NOTES 3
KINEMATICS OF JOURNAL BEARINGS

Lecture 3 introduces the analysis of fluid flow in a (simple & ideal) cylindrical journal bearing whose film thickness is a function of the radial clearance and the instantaneous journal center displacements (eccentricity vector). The thin film is described with either of two coordinate systems, one inertial (fixed) and the other one *(r, t)* moving with the journal center. Corresponding expressions for classical Reynolds equation follow. In a coordinate system rotating with ½ the journal angular speed, the journal motion (translation and rotation) is seen as a pure squeeze type motion, thus enabling a simpler understanding of the physical terms in Reynolds equation.

Nomenclature

C	Bearing radial clearance. = R_B -R_J [m]
e	$\sqrt{e_X^2+e_Y^2}$. Journal center eccentricity [m]
F	Fluid film reaction force (acting on journal) $F=\sqrt{F_X^2+F_Y^2}=\sqrt{F_r^2+F_r^2}$ [N]
F_X, F_Y	Components of fluid film force along fixed *X,Y* axes
F_r , F_t	Components of fluid film force along fixed *r, t* axes [N]
h	$C+e\cos\theta = C + e_X\cos\Theta + e_Y\sin\Theta$. Film thickness [m]
L	Bearing axial length [m]
M_x, M_z	$\int_0^h \rho V_x\,dy$, $\int_0^h \rho V_z\,dy$. Mass flow rates per unit length [kg/(m-s)]
P	Hydrodynamic pressure [N/m^2]
P_{cav}	Liquid cavitation pressure
R_B , R_J	Bearing and Journal Radii [m]
t	Time [s]
$\bar{V}_x, \bar{V}_z$	$\frac{M_x}{\rho_A h}, \frac{M_z}{\rho_A h}$. Mean flow velocities [m/s]
V_X, V_Y	$\dot{e}_X, \dot{e}_Y$. Components of journal velocity along *X,Y* axes [m/s]
V_r, V_t	$\dot{e}, e\dot{\phi}$. Components of journal velocity along *r,t* axes [m/s]
V_S	(pure) squeeze film velocity [m/s]
(X,Y) & (r,t)	Fixed coordinate system, moving coordinate system
$\Theta=x/R$, y, z	Coordinate system on plane of bearing
ϕ	$\tan\left(e_Y/e_X\right)$. Journal attitude angle
ρ	Fluid density [kg/m^3]
μ	Fluid absolute viscosity [N.s/m^2]
ω	Whirl frequency [rad/s]
Ω	Journal angular speed [rad/s]

The global mass conservation Eqn. in thin film flows is:

$$\frac{\partial(\rho h)}{\partial t}+\frac{\partial M_x}{\partial x}+\frac{\partial M_z}{\partial z}=0 \qquad (3.1)$$

or, in terms of the mean flow velocities:

$$\frac{\partial}{\partial t}\{\rho h\}+\frac{\partial}{\partial x}\{\rho h \bar{V}_x\}+\frac{\partial}{\partial z}\{\rho h \bar{V}_z\}=0 \qquad (3.2)$$

Substitution of the mean velocities

$$\bar{V}_x=-\frac{h^2}{12\mu}\frac{\partial P}{\partial x}+\frac{U}{2}; \quad \bar{V}_z=-\frac{h^2}{12\mu}\frac{\partial P}{\partial z} \qquad (3.3)$$

leads to **Reynolds Equation** of Classical Lubrication Theory,

$$\frac{\partial}{\partial t}\{\rho h\}+\frac{1}{2}\frac{\partial}{\partial x}\{\rho h U\}=\frac{\partial}{\partial x}\left\{\frac{\rho h^3}{12\mu}\frac{\partial P}{\partial x}\right\}+\frac{\partial}{\partial z}\left\{\frac{\rho h^3}{12\mu}\frac{\partial P}{\partial z}\right\} \qquad (3.4)$$

Recall that the wall shear stress differences are given as functions of the mean flow components,

$$\begin{aligned}\Delta\tau_{xy}&=\left\langle\tau_{xy}\right|_{y=0}^{y=h}=h\frac{\partial P}{\partial x}=-\frac{12\mu}{h}\left(\bar{V}_x-\frac{U}{2}\right);\\ \Delta\tau_{zy}&=\left\langle\tau_{zy}\right|_{y=0}^{y=h}=h\frac{\partial P}{\partial z}=-\frac{12\mu}{h}\left(\bar{V}_z\right)\end{aligned} \qquad (3.5)$$

