Lecture 2

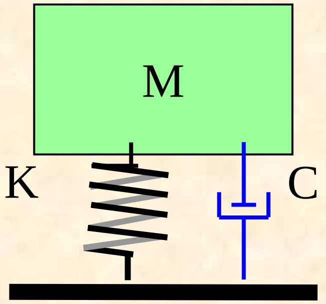
Read textbook CHAPTER 1.4, Apps B&D
Today: Derive EOMs & Linearization
Fundamental equation of motion for mass-spring-damper system (1DOF). Linear and nonlinear system. Examples of derivation of EOMs **Appendix A Equivalence of principles of conservation of mechanical energy and conservation of linear momentum.** **Appendix B: Linearization**
Work problems: **Chapter 1**: 5, 8, 13, 14, 15, 20, 43, 44, 56

Lecture 2
ME617

Kinetics of 1-DOF mechanical systems

The fundamental elements in a mechanical system and the process to set a coordinate system and derive an equation of motion.

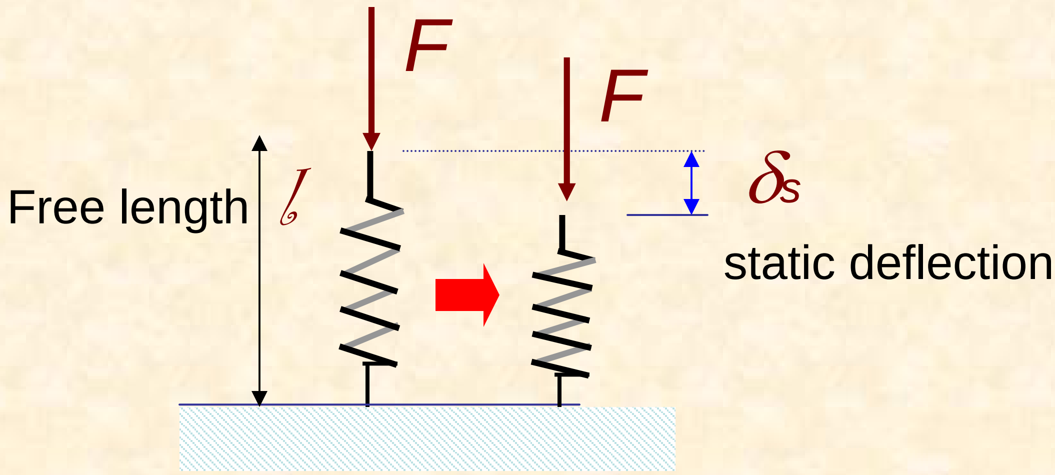
LINEARIZATION included.

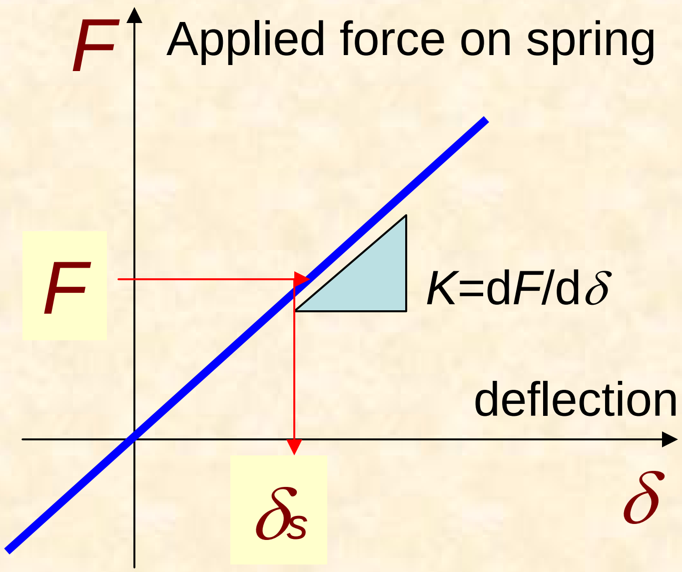


Luis San Andres
Texas A&M University

A system with an elastic element (linear spring)

Linear elastic element (spring-like)





Notes:

a) Spring element is regarded as massless

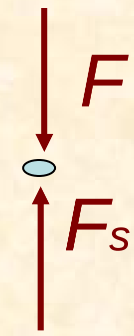
b) Applied force is STATIC

c) Spring reacts with a force proportional to deflection, *$F_s = -K\delta_s$*, and stores potential energy

d) *K* = stiffness coefficient [N/m or lbf/in] is constant

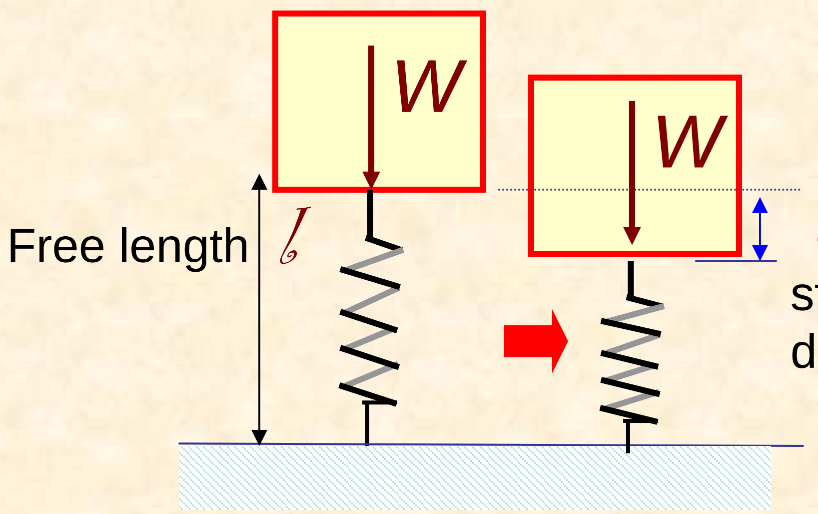
Balance of static forces

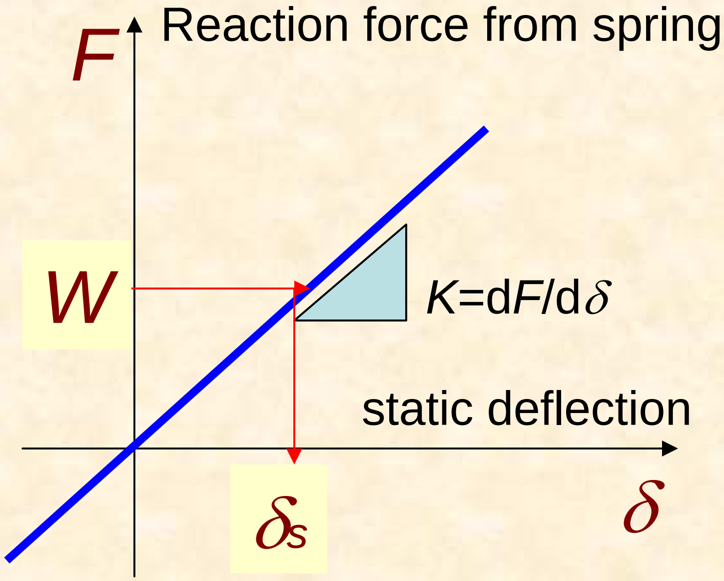
$$F_s = -K\ \delta_s$$



Statics of system with elastic & mass elements

Linear spring + added weight





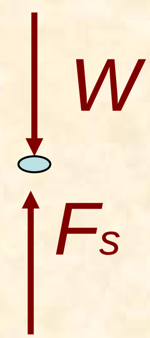
Notes:

