IMBALANCE RESPONSE OF A RIGID ROTOR SUPPORTED ON SHORT LENGTH OPEN-ENDED SFDs AND ELASTIC SUPPORTS

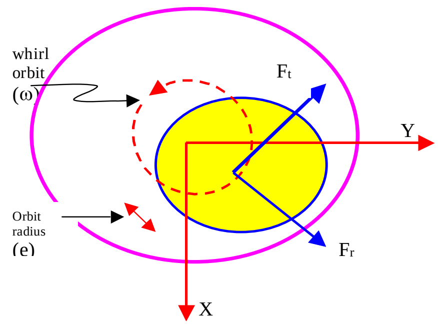
WITH FLUID INERTIA EFFECTS

(c) Dr. L. San Andres, TAMU 11/2008, 2009

The EOMs for a rigid rotor of mas (2M) supported on two identical SFDs with isotropic elastic supports (squirrel cage) are:

$$M \cdot a_r + K \cdot e = -F_r + M \cdot U \cdot \omega^2 \cdot \cos(\omega t)$$

$$M \cdot a_t = -F_t + M \cdot U \cdot \omega^2 \cdot \sin(\omega t)$$



Forces in a SFD describing circular centered motions

where K is the stiffness of the squirrel cage support. (F_r,F_t) are the SFD reaction forces and U is the mass imbalance displacement. e is the orbit radius (mistakenly called a journal eccentricity), and (v_t, a_r) are the journal center tangential velocity and radial acceleration in the (r,t) coordinate system.

$$v_t = e \cdot \omega \quad \& \quad a_r = -e \cdot \omega^2$$

The rotor describes circular centered orbits of amplitude (e) at whirl frequency (ω) synchronous with the rotor speed (ω=Ω). The SFD forces are stated as

$$-F_r = C_{rt} \cdot v_t + M_{rr} \cdot a_r \qquad -F_t = C_{tt} \cdot v_t + M_{tr} \cdot a_r$$

$ORIGIN := 1$

(C_{rt},C_{tt}) and (M_{rr},M_{tr}) are the SFD damping and inertia force coefficients defined below

(R,L,C) are the SFD length, diameter and clearance, (μ, ρ) are the lubricant viscosity and density.

Define the following:

SFD bearing parameter: $B = (\mu \cdot R) \cdot \left(\frac{L}{C}\right)^3 \cdot \frac{1}{M \cdot \omega_n}$

Imbalance [-] $u = \frac{U}{C}$

Rotor-support spring natural frequency $\omega_n = \sqrt{\frac{K}{M}}$

frequency ratio [-] $f = \frac{\omega}{\omega_n}$

Nominal squeeze film Reynolds # $Re_s = \left(\frac{\rho}{\mu}\right) \cdot \omega_n \cdot C^2$ at the system natural frequency

For small amplitude motions (i.e. small imbalances u<<1), $B = \frac{4}{\pi} \cdot \zeta$ is "just like" a viscous damping ratio ζ: $\zeta = \frac{C_{tt}}{2 \cdot M \cdot \omega_n}$

SET: $a := 1$ Set: a=2 for full film SFD, a=1 for PI-film SFD

▸ Forces & EQN

▸ Solve nonlinear function: find roots

Examples

$u := .40$ $B := .125$

$a = 1$ PI-film model

$\zeta := B \cdot \frac{\pi}{4} \cdot a$

$Res := 0$

$\zeta = 0.098$

search and order

for comparison find Imbalance response of a LINEAR SYSTEM with damping ratio ζ

$$a_L(u,f) := \frac{u \cdot f^2}{\left[\left(1-f^2\right)^2 + (2\cdot\zeta\cdot f)^2\right]^{.5}}$$