Fluid Flow in a Cylindrical Journal Bearing

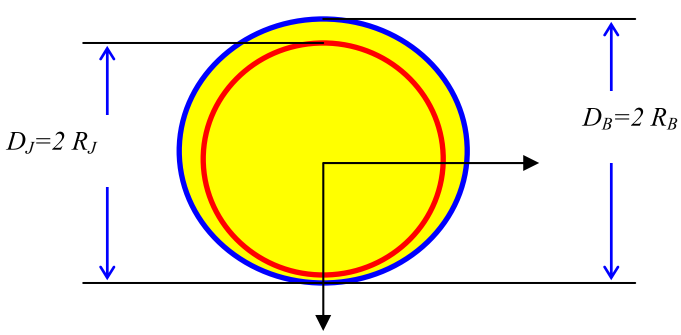

Figure 3.1 Schematic of cylindrical bearing

Cylindrical fluid film bearings are commonly used to support loads, static and dynamic, in rotating machinery. These lubricated bearings also introduce viscous damping that aids in reducing the amplitude of vibrations in operating machinery.

A depicted in Figure 3.1, a **plain** cylindrical journal bearing comprises of an inner rotating cylinder (JOURNAL) of radius R_J and an outer cylinder (BEARING) of radius R_B ($>R_J$). The two cylinders are closely spaced and the annular gap between the two cylinders is filled with a lubricant. The radial clearance $C = (R_B - R_J)$ is very small, typically $C/R_J \approx 0(10^{-3})$ in mineral oil lubricated bearings.

The journal spins with angular speed (Ω). The journal center, denoted by O_J, may also perform translational motions within the bearing clearance. The bearing or housing is stationary (not moving) in most applications. Notable exceptions are those of floating ring journal bearings and crankshaft support bearings in reciprocating engines.

The smallness of the film thickness ratio, $h/C<<1$, allows for a Cartesian coordinate[1] $(x=R\Theta, y, z)$ be located on the bearing surface (see Figure 3.2). Then, the Reynolds equation describing the flow in the journal bearing becomes

$$\frac{1}{R^2}\frac{\partial}{\partial \Theta}\left\{\frac{\rho\, h^3}{12\,\mu}\frac{\partial P}{\partial \Theta}\right\}+\frac{\partial}{\partial z}\left\{\frac{\rho\, h^3}{12\,\mu}\frac{\partial P}{\partial z}\right\}=\frac{\partial}{\partial t}\left\{\rho h\right\}+\frac{\Omega}{2}\frac{\partial}{\partial \Theta}\left\{\rho\, h\right\} \tag{3.8}$$

in the flow domain $\{0 \le \Theta \le 2\pi\,,\ -\tfrac{1}{2}L \le z \le \tfrac{1}{2}L\}$, where $h(\Theta, z, t)$ is the film thickness, L is the bearing axial length, and $U = \Omega R_J$ is the journal surface speed.

The boundary conditions for the hydrodynamic pressure in the **plain** cylindrical bearing are[2]:

a) The pressure is continuous and periodic in the circumferential direction (Θ), i.e.

$$P(\Theta, z ,t) = P(\Theta + 2\pi, z, t) \tag{3.9}$$

b) At the bearing sides or axial ends, the pressure equals the discharge or atmospheric value

$$P(\Theta, \tfrac{1}{2} L\ , t) = P(\Theta, - \tfrac{1}{2} L, t) = P_a \tag{3.10}$$

Alternatively, in the absence of journal misalignments the flow domain is symmetric about the plane $z = 0$ and $P(z)=P(-z)$. Hence, the axial flow rate is nil at the bearing mid-plane (z=0), i.e.

$$\partial P/\partial z = 0 \quad \text{at } z = 0 \quad \text{for all } (\Theta,\ t) \tag{3.11}$$

As a constraint, everywhere in the flow domain, the hydrodynamic pressure must be above (greater than) the liquid cavitation pressure, i.e.

$$P \ge P_{cav} \quad \text{in } 0 \le \Theta \le 2\pi\,,\ -\tfrac{1}{2} L \le z \le \tfrac{1}{2} L \tag{3.12}$$

Here P_{cav} represents the lubricant saturation pressure or the saturation pressure for release of dissolved gases, typically ambient pressure. In practice, no distinction is made between these two types of pressures since hydrodynamic film pressures can be one to two orders of magnitude larger than ambient.