a) Block has weight ***W= Mg***

b) Block is regarded as a point mass

Balance of static forces

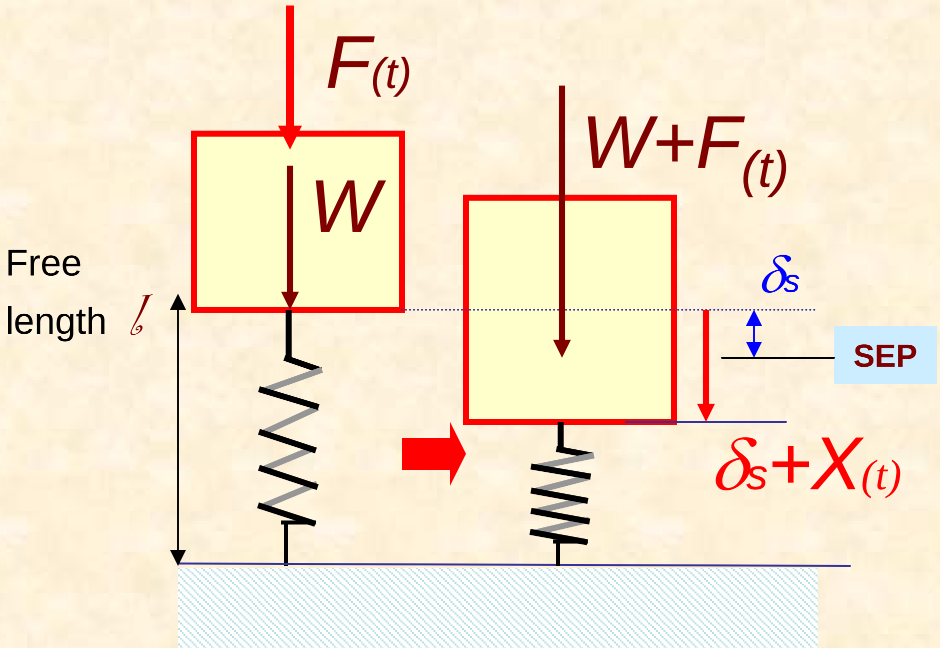
$$W = F_s = K\,\delta_s$$

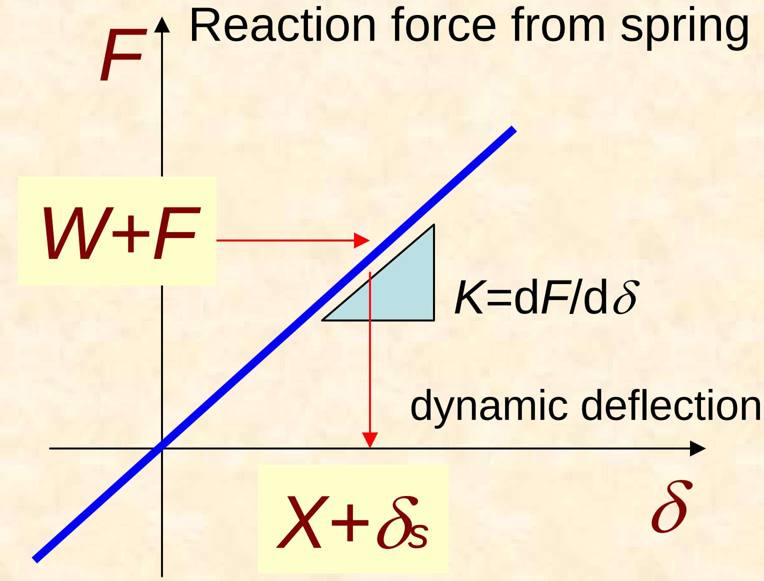


Dynamics of system with elastic & mass elements

Derive the equation of motion (EOM) for the system

Linear spring + weight (mass x g) + force $F_{(t)}$





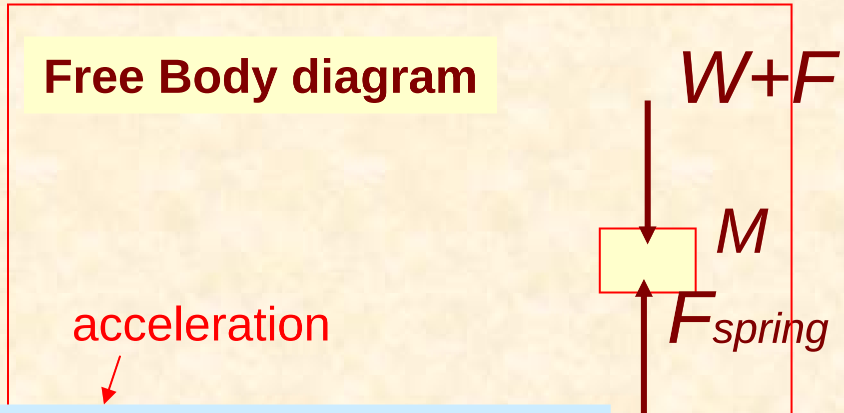
Notes:

a) Coordinate ***X*** describing motion has origin at **Static Equilibrium Position** (SEP)

b) For free body diagram, assume state of motion, for example $X_{(t)}>0$

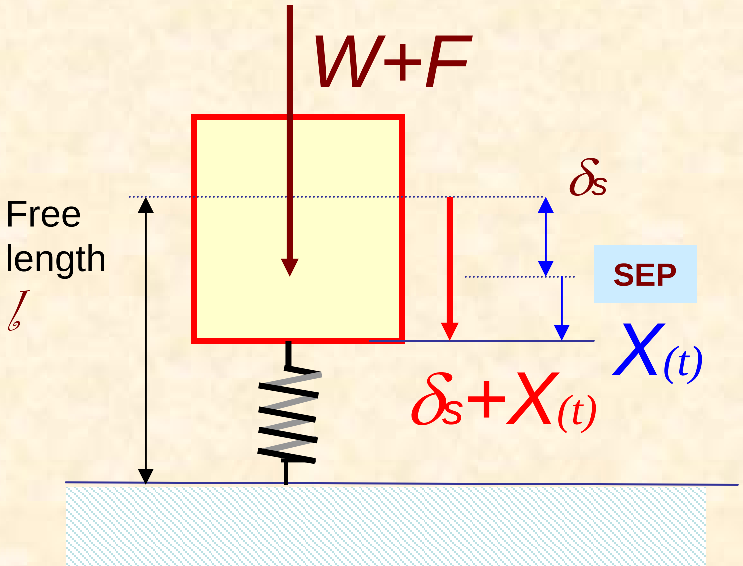
c) Then, **state** Newton's equation of motion

d) Assume no lateral (side motions)