No fluid inertia

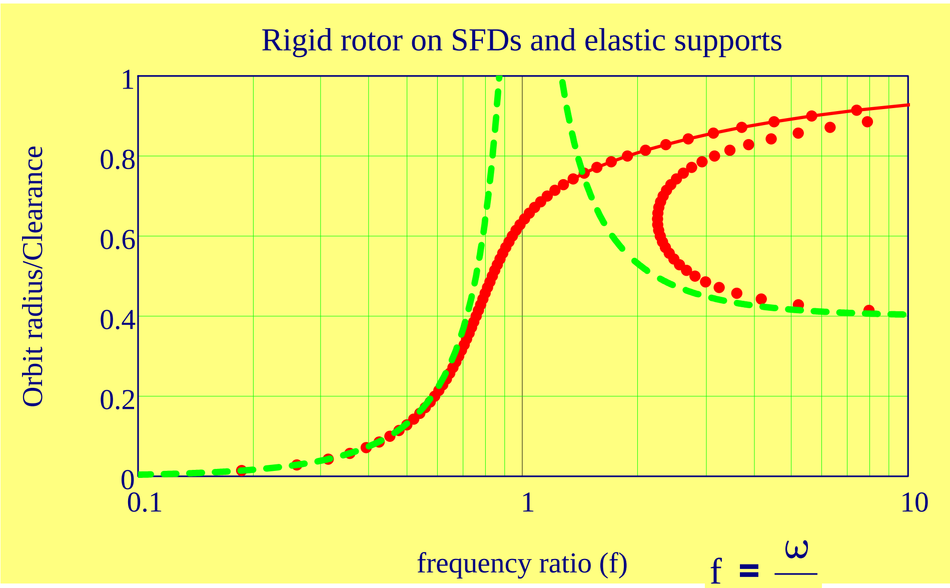
$Res = 0$

approximate linear response at f=1

$$LR := \frac{u}{2\cdot\zeta}$$

$LR = 2.037$

$\zeta = 0.098$



$$f = \frac{\omega}{\omega_n}$$

A **multiple valued rotor response** appears for f=f* >3. There are three possible orbits:

a) one with small amplitude (bottom) and also low transmitted forces, (desirable operating mode)

b) one with large amplitude (top) and large transmitted forces, (undesirable mode of operation)

c) one intermediate orbit which is UNSTABLE, i.e. it becomes either (a) or (b), the stable responses.

Note that rotor jump-down and jump-up may occur while accelerating above f* and decelerating towards f*, respectively.

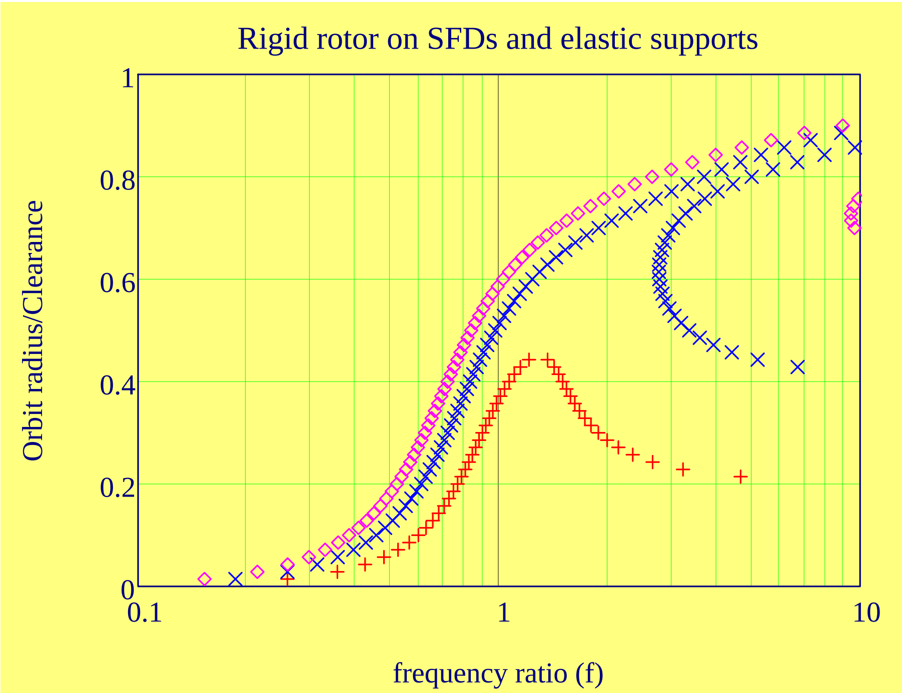
Rotor jumps are rarely reported in actual rotor-SFD applications. Why?

Imbalance varies

$u := \begin{pmatrix} 0.2 \\ 0.4 \\ 0.6 \end{pmatrix}$ $B := .25$ $Res := 0$

$\zeta := B \cdot \frac{\pi}{4} \cdot a$

CURVE colors = vector rows
RED: first row
BLUE: second row
PURPLE third row



Response if linear system at f=1

$LR := \frac{u}{2 \cdot \zeta}$

$B = 0.25$

$\zeta = 0.196$

$u = \begin{pmatrix} 0.2 \\ 0.4 \\ 0.6 \end{pmatrix}$ $LR = \begin{pmatrix} 0.509 \\ 1.019 \\ 1.528 \end{pmatrix}$

$f = \frac{\omega}{\omega_n}$

For small imbalance, u=0.2, the rotor response is lsingle valued and appears ~ linear, i.e. it has a peak at f nearby 1 with magnitude ~equal to u/(2ζ). For f >>1, ε approaches u (imbalance displacement).

