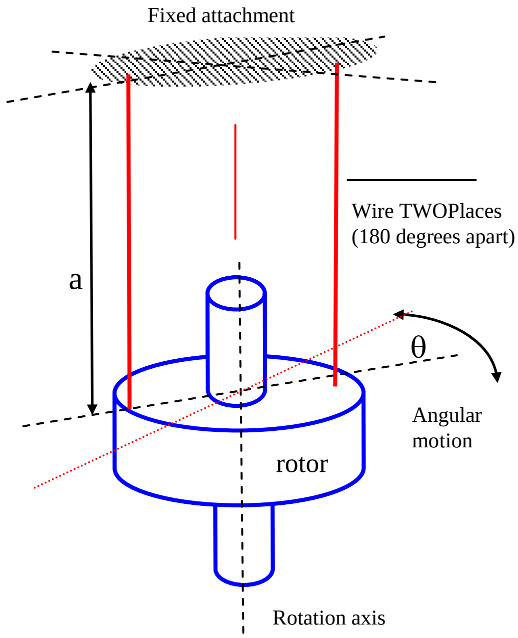
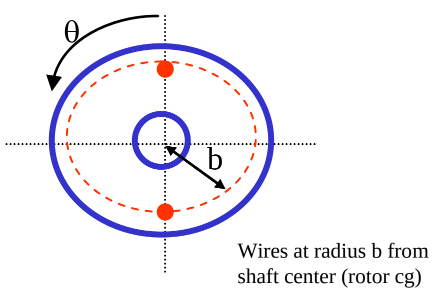
MEEN 363 QUIZ 10 (30' max) Names_________





View from top

DRAWINGS NOT TO SCALE

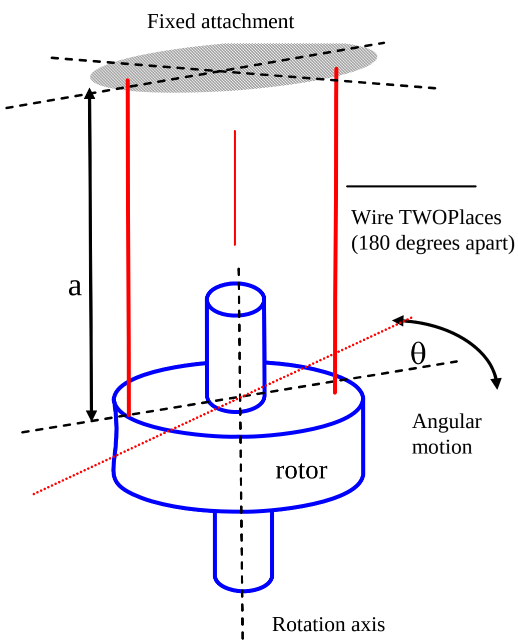
The figure shows a pendulum-like arrangement for measuring the radius of gyration (*k*) of a rotor (i.e. polar mass moment of inertia $I_p = m\,k^2$). The rotor is suspended from massless and inextensible wires. Each wire of length *(a)* is fixed at radius *(b)* from the rotor center of mass. From <u>small amplitude</u> motions $\theta(t)$, the procedure requires recording the natural period of motion and then extracts the radius of gyration from a simple engineering formula.

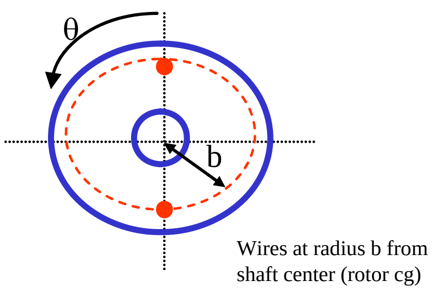
a) **Clearly outlining** physical considerations and assumptions **DERIVE** the equation of motion for the arrangement shown, i.e. $I_P \ddot{\theta} + K_\theta \theta = 0$, where $K_\theta = W\,(b^2/a)$ is a torsional "stiffness" depending on the wires' length and disposition and the rotor weight. Show a rigorous procedure, define variables, establish principles used, note assumptions, and highlight steps in modeling and analysis.

b) Measurements were conducted with a single stage compressor rotor. The rotor weighs 150 lb_f, and a=100 inch, $b=10$ inch. The rotor performed 10 oscillations in 30 seconds. Determine, the system natural frequency [rad/s], the rotor polar mass moment of inertia I_p[lb_f-inch-s^2], and the radius of gyration k [inch]

NOTE: Neatness, organization, explanations in complete sentences, and clarity of procedure are a must for full grade.