$$M\,Ax = W+F - F_{spring}$$

Derive the equation of motion



$$M\,Ax = W+F - F_{spring}$$

$$F_{spring} = K(X+\delta_s)$$

$$M\,Ax = W+F - K(X+\delta_s)$$

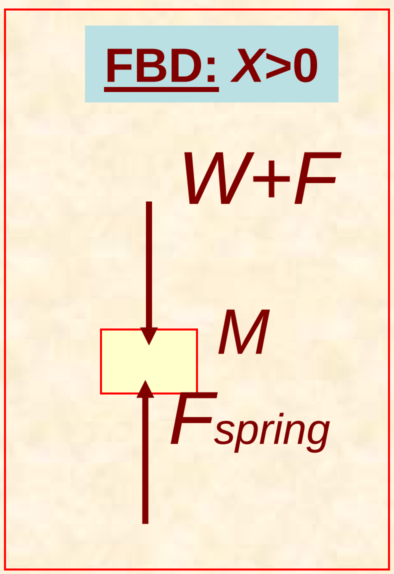
$$M\,Ax = (W-K\delta_s) + F - KX$$

Cancel terms from force balance at SEP to get

$$M\,Ax = +F - KX$$

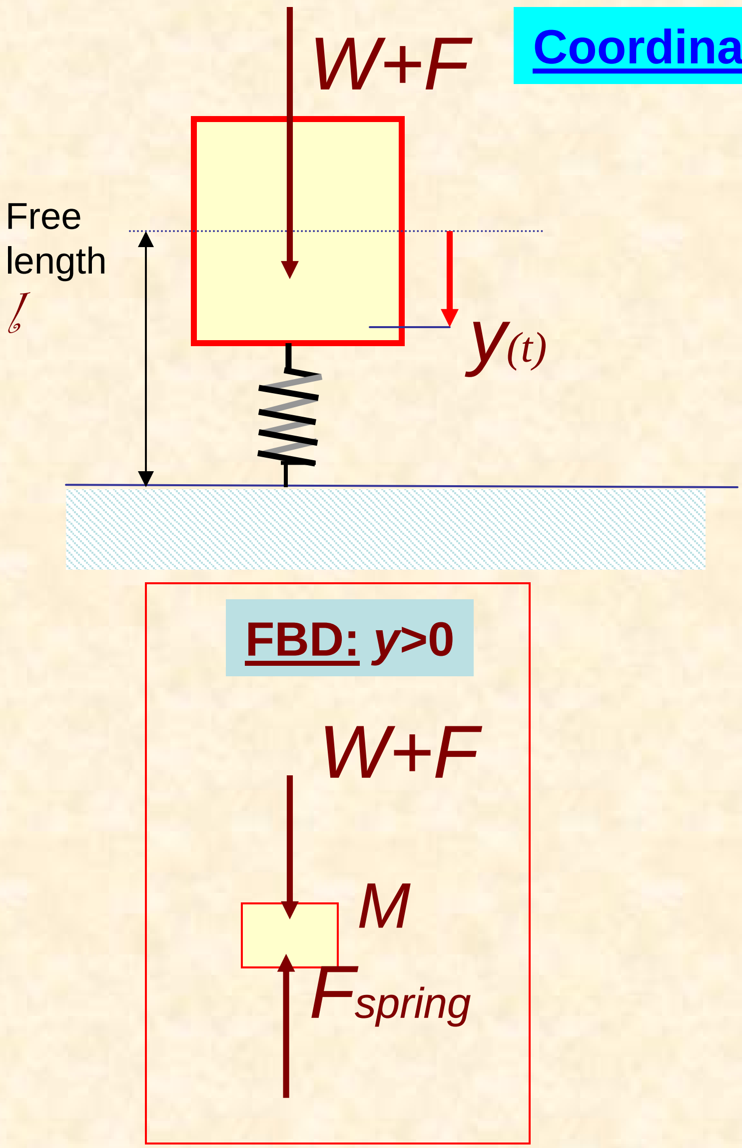
$$M\,Ax + K\,X = F_{(t)}$$

$$A_X = \frac{d^2X}{dt^2} = \ddot{X}$$



Note: Motions from **SEP**

Another choice of coordinate system



Coordinate y has origin at free length of spring element.

$$M\,A_y = W + F - F_{spring}$$

$$F_{spring} = K y$$

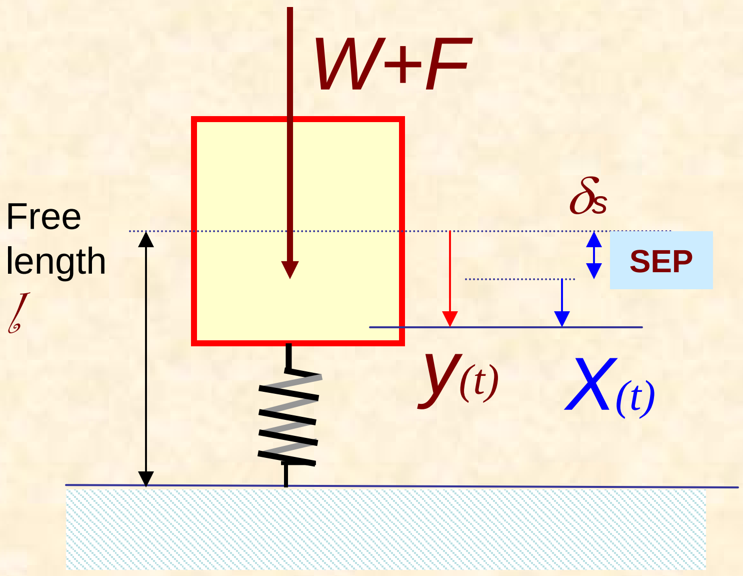
$$M\,A_y = W + F - K\,y$$

Weight (static force) remains in equation

$$M\,A_y + K\,y = W + F_{(t)}$$

$$A_y = \frac{d^2 y}{d\,t^2} = \ddot{y}$$

EOM in two coordinate systems



$y = X + \delta_s$ $\ddot{y} = \ddot{X}$

$F_{spring} = Ky = K(X + \delta_s)$

$$M\,A_y + K\,y = W + F_{(t)} \quad (1)$$

$$A_y = \frac{d^2 y}{d t^2} = \ddot{y}$$

$$M\,A_x + K\,X = F_{(t)} \quad (2)$$

$$A_X = \frac{d^2 X}{d t^2} = \ddot{X}$$

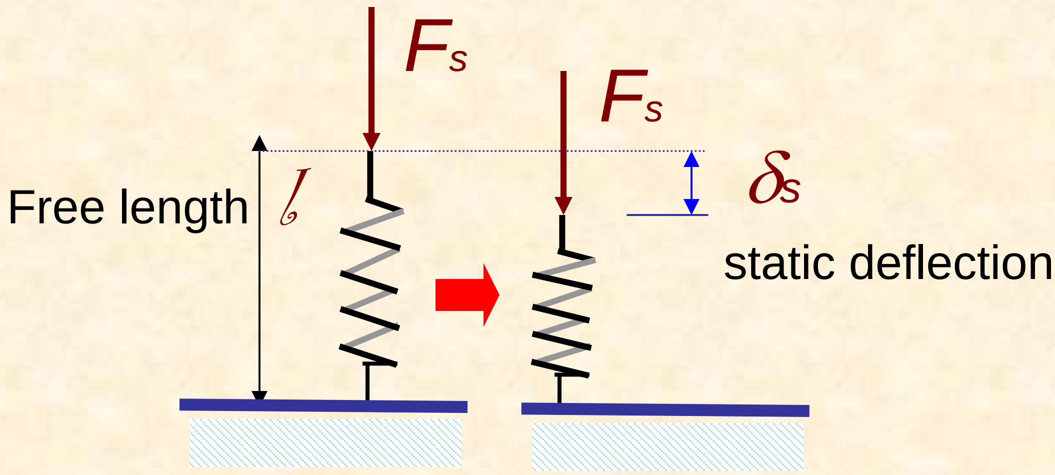
Coordinate *y* has origin at free length of spring element. *X* has origin at SEP.

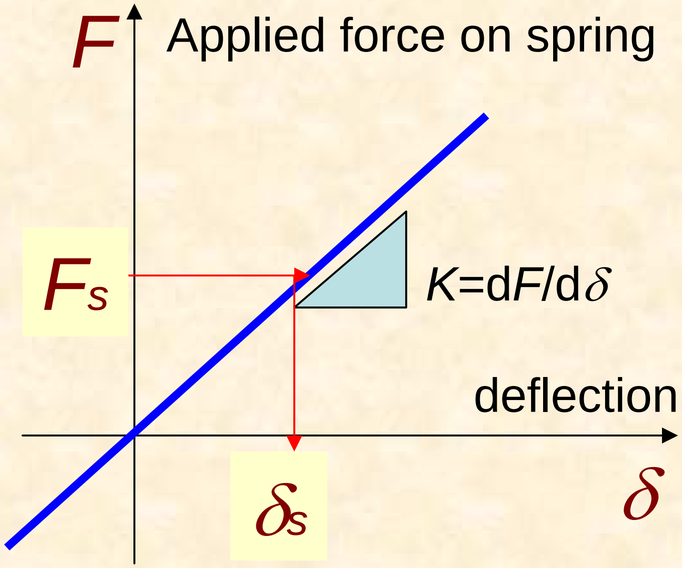
(?) Are Eqs. (1) and (2) the same?
(?) Do Eqs. (1) and (2) represent the same system?
(?) Does the weight disappear?
(?) Can I just cancel the weight?