For larger imbalances, u >= 0.34, the multiple valued response becomes evident for f >1, with jump-up (or down).

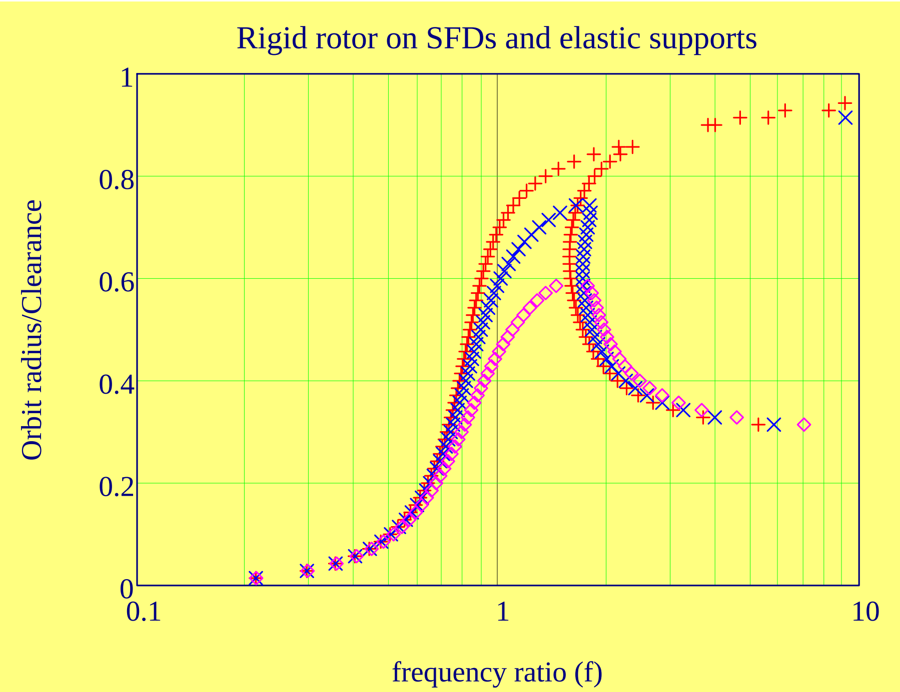
The larger the imbalance u, the larger the rotor amplitude of motion. The response shows a characteristic non-linear stiffnening or hardening effect due to the cross-coupling damping, i.r. Krt = Crt ω

Bearing parameter varies

$$B := \begin{pmatrix} 0.065 \\ .125 \\ .25 \end{pmatrix} \cdot 1 \qquad u := 0.30 \qquad Re_s := 0.00$$

$$\zeta := B \cdot \frac{\pi}{4}$$

Linear damping ratios $\zeta^T = (0.051 \quad 0.098 \quad 0.196)$



$$f = \frac{\omega}{\omega_n}$$

Response
if linear system at f=1

$$LR := \frac{u}{2 \cdot \zeta}$$

$$B = \begin{pmatrix} 0.065 \\ 0.125 \\ 0.25 \end{pmatrix} \qquad LR = \begin{pmatrix} 2.938 \\ 1.528 \\ 0.764 \end{pmatrix}$$

$$u = 0.3$$

For large Bearing numbers (B), i.e. large viscous damping, the rotor response appears linear, i.e. it has a peak at f nearby ~1 with amplitude ~= u/(2ζ).

For smaller Bearing numbers, B < 0.1, multiple valued rotor response becomes evident with a jump (up or down).

