MEEN 363 / 617 Example: Derive EOM for a simple system

A cable and pulleys' system holds the two masses m_B and m_C. The cable is not extensible and attaches to ground through stiffnesses k_B and k_C. A dashpot (damping coefficient c_B) also connects block B to **ground**. **Assume a state of <u>motion</u> for both blocks**

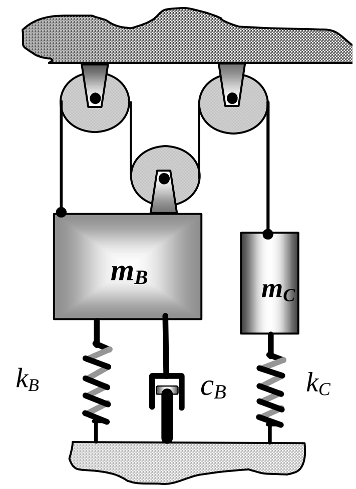
a) Define coordinate systems for the motion of each block. Explain in words, do not just show arrows [5]

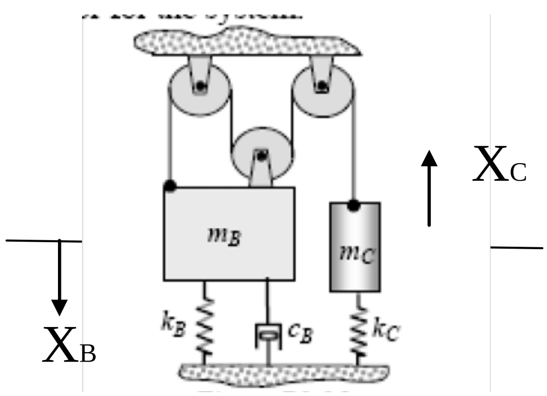
b) Draw a complete free body diagram for the system components, define all forces [10]

c) Identify the kinematic constraint relating the motion of both blocks. Explain, do not just write a formula [5].

d) State EOMs for each block, use the kinematic constraint, and **derive a single system EOM in terms of the coordinate for motion of block B.** Write steps in your analysis [10]

e) Determine the static deflection of spring *B* in terms of the weights and spring coefficients [5]





LET:

Origin of coordinate systems, X_A and X_B, is at natural length (not stretched) of springs B & C

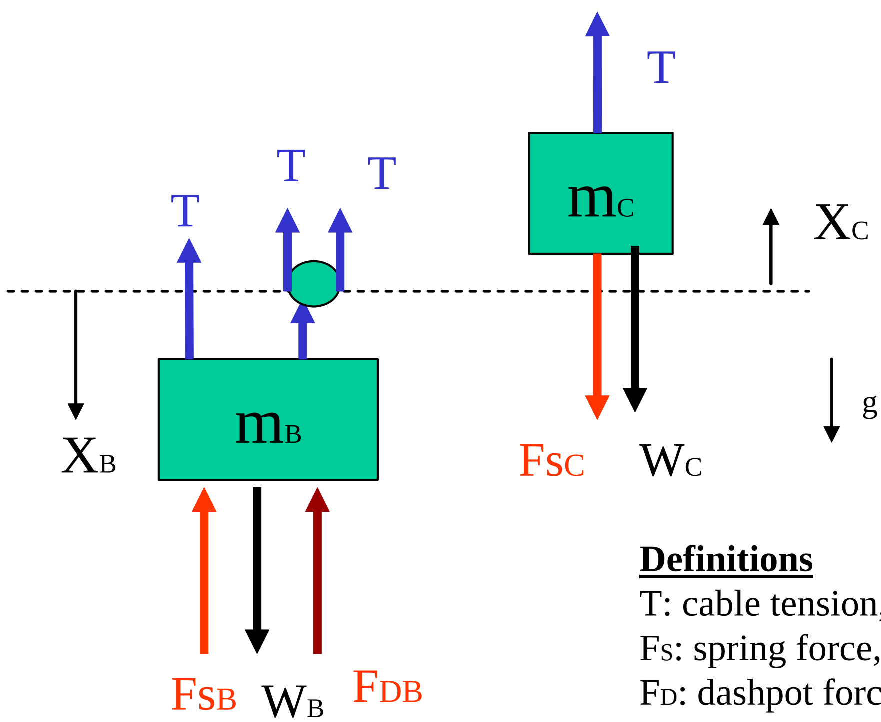
Kinematic constraint: **$3\,T\,\delta X_B = T\,\delta X_C$**