QUIZ 10 **Determining mass moments of inertia**





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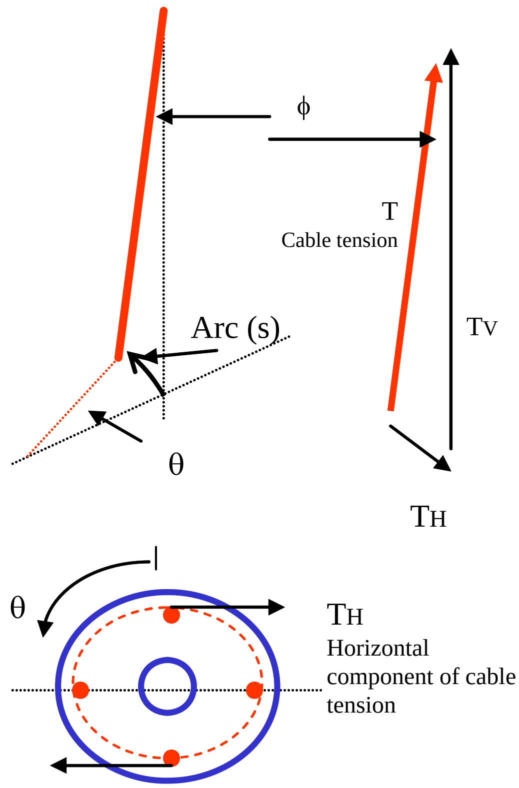

Kinematics and FBD of rotor for angular motions

Constraint: arc length $s = \phi\, a = \theta\, b$

Assumptions :
no friction, small angular motions, rotations about cg, cables are massless and inextensible.

kinematic constraint:
arc motion (s) described by rotor motion is given by

$$s = a\cdot\phi = b\cdot\theta \qquad [1]$$

with ϕ as the angle between a support cable and the vertical plane

Let: $\theta'' = \dfrac{d^2}{dt^2}\theta$

For small angular motions $\theta(t)$ about the center of mass, the equation of motion is

$$I_P\cdot\theta'' = -2T_H\cdot b \qquad [2]$$

where T_H is the component of the cable tension in the horizontal plane

from the EOM for vertical motions of rotor cg: $M\cdot y'' = 2\cdot T_V - W$ [3]

where T_V is the component of the cable tension in the vertical plane

For small amplitude angular motions. From [1], $\phi = \dfrac{b}{a}\cdot\theta$ [4a] and: $y'' := 0$

.Thus,

$$T_v := \frac{W}{2} \quad \text{and} \quad T_H = T_V \cdot \frac{\sin\phi}{\cos\phi} \approx \frac{W}{2}\cdot\phi = \frac{W}{2}\cdot\frac{b}{a}\cdot\theta \qquad [4b]$$

Substitution of [4b] into [2] renders the desired EOM:

$$I_p\cdot\theta'' + k_\theta\cdot\theta = 0 \qquad [6a] \qquad \text{where} \quad k_\theta = W\cdot\frac{b^2}{a}$$

since: $W := m\cdot g$ and $I_P = m\cdot r_k^2$

. Then [6a] reduces to:

$$\theta'' + g\cdot\frac{b^2}{a\cdot r_k^2}\cdot\theta = 0 \qquad [6b]$$

The natural frequency of the oscillatory system is defined as: $\omega_n = \left(\frac{k_\theta}{I_p}\right)^{.5}$ [7]

and the natural period of motion is: $T_n = \frac{2\cdot\pi}{\omega_n}$

(b) Engineering calculations

$T_n := \frac{30}{10}\cdot s \quad \omega_n := \frac{2\cdot\pi}{T_n} \quad \omega_n = 2.094\,\frac{rad}{s}$

$a := 100\cdot in \quad W := 150\cdot lb$

$b := 10\cdot in$

$k_\theta := W\cdot\frac{b^2}{a} \quad k_\theta = 150\,lb\cdot in$ stiffness

from [7], the polar moment of inertia is $I_P := \frac{k_\theta}{\omega_n^2}$ $\quad I_P = 34.196\,lb\cdot in\cdot s^2$

and the radius of gyration is $r_k := \left(\frac{I_P\cdot g}{W}\right)^{.5} \quad r_k = 9.382\,in$

The equation of motion can be easily derived using conservation of mechanical energy:

$T = \frac{1}{2} \cdot I_P \cdot \left(\frac{d}{dt} \theta \right)^2$ kinetic energy

$V = m \cdot g \cdot h$ change in potential energy

$h = a \cdot (1 - \cos(\phi)) = a \cdot \frac{\phi^2}{2}$

for small angles, and $\phi = \frac{b}{a} \cdot \theta$

Thus, $V = m \cdot g \cdot \frac{b^2}{a} \cdot \frac{\theta^2}{2} = \frac{1}{2} \cdot k_\theta \cdot \theta^2$ where $k_\theta = W \cdot \frac{b^2}{a}$

Hence from $\frac{d}{dt}(T + V) = 0$

$$I_P \cdot \frac{d^2}{dt^2}\theta + k_\theta \cdot \theta = 0$$