K-M system with dissipative element (a viscous dashpot)

Derive the equation of motion (EOM) for the system

Recall: a linear elastic element





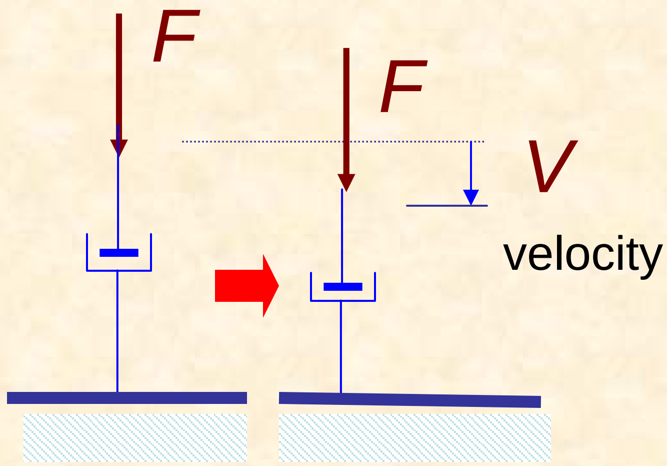
Notes:

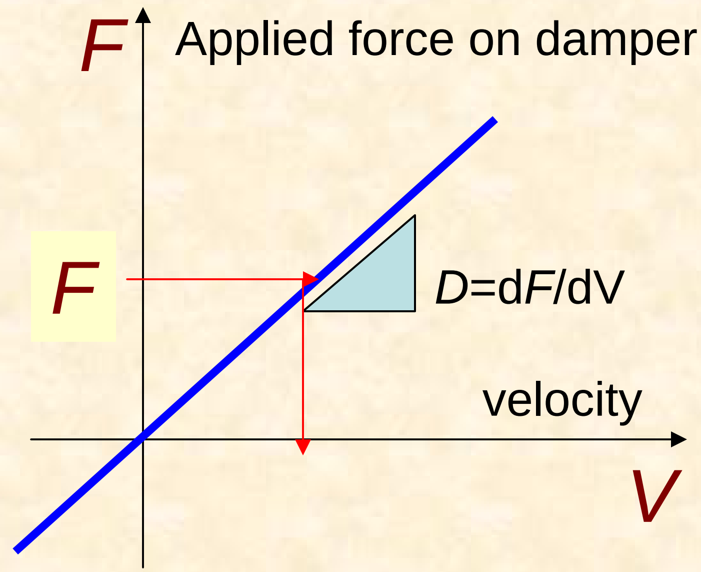
a) Spring element is regarded as massless

b) K = stiffness coefficient is constant

c) Spring reacts with a force proportional to deflection and stores potential energy

(Linear) viscous damper element





Notes:

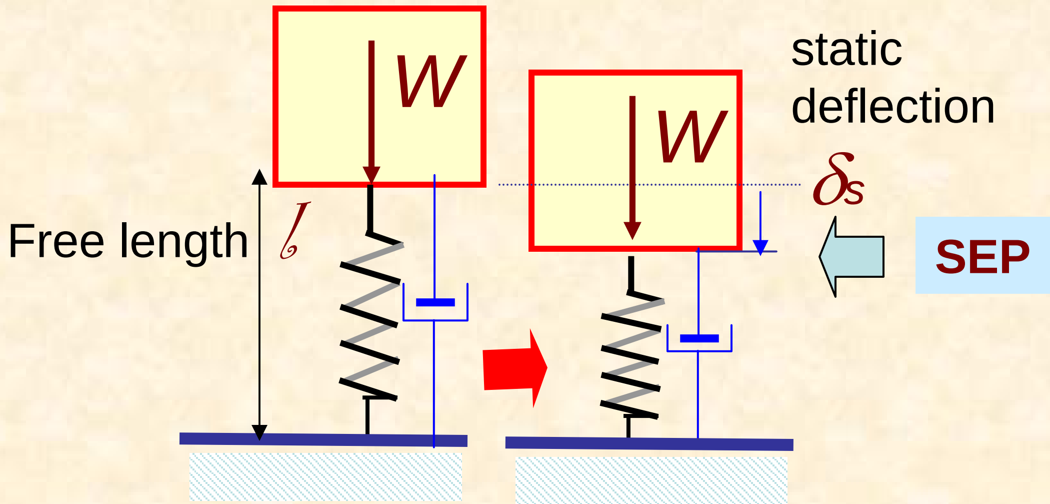
a) Dashpot element (damper) is regarded as massless

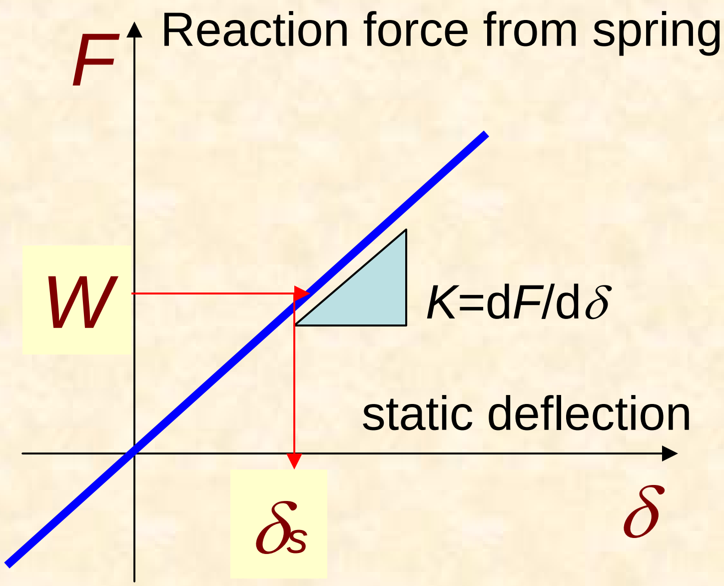
b) *D* = **damping coefficient** [N/(m/s) or lbf/(in/s)] is constant

c) A damper needs velocity to work; otherwise it **can not** dissipate mechanical energy

d) Under static conditions (no motion), a damper does NOT react with a force

Spring + Damper + Added weight





Notes:

a) Block has weight *W= Mg* and is regarded as a point mass.

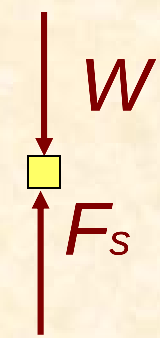
b) Weight applied very slowly – static condition.