1 Surface curvature effects in the fluid flow are negligible in most bearing configurations.

2 The following simple model does not account for feeding holes or axial grooves for supply of the lubricant into the bearing.

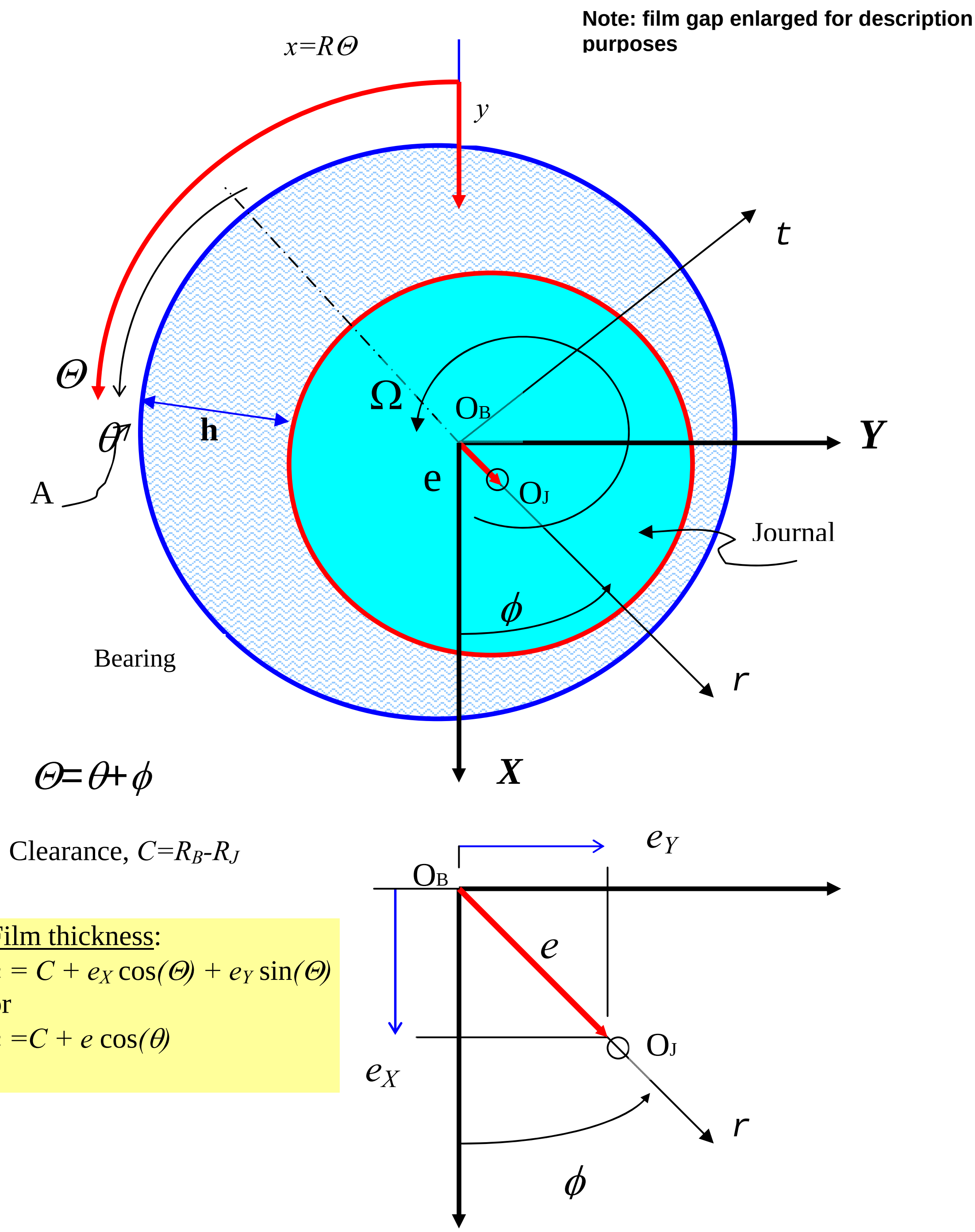

Figure 3.2. Schematic view of a cylindrical journal bearing. Coordinate systems: fixed (*X*,*Y*) and moving (*r*,*t*)

Film thickness in a cylindrical journal bearing

Figure 3.2 shows the film thickness (h) expressed in two coordinate systems (Θ, z) and (θ, z), both located on the bearing surface. The angular coordinate Θ has its origin at the line of $-X$ axis, as shown in the Figure, while the coordinate θ starts from the position of maximum film thickness.