The smaller the Bearing number B, the larger the rotor amplitude thus showing a characteristic non-linear stiffnening effect due to the cross-coupling damping, i.r. Krt = Crt ω

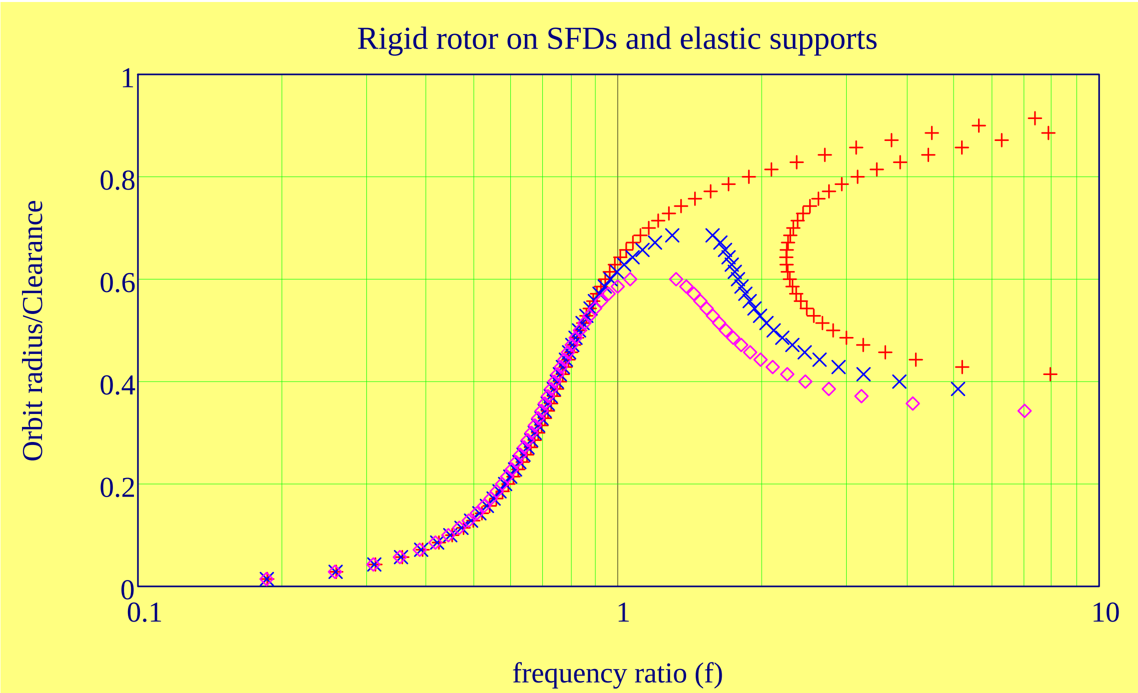
Note that too large values of B are not recommended since too much damping could lock the elastic support (pin-pin suppports) and actually making worse the esponse of other (flexible) rotor modes of vibration.

Influence of fluid inertia on rotor-SFD imbalance response

Reynolds # varies

$u := 0.4$ $B := .125$ $Re_s := \begin{pmatrix} 0 \\ 6 \\ 12 \end{pmatrix} \cdot 1$

squeeze film Reynolds number: $Re_s = \left(\frac{\rho}{\mu}\right) \cdot \omega_n \cdot C^2$



$B = 0.125$

$u = 0.4$

$\zeta = 0.098$

$\frac{u}{2 \cdot \zeta} = 2.037$

$Re_s = \begin{pmatrix} 0 \\ 6 \\ 12 \end{pmatrix}$

Fluid inertia reduces the likelihood of multiple valued response and hence the (near) absence of rotor jumps (up or down). Fluid inertia acts in two ways, since it increases the effective direct damping coefficients and reduces the cross-coupled damping, i.e. lowers the SFD "stiffness".

Recall that the SFD forces for circular centered orbits are:

$$-F_r = C_{rt} \cdot v_t + M_{rr} \cdot a_r \qquad -F_t = C_{tt} \cdot v_t + M_{tr} \cdot a_r \qquad M_{rr} > 0 \qquad M_{tr} < 0$$

with: $v_t = e \cdot \omega$ $\quad a_r = -e \cdot \omega^2$

Thus $\frac{-F_r}{e} = \left(C_{rt} \cdot \omega - M_{rr} \cdot \omega^2\right) = K_{rr}^{\langle effective \rangle}$ $\quad \frac{-F_t}{e \cdot \omega} = \left(C_{tt} - M_{tr} \cdot \omega\right) = C_{tt}^{\langle effective \rangle}$

Most importantly, the direct added mass coefficient Mrr can be large enough that it reduces considerably the critical speed of the rotor-SFD system.

Rotors supported on large clearance SFDs with light viscosity lubricants (as those used in jet engine applications) do evidence the effects of fluid inertia; namely, a notable reduction in criticaql speeds (natural frequncies), single valued response (no jumps reported), and little damping, in particular at high frequencies where air ingestion tends to dominate damper forced performance.

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When observed, what do these r**otor jumps mean? where do they come from?**

$$f4_{kk} = 2.343$$