derived from conservation of work **$3\,\delta X_B = \delta X_C$**

or

cable length is constant

$L = L_o + 3 \cdot X_B - X_C$



Free Body Diagram

<u>Definitions</u>

T: cable tension,

F_S: spring force,

F_D: dashpot force,

W: weight

m: mass

State Equations of motion for each block

Block B:

$$m_B \cdot \frac{d^2}{dt^2} X_B = -3 \cdot T + W_B - F_{SB} - F_{DB} \qquad [1a]$$

weights:

$$W_B = m_B \cdot g$$

where

$$F_{SB} = k_B \cdot X_B \qquad \text{spring B force} \qquad [1b]$$

$$F_{DB} = c_B \cdot \frac{d}{dt} X_B \qquad \text{dashpot B force} \qquad [1c]$$

Block C:

$$m_C \cdot \frac{d^2}{dt^2} X_C = 1 \cdot T - W_C - F_{SC} \qquad [2a]$$

$$W_C = m_C \cdot g$$

where

$$F_{SC} = k_C \cdot X_C \qquad \text{spring B force} \qquad [2b]$$

Constraint:

$$X_C = 3 \cdot X_B \qquad [3]$$

Multiply Eq (2a) by 3 and add to equation (1a) to eliiminate the tensions and to obtain:

$$m_B \cdot \frac{d^2}{dt^2} X_B + 3 \cdot m_C \cdot \frac{d^2}{dt^2} X_C = -3 \cdot T + 3 \cdot T + W_B - F_{SB} - F_{DB} - 3 \cdot W_C - 3 \cdot F_{SC}$$

$$m_B \cdot \frac{d^2}{dt^2} X_B + 9 \cdot m_C \cdot \frac{d^2}{dt^2} X_B = W_B - F_{SB} - F_{DB} - 3 \cdot W_C - 3 \cdot F_{SC} \qquad [4]$$

substitute the constraint, Eqn. [3] to obtain:

$$\left(m_B + 9 \cdot m_C\right) \cdot \frac{d^2}{dt^2} X_B = -k_B \cdot X_B - 9 \cdot k_C \cdot X_B + W_B - 3 \cdot W_C - c_B \cdot \frac{d}{dt} X_B$$

arranging terms, the final EOM is:

$$m_E \cdot \frac{d^2}{dt^2} X_B + k_E \cdot X_B + c_B \cdot \frac{d}{dt} X_B = W_B - 3 \cdot W_C \qquad [5]$$

$$m_E = m_B + 9 \cdot m_C \qquad k_E = k_B + 9 \cdot k_C$$

System natural frequency and damping ratio:

$$\omega_n = \left(\frac{k_E}{m_E}\right)^{.5} = \left(\frac{k_B + 9 \cdot k_C}{m_B + 9 \cdot m_C}\right)^{.5} \qquad \zeta = \frac{c_B}{2 \cdot \left(k_E \cdot m_E\right)^{.5}} \qquad [6]$$

Static deflection of spring B: under static conditions there is no motion; hence set acceleration and velocity equal to zero.

$$\frac{d^2}{dt^2} X_B = 0 \qquad \frac{d}{dt} X_B = 0$$

$$X_{B\_s} = \frac{W_B - 3 \cdot W_C}{k_E} \quad [7]$$

spring B could be stretched or compressed depending on relationship between weights B and C.

$$X_{C\_s} = 3 \cdot X_{B\_static}$$

spring forces - static

$$F_{SB\_s} = k_B \left(X_{B\_s} \right) \qquad F_{SC\_s} = k_C \left(X_{c\_s} \right)$$