The journal center O_J is displaced a distance (e) from the bearing center O_B. This offset distance is known as the **journal eccentricity,** and it may vary with time depending upon the imposed external load on the bearing and the journal rotational speed (Ω). The journal eccentricity cannot exceed the bearing clearance, i.e. e<C; otherwise, solid-solid contact and potential catastrophic failure may occur.

For a journal eccentric displacement e *(≤ C),* as shown in Figure 3.2, the following relation becomes apparent from triangle (0_B - 0_J –A*),*

$$(R_J + C)^2 + e^2 - 2(R_J + C)e\cos(\pi - \theta) = (R_J + h)^2 \qquad (3.13)$$

where R_J is the journal radius, h is the film thickness and θ is the angle measured from the location of maximum film thickness. Expansion of the formula above gives,

$$R_J^{\,2} + 2R_J \cdot C + C^2 + e^2 + 2(R_J + C)e\cos\theta = R_J^{\,2} + 2R_J \cdot h + h^2$$

and dividing by $(2\,R_J)$,

$$h = C + e\cos(\theta) + \frac{C}{2R_J}\left\{C + \frac{e^2}{C} + 2e\cos(\theta) - \frac{h^2}{C}\right\} \qquad (3.14)$$

and since the ratio (C/R_J) is very small, then the film thickness is just

$$h = C + e\cos(\theta) \qquad (3.15)$$

This formula is accurate for (C/R_J) ratios as large as 0.10. The film thickness formula derived assumes:

a) no journal misalignment,
b) a uniform axial and circumferential clearance,
c) rigid bearing and journal surfaces.

The components of the eccentricity (e) along the (X,Y) axes are

$$e_X = e\cos(\phi);\;\; e_Y = e\sin(\phi) \qquad (3.16)$$

where ϕ is known as the journal attitude angle, and $\Theta = \theta + \phi$. Then, the film thickness is also equal to:

$$h = C + e_X \cos\Theta + e_Y \sin\Theta = C + e\cos\theta \quad (3.17)$$

and

$$\frac{\partial h}{\partial \Theta} = -e_X \sin\Theta + e_Y \cos\Theta; \qquad \frac{\partial h}{\partial t} = \dot{e}_X \cos\Theta + \dot{e}_Y \sin\Theta \quad (3.18)$$

where ($\dot{}$) denotes differentiation with respect to time, i.e. $(\partial/\partial t)$.

Reynolds equations for plain cylindrical journal bearings

Substitution of the film thickness (h) and its gradients into Reynolds equation (3.8) renders the following PDE for an incompressible and isoviscous fluid:

$$\frac{1}{R^2}\frac{\partial}{\partial\Theta}\left\{\frac{h^3}{12\mu}\frac{\partial P}{\partial\Theta}\right\} + \frac{\partial}{\partial z}\left\{\frac{h^3}{12\mu}\frac{\partial P}{\partial z}\right\} = \left\{\dot{e}_X + e_Y\frac{\Omega}{2}\right\}\cos\Theta + \left\{\dot{e}_Y - \frac{\Omega}{2}e_X\right\}\sin\Theta \quad (3.19)$$

with $h = C + e_X\cos\Theta + e_Y\sin\Theta$ in the flow domain $\{0 \le \Theta \le 2\pi, -½ L \le z \le ½ L\}$.

An alternative form of Reynolds equation arises when using the angular coordinate (θ) whose origin is at the location of maximum film thickness. A coordinate system with radial and tangential *(r, t)* axes is conveniently defined; the radial coordinate joins the bearing and journal centers.

