INTRODUCTION TO THE ANALYSIS OF VIBRATIONS IN MECHANICAL SYSTEMS

The only constant is change!

Motion (i.e. time varying changes) is ubiquitous in nature. All systems, small and large, simple or complex, describing natural, physical, biological or social phenomena, are subject to change and variation.

A system is an ensemble of components acting as a whole. In the case of a **mechanical system**, this is described by relationships of energy transfer between its components and also with its surroundings.

The system components are designed to satisfy a technical goal, i.e. convert one type of energy to another such as in a steam turbine, an electric motor, a compressor, etc. The basic mechanical components are shafts, bearings, wheels, couplings, gears, belts, chains, cams, structural support elements, piping, etc.

The study of the dynamic response of a system includes:

Design: Conceptualize system of interest, describe its functions, and separate it from others.

Analysis: Create a model that defines as closely as possible the nature of the relationships between its parts, and determine the dynamic response to a set of realistic conditions.

Testing: Measurement of the dynamic response on a real system or prototype to confirm analytical predictions.

STEPS in Modeling a Mechanical System:

The steps to follow in the analysis of mechanical systems are:

a) **Establish the necessary assumptions, provide pictorial representations, free body diagrams, determine similarities/difference from other systems, etc., i.e. bring the REAL PROBLEM into an ANALYTICAL MODEL**

b) **MATHEMATICAL MODEL: identify constraints & establish degrees of freedom, apply the fundamental principles of motion to the analytical model and derive equations of motion governing the response of the system. Give attention to initial conditions and external forcing functions.**
The principles used are:
   - **Conservation of linear momentum and angular momentum, or**
   - **Conservation of mechanical energy**

c) **Find the dynamic behavior, i.e. solve the mathematical model within the range of parameters of interest and determine the goodness or badness of proposed design.**

A more detailed (systemic) approach is given by

1. Define the system and its fundamental components.
2. List all assumptions and constraints.
3. Select significant input (excitations) and outputs (responses).
4. Model the system components constitutive equations.
5. Model the system (define relationships between components).
6. Solve for required responses given a set of known excitations for a system configuration.
7. Check for consistency of results (no violation of item 2).
8. Interpret the system response.
9. Provide recommendations, design changes and conclusions.

THE STUDY OF THE DYNAMIC RESPONSE OF MECHANICAL SYSTEMS

The mathematical models of mechanical systems are of two classes:

1. **Continuous Models**: represented by an infinite number of degrees of freedom and usually described by partial differential equations, and

2. **Discrete-Parameter or Lumped-Mass Models**: represented by a finite number of degrees of freedom and described by ordinary differential equations (as given above).

What model is the most adequate to select? This is determined by the type of behavior the system is expected to show (or desired to perform) for the conditions of interest. Simplicity is most desirable but model must always replicate the physics of the system.

In the study of mechanical systems we will concentrate on the dynamics of (lumped parameter) **linear systems**. These systems are deterministic and where the **principle of superposition** holds. In a system there is a specified set of dynamic variables called INPUTs (or excitations) and a dependent set called OUTPUTs (or responses).

For **nonlinear** systems, whose behavior depends greatly on their initial state, we will generally consider small amplitude motions or changes about an equilibrium position. This assumption brings, most often, linearity into the dynamics of the system of interest.

We will learn that lumped-parameter mechanical systems undergoing oscillatory motions (vibrations) can be modeled as Second Order Systems described by ordinary differential equations (ODEs):

$$\mathbf{M}\,\ddot{\mathbf{X}} + \mathbf{K}\,\mathbf{X} + \mathbf{D}\,\dot{\mathbf{X}} = \mathbf{F}(t)$$

and initial conditions, at time t = 0 $\rightarrow \mathbf{X}_{(t=0)} = \mathbf{X_0}\,;\ \dot{\mathbf{X}}_{(t=0)} = \dot{\mathbf{X}}_\mathbf{o} = \mathbf{V_o}$

where **M, D, K** are generalized inertia, damping and stiffness elements describing the system, and **X**(t) and **F**(t) are generalized time dependent displacement and external forces, respectively.

We are interested in the study of the dynamic response (the time dependent changes) of a mechanical vibratory system due primarily to two types of considerations:

a) **Free Response**: Motion resulting from specified initial conditions (disturbances to an equilibrium or steady-state configuration).

b) **Forced Response**: Motion resulting from specified external inputs or load excitations to the system.

In the last decade, high speed computers and advanced numerical modeling techniques have helped in understanding the dynamic response of complex systems with large number of degrees of freedom. However, you need to always remember that **a computer is just a tool to solve, not to understand, a problem**!

In MEEN 617 you will tackle the analysis of systems not by computer!!! You will be successful if you can are able to master:

- knowledge of the physical laws of motion,
- identification of the fundamental parameters or elements describing the system,
- understanding of the equations of motion governing the system behavior, and
- understanding the beauty of simple, yet accurate, solutions valid for systems of any complexity.

We will devote considerable time to obtain the dynamic response (solution) of the equations above for many different mechanical systems. To this end, we will learn the necessary analytical tools to derive the equations above and also the effective means to obtain solutions predicting the dynamic response of a vibratory system.

Independent of the physical details of the system considered, the analysis of the dynamic response of a vibratory mechanical system should aid to answer the following fundamental questions:

- ⇒**How does the system respond with time for any particular type of disturbance?**

- ⇒**How long will it take for the dynamic action to dissipate if the disturbance is briefly applied and then removed?**

- ⇒**Whether the system is stable or if its oscillations will increase in magnitude with time even after the disturbance has been removed.**

- ⇒**What modifications can be made to the system to improve its dynamic characteristics with regard to some specific application**

That is, **the ultimate purpose of the modeling/analysis is t**o answer relevant questions about the **design & performance** of a mechanical system, such as:

Will the system operate? Will it have the rated (design) performance?
Will operation be stable (static and dynamic)?
Does it meet vibration characteristics?
Will any part break under "normal" operation?
Will it be reliable? With life how long?
What are its operating limits? And for how long can operate safely?
Why does part ***X*** keep breaking while system operates?
Is modification ***Y*** able to improve performance?