c) Damper **does NOT react** to a static action, i.e. F_D=0

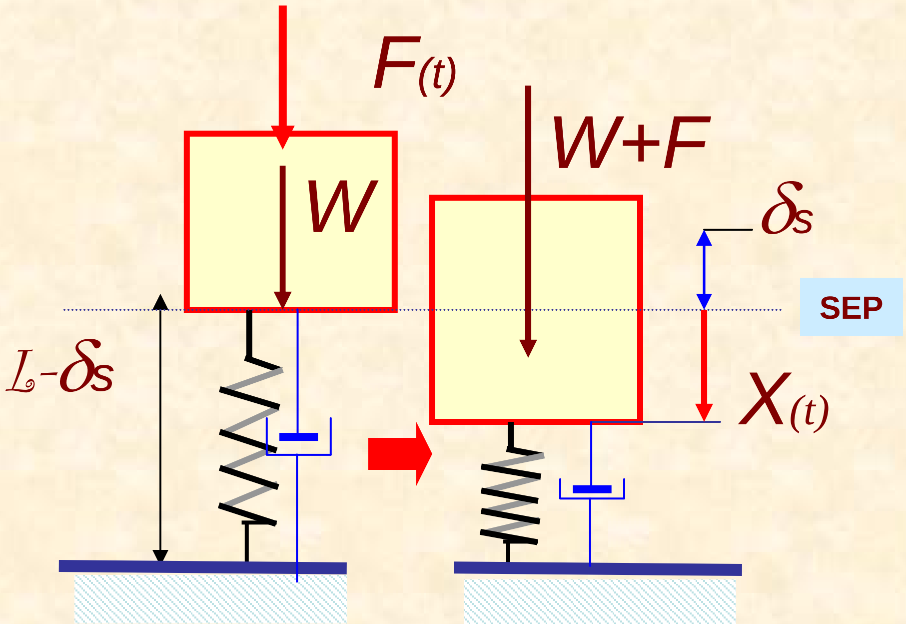
SEP: static equilibrium position

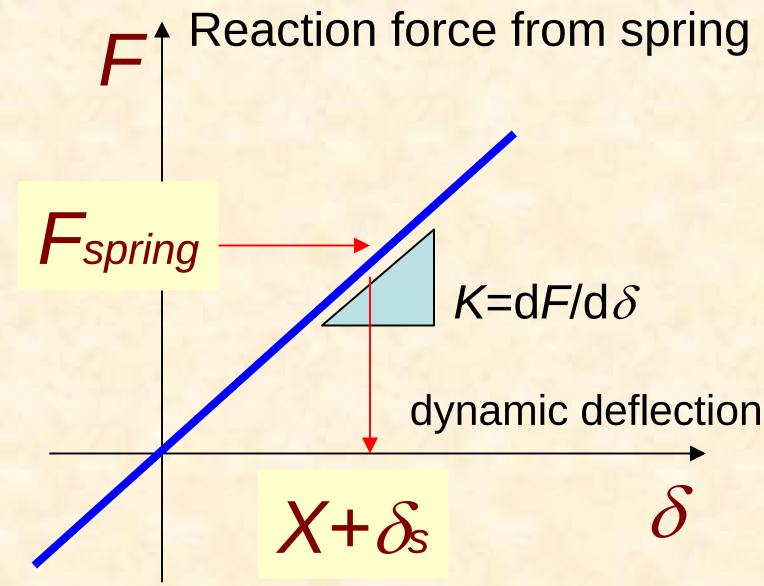
Balance of static forces

$$W = F_s = K\,\delta_s$$



Spring + Damper + Weight + force $F_{(t)}$





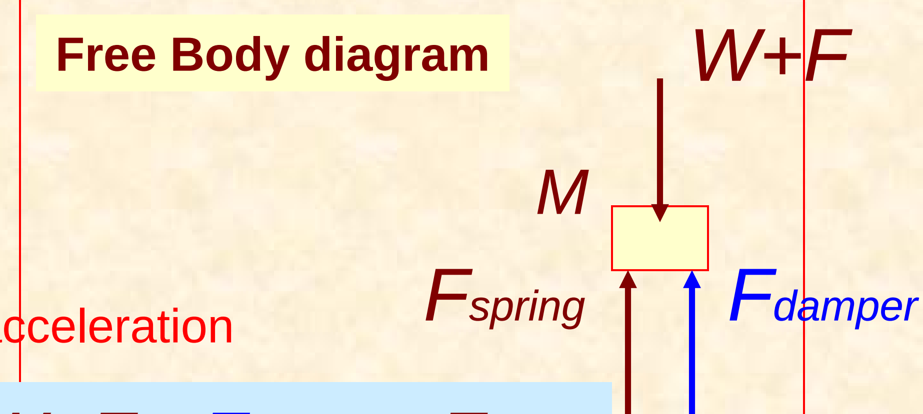
Notes:

a) Coordinate X describes motion from **Static Equilibrium Position** (SEP)

b) For free body diagram, assume a state of motion $X_{(t)}>0$

c) **State** Newton's equation of motion

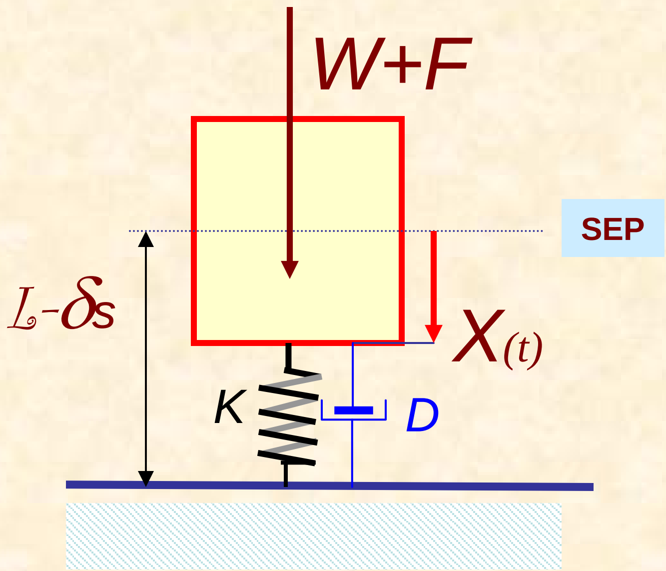
d) Assumed no lateral (side motions)



acceleration

$$M\,Ax = W+F - F_{damper} - F_{spring}$$

Equation of motion: spring-damper-mass



$$M\,Ax = W + F_{(t)} - F_{damper} - F_{spring}$$

$$A_X = \frac{d^2X}{dt^2} = \ddot{X}$$

$$V_X = \frac{dX}{dt} = \dot{X}$$

$$F_{damper} = D\,V_X$$

$$F_{spring} = K(X + \delta_s)$$

$$M\,Ax = W + F - K(X+\delta_s) - D\,V_X$$

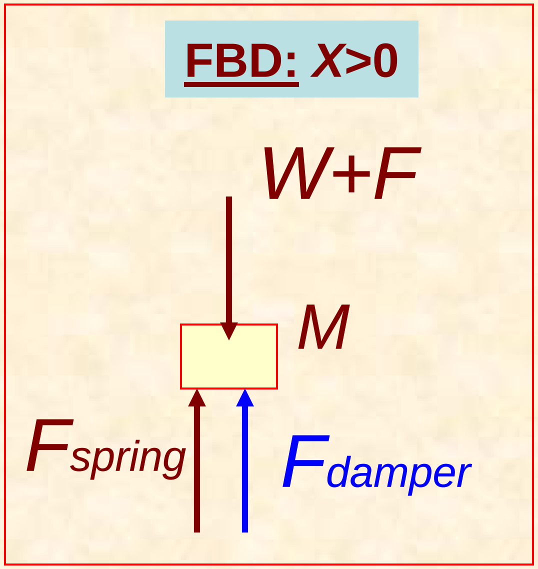
$$M\,Ax = (W - K\delta_s) + F - KX - DV_X$$

Cancel terms from force balance at SEP to get

$$M\,Ax = +F - KX - D\,V_X$$

$$M\,Ax + DV_X + K\,X = F_{(t)}$$

EOM: $$M\,\ddot{X} + D\,\dot{X} + K\,X = F_{(t)}$$

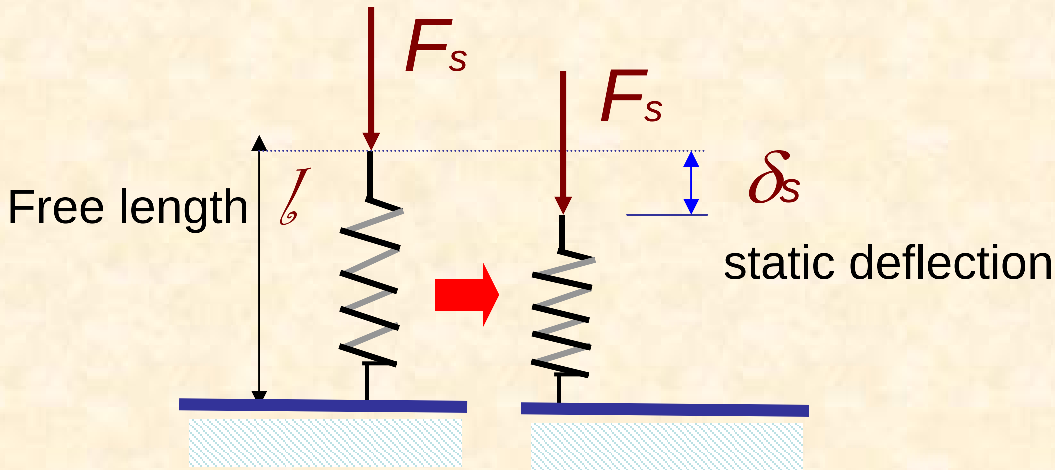


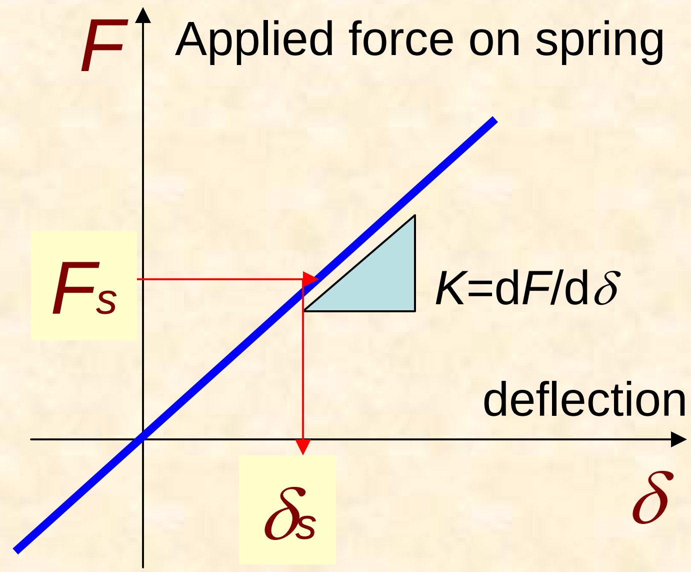
Notes: Motions from SEP

Simple nonlinear mechanical system

Derive the equation of motion (EOM) for the system and **linearize EOM about SEP**