Recall that $e_X = e\cos(\phi)$; $e_Y = e\sin(\phi)$, and $e^2 = e_X^2 + e_Y^2$. The journal center velocities in the *(X,Y)* and *(r,t)* coordinate systems are related by the linear transformation

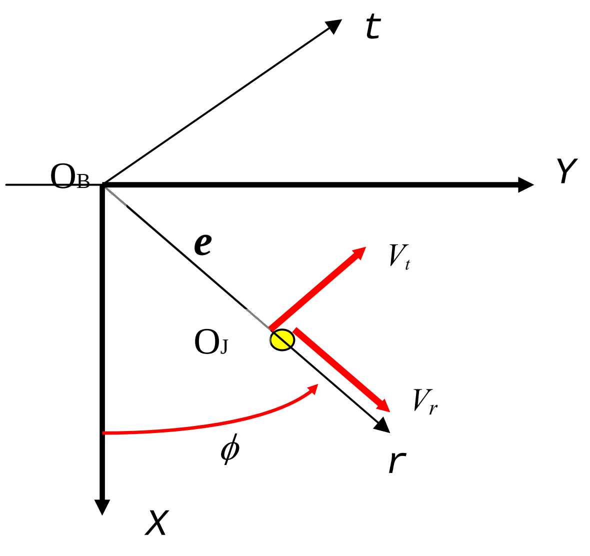

Fig. 3.3 Velocity components of journal center

$$\begin{bmatrix}\dot{e}_X \\ \dot{e}_Y\end{bmatrix} = \begin{bmatrix}\cos\phi & -\sin\phi \\ \sin\phi & \cos\phi\end{bmatrix}\begin{bmatrix}\dot{e} \\ e\dot{\phi}\end{bmatrix} \quad (3.20)$$

where $V_r = \frac{de}{dt} = \dot{e}$; $V_t = e\frac{d\phi}{dt} = e\dot{\phi}$ are the radial and tangential components of the journal center translational velocity, respectively, as shown in Fig 3.3.

From the film thickness expression $h = C + \boldsymbol{e}\cos\theta$, it follows that

$$\frac{\partial h}{\partial \theta} = -e \sin\theta$$

$$\frac{\partial h}{\partial t} = \dot{e}\cos\theta - e\frac{\partial \theta}{\partial t}\sin\theta \cdot = \dot{e}\cos\theta + e\dot{\phi}\,\sin\theta \cdot = V_r\cos\theta + V_t\sin\theta \tag{3.21}$$

Thus, Reynolds equation (3.8) for an incompressible and isoviscous fluid is also expressed as

$$\frac{1}{R^2}\frac{\partial}{\partial \theta}\left\{\frac{h^3}{12\mu}\frac{\partial P}{\partial \theta}\right\} + \frac{\partial}{\partial z}\left\{\frac{h^3}{12\mu}\frac{\partial P}{\partial z}\right\} = \dot{e}\cos\theta + e\left\{\dot{\phi} - \frac{\Omega}{2}\right\}\sin\theta \tag{3.22}$$

with $h = C + \boldsymbol{e}\cos\theta$ in the flow domain $\{0 \le \theta \le 2\pi,\ -½\,L \le z \le ½\,L\}$. Note that the *(r,t)* coordinate system may be moving since the journal center can move due to imposed dynamic loads, for example.

Fluid film forces

Integration of the pressure field on the journal surface produces a fluid film reaction force (*F*), as shown in Figure 3.4. An equal though opposing fluid film force acts on the bearing, i.e. the applied load transfers to the bearing casing.

Under static conditions, the reaction force *F* balances the applied external force *W*. Under dynamic load conditions, when the journal displaces in time, equations of motion that include the journal mass times its acceleration need be satisfied. The fluid reaction force can be written in terms of its components in either (*X, Y*) or (*r, t*) axes, i.e.

