\omega_{t\_}}{\omega_{Mss}} - 1 = -e^{\frac{-t\_}{\tau}}$$

Take natural log from both sides to find

$$t\_ := -\ln\left(1 - \frac{\omega_{t\_}}{\omega_{Mss}}\right) \cdot \tau$$

$$\frac{t\_}{\tau} = 4.605$$

$$t\_ = 4.633\,s$$