Recall a linear spring element





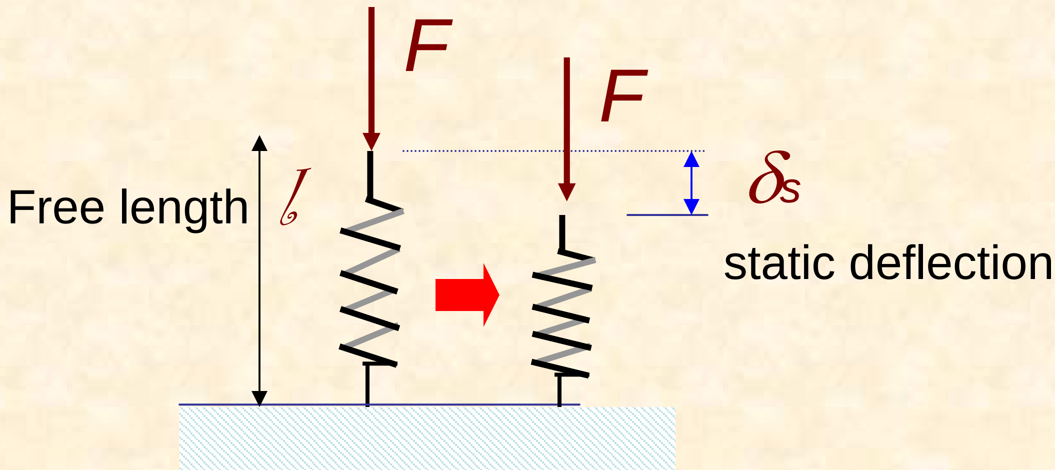
Notes:

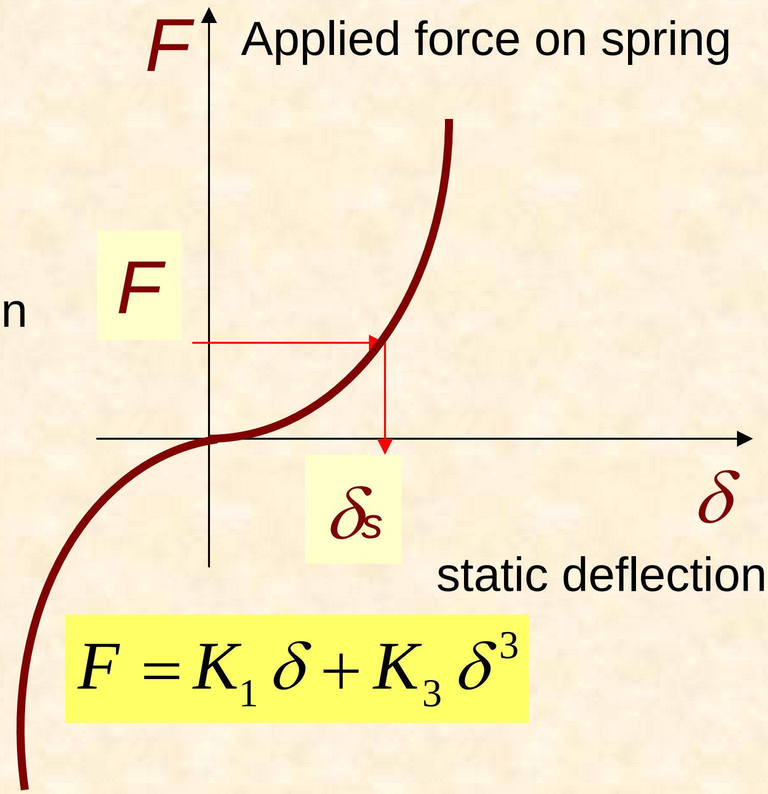
a) Spring element is regarded as massless

b) K = stiffness coefficient is constant

c) Spring reacts with a force proportional to deflection and stores potential energy

A nonlinear spring element



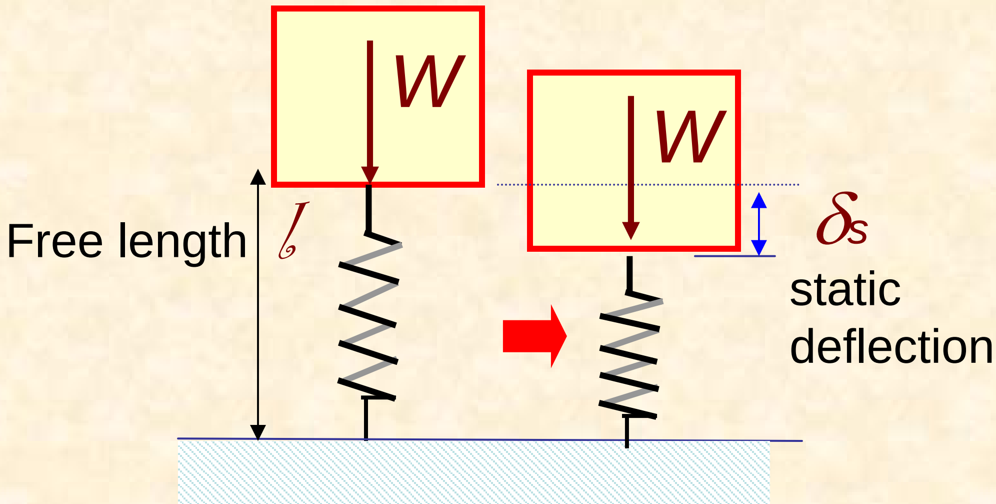


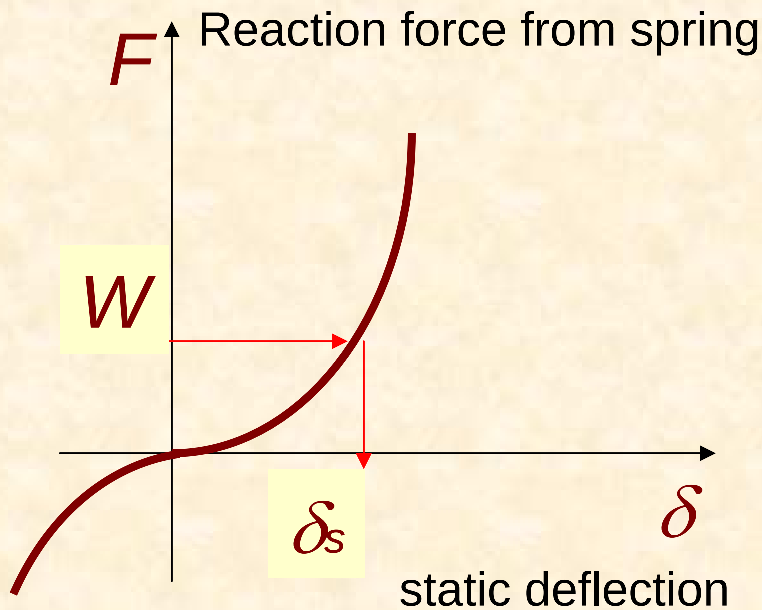
$$F = K_1 \delta + K_3 \delta^3$$

Notes:

a) Spring element is massless
b) Force vs. deflection curve is **NON linear**
c) K_1 and K_3 are material parameters [N/m, N/m^3]

Linear spring + added weight



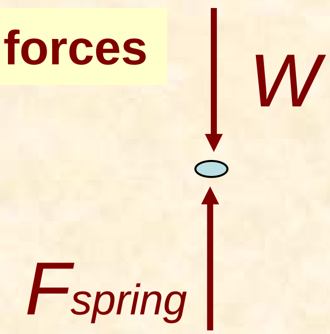


Notes:

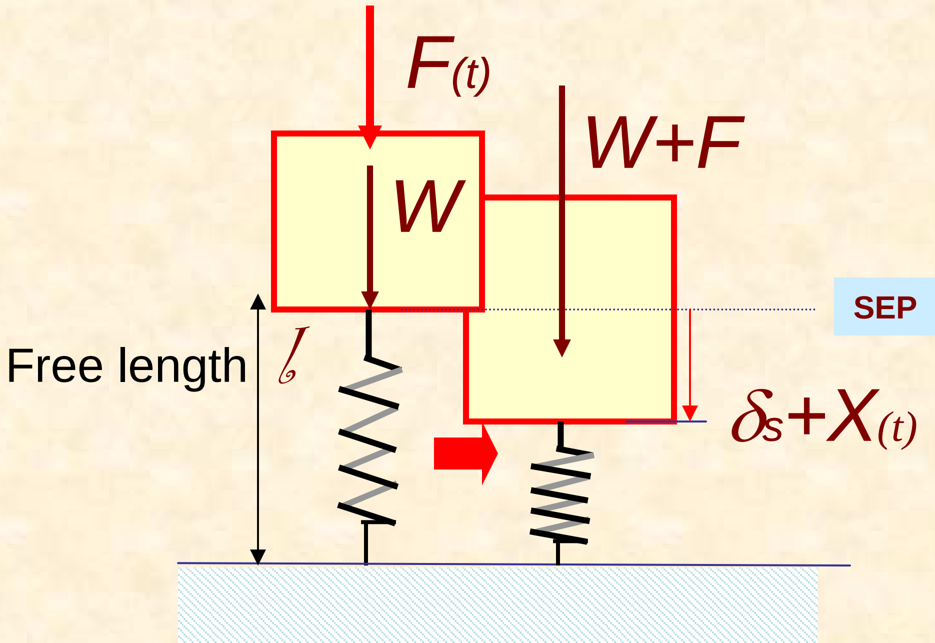
a) Block has weight *W= Mg*
b) Block is regarded as a point mass
c) Static deflection δ_s found from solving nonlinear equation:

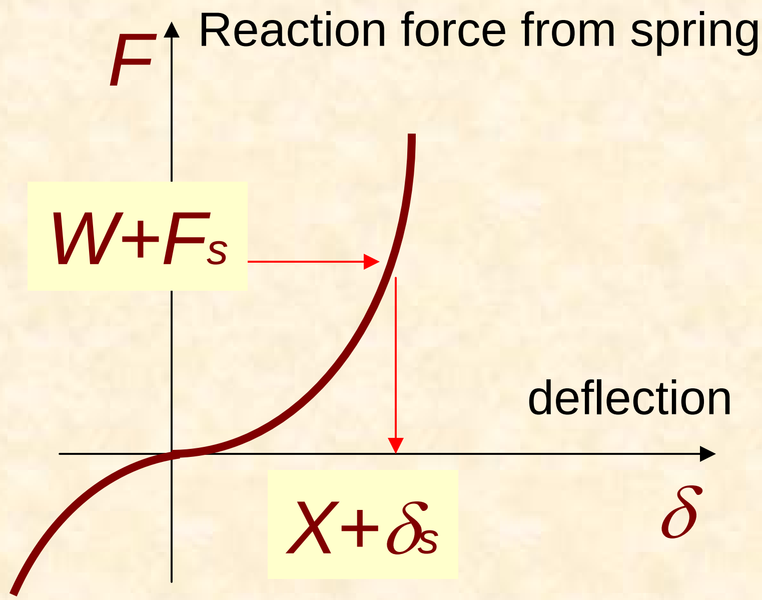
$$W = F_{spring} = K_1\,\delta_s + K_3\,\delta_s^3$$

Balance of static forces



Nonlinear spring + weight + force $F_{(t)}$





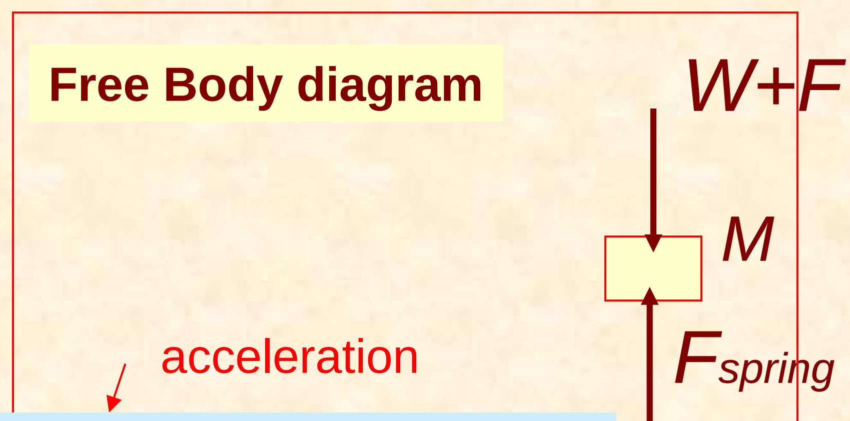
Notes:

a) Coordinate X describing motion has origin at **Static Equilibrium Position** (SEP)

b) For free body diagram, assume state of motion, for example $X_{(t)}>0$

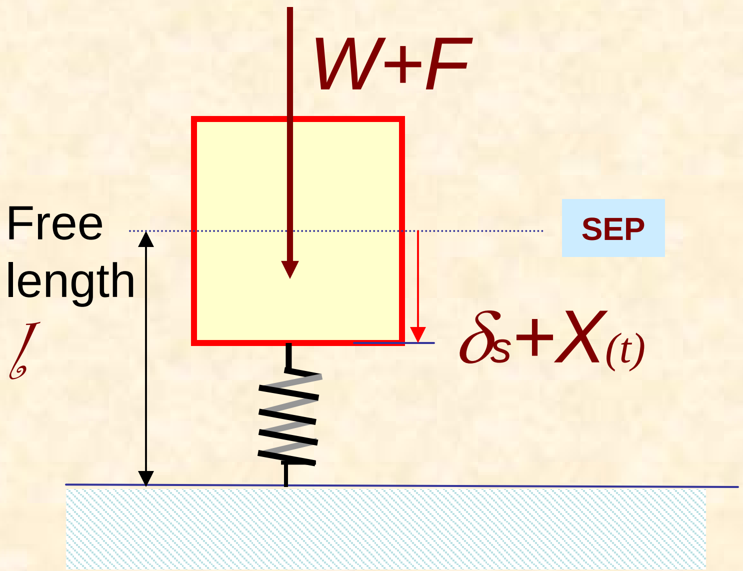
c) Then, **state** Newton's equation of motion

d) Assume no lateral (side motions)



$$M\,Ax = W+F - F_{spring}$$

Nonlinear Equation of motion



$$M\,\ddot{X} = F_{(t)} + W - F_{spring}$$

$$A_X = \frac{d^2X}{dt^2} = \ddot{X}$$

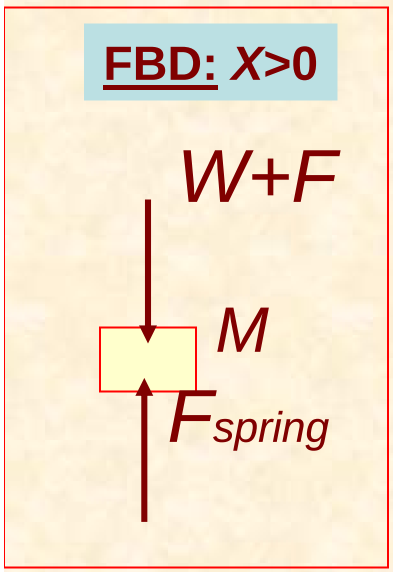
$$F_{spring} = K_1\left(\delta_s + X\right) + K_3\left(\delta_s + X\right)^3$$

$$M\,\ddot{X} = F_{(t)} + W - K_1\left(\delta_s + X\right) - K_3\left(\delta_s + X\right)^3$$

Expand RHS:

$$M\,\ddot{X} = F_{(t)} + W - K_1\left(\delta_s + X\right) - K_3\left(\delta_s^3 + 3\delta_s^2 X + 3\delta_s X^2 + X^3\right)$$

$$M\,\ddot{X} = F_{(t)} + \left(W - K_1\delta_s - K_3\delta_s^3\right) - \left(K_1 + 3K_3\delta_s^2\right)X - K_3\left(3\delta_s X^2 + X^3\right)$$

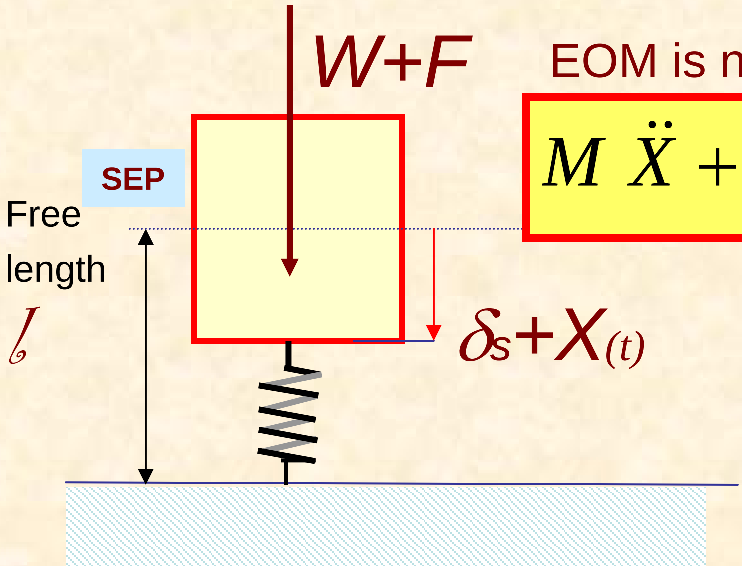


Cancel terms from force balance at SEP to get

$$M\,\ddot{X} + \left(K_1 + 3K_3\delta_s^2\right)X + K_3\left(3\delta_s X^2 + X^3\right) = F_{(t)}$$

Notes: Motions are from SEP

Linearize equation of motion



EOM is nonlinear:

$$M\ddot{X} + \left(K_1 + 3K_3\delta_s^2\right)X + K_3\left(3\delta_s X^2 + X^3\right) = F_{(t)}$$

Assume motions $X(t) << \delta_s$

which means $|F(t)| << W$

and set $X^2 \sim 0$, $X^3 \sim 0$

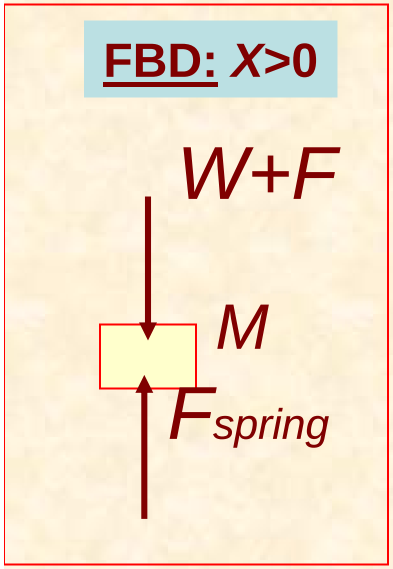
to obtain the linear EOM:

$$M\ddot{X} + K_e X = F_{(t)}$$

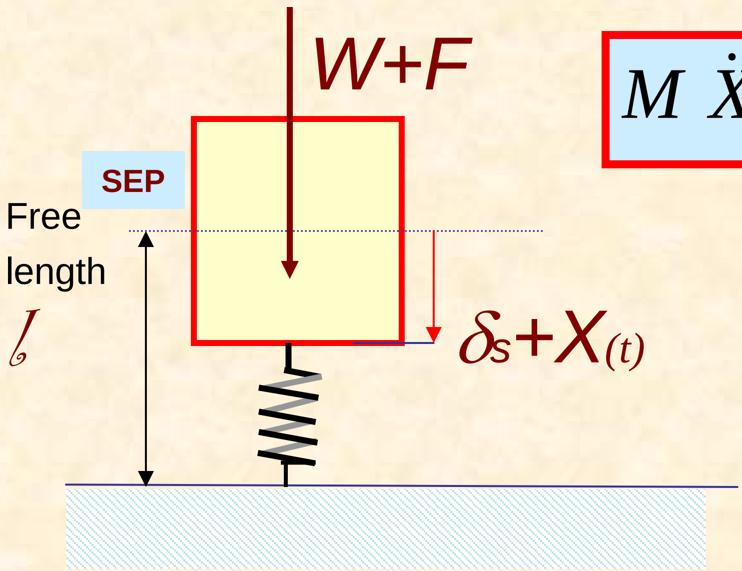
where the linearized stiffness $K_e = \left(K_1 + 3K_3\delta_s^2\right)$

is a function of the static condition

$$K_e = \left[\frac{dF_{spring}}{d\delta}\right] = \left.\frac{d\left(K_1\delta + K_3\delta^3\right)}{d\delta}\right|_{\delta_s} = K_1 + 3K_3\delta_s^2$$

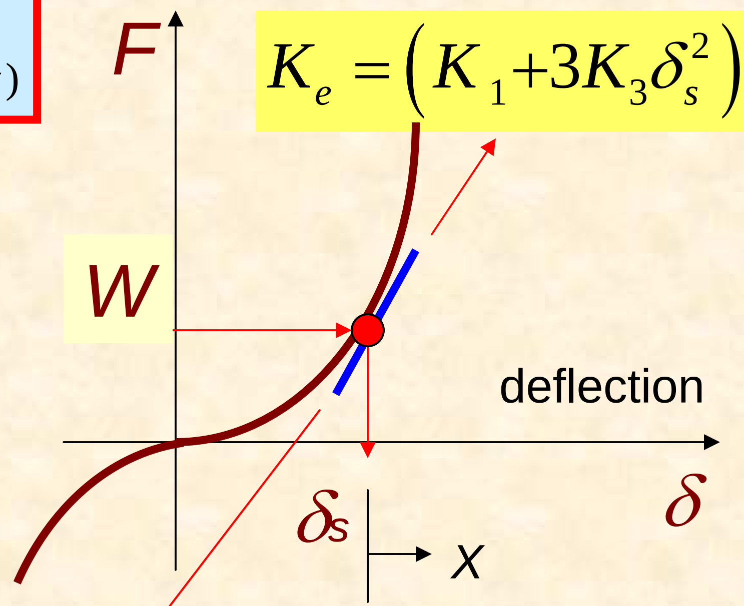


Linear equation of motion

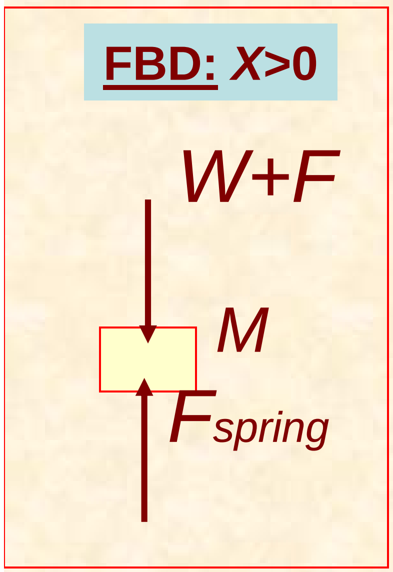


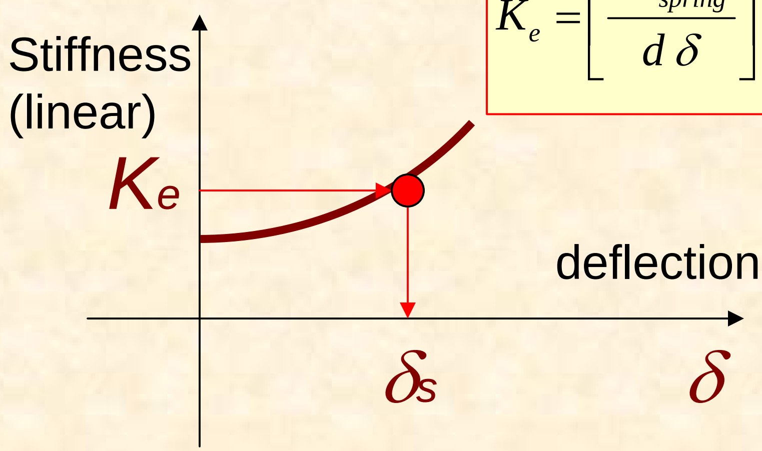
$$M\ddot{X} + K_e X = F_{(t)}$$

$$K_e = \left(K_1 + 3K_3\delta_s^2\right)$$



$$K_e = \left[\frac{dF_{spring}}{d\,\delta}\right] = \frac{d\left(K_1\,\delta + K_3\,\delta^3\right)}{d\,\delta}\Bigg|_{\delta_s} = K_1 + 3K_3\,\delta_s^2$$





Read & rework

Examples of derivation of EOMS for physical systems available on class URL site(s)