$$F = \sqrt{F_X^2 + F_Y^2} = \sqrt{F_r^2 + F_r^2} \tag{3.23}$$

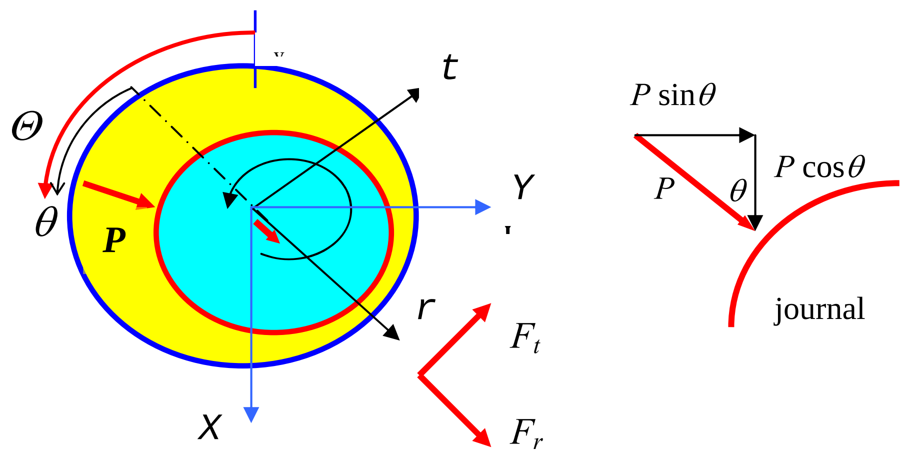

Figure 3.4 Fluid film forces acting on journal

With reference to the *(r, t)* coordinate system, the radial and tangential components of the fluid film reaction force are

$$\begin{bmatrix} F_r \\ F_t \end{bmatrix} = \int_{-L/2}^{L/2} \int_0^{2\pi} P(\theta, z, t) \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} R \cdot d\theta \, dz \tag{3.24a}$$

With reference to the fixed *(X, Y)* system, the vertical and horizontal components of the (same) fluid film force are

$$\begin{bmatrix} F_X \\ F_Y \end{bmatrix} = \int_{-L/2}^{L/2} \int_0^{2\pi} P(\Theta, z, t) \begin{bmatrix} \cos\Theta \\ \sin\Theta \end{bmatrix} R \cdot d\Theta \, dz \tag{3.24b}$$

The relationship between the components of the fluid film force in both coordinate systems is given by the linear transformation

$$\begin{bmatrix} F_X \\ F_Y \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} F_r \\ F_t \end{bmatrix} \tag{3.25}$$

In general, the fluid film reaction force is a function of the journal rotational speed (Ω),the journal center eccentricity vector (e, ϕ), and the journal center velocity with components $(\dot{e}, \dot{\phi})$, *i.e.*

$$F_\alpha = F_\alpha \left(\Omega, e_X, e_Y, \dot{e}_X, \dot{e}_Y\right)_{\alpha = X,Y} = F_\alpha \left(e, \varphi, \dot{e}, e\left[\dot{\phi} - \frac{\Omega}{2}\right]\right)_{\alpha = r,t} \tag{3.26}$$

Kinematics of journal motion[3]

In vector form the journal center velocity is

$$\vec{V}_J = \frac{d\vec{e}}{dt} = \dot{e}_X \, \vec{i} + \dot{e}_Y \, \vec{j} = \dot{e} \vec{u}_r + e\dot{\phi} \, \vec{u}_t \tag{3.27}$$

where $(\vec{i}, \vec{j})$ and $(\vec{u}_r, \vec{u}_t)$ are unit vectors in the *(X, Y)* and *(r, t)* coordinate systems, respectively, as shown in Fig. 3.5.

Define V_S as a velocity equaling the time rate of change of the vector $\vec{e}$ relative to a coordinate system that has angular velocity (½ Ω)·$\vec{k}$ with respect to the fixed coordinate system *(X, Y)*. V_S equals

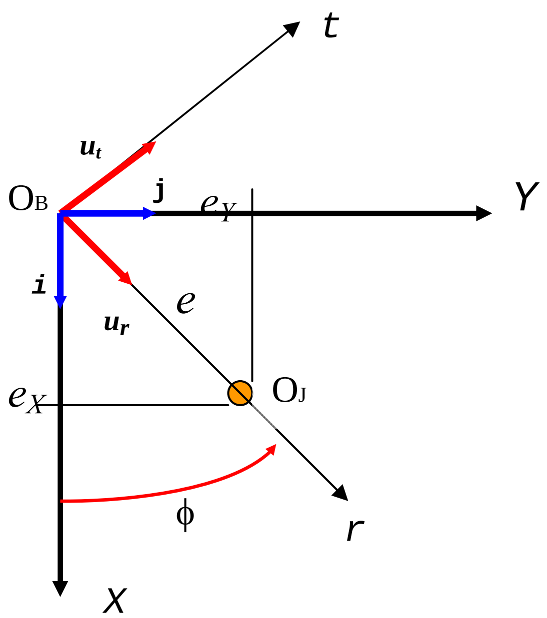

Fig 3.5 Unit vectors in (*r*,*t*) and (*X*,*Y*) CSs

[3] Follows the description given by D. Childs in "Turbomachinery Rotordynamics", Wiley Inter-Science Pub., 1993.

$$\vec{V}_S = \vec{V}_J - \frac{\Omega}{2}\cdot\vec{\boldsymbol{k}}\times\vec{\boldsymbol{e}} = \left(\dot{e}\,\vec{\boldsymbol{u}}_r + e\dot{\phi}\,\vec{\boldsymbol{u}}_t\right) - \frac{\Omega}{2}\vec{\boldsymbol{k}}\times e\,\vec{\boldsymbol{u}}_r = \dot{e}\,\vec{\boldsymbol{u}}_r + e\left(\dot{\phi} - \frac{\Omega}{2}\right)\vec{\boldsymbol{u}}_t \qquad (3.28a)$$

And since, $\vec{e} = e\cos\phi\;\vec{i} + e\sin\phi\;\vec{j}$, then also,

$$\vec{V}_S = \left\{\dot{e}_X + e_Y\frac{\Omega}{2}\right\}\vec{i} + \left\{\dot{e}_Y - e_X\frac{\Omega}{2}\right\}\vec{j} \qquad (3.28b)$$

Hence, any journal motion (translation and rotation) always appears as a state of pure squeezing in the defined rotating coordinate system, as shown in Figure 3.6. Thus, V_S is best known as a **pure squeeze velocity.**

From equation (3.28a),

$$\vec{V}_S = \dot{e}\,\vec{\boldsymbol{u}}_r + e\left(\dot{\phi} - \frac{\Omega}{2}\right)\vec{\boldsymbol{u}}_t = V_s\left\{\cos\alpha\;\vec{\boldsymbol{u}}_r - \sin\alpha\;\;\vec{\boldsymbol{u}}_t\right\} \qquad (3.29)$$

where V_S and α are the magnitude and lead angle of the squeeze velocity vector relative to the *(r, t)* coordinate system, i.e.

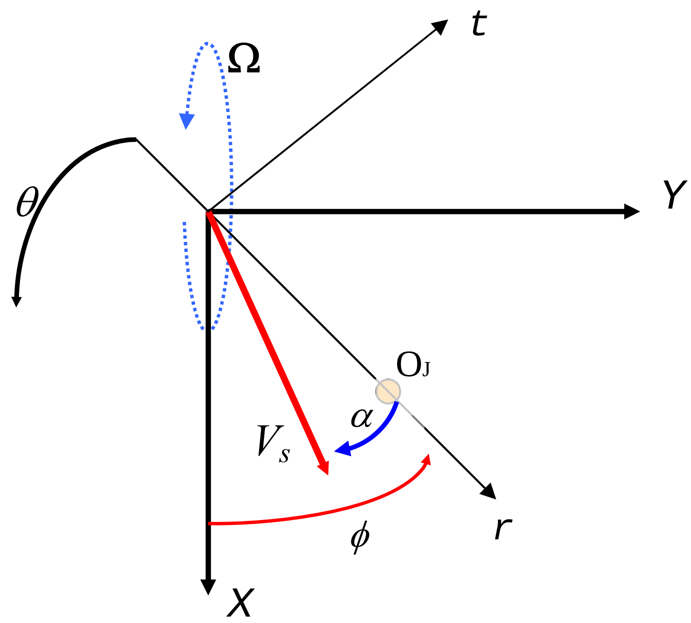

Figure 3.6 Pure squeeze film velocity V_s in rotating coordinate system

$$V_S \cos\alpha = \dot{e}; \quad -V_S \sin\alpha = e\left\{\dot{\phi} - \frac{\Omega}{2}\right\}$$

$$V_S^2 = \dot{e}^2 + e^2\left(\dot{\phi} - \frac{\Omega}{2}\right)^2; \quad \tan(\alpha) = -\frac{e\left\{\dot{\phi} - \frac{\Omega}{2}\right\}}{\dot{e}} \tag{3.30}$$

Also, from (3.28b)

$$\vec{V}_S = \left\{\dot{e}_X + e_Y \frac{\Omega}{2}\right\}\vec{i} + \left\{\dot{e}_Y - e_X \frac{\Omega}{2}\right\}\vec{j} = V_s \left\{\cos\eta\, \vec{i} - \sin\eta\, \vec{j}\right\} \tag{3.31}$$

with $V_S \cos\eta = \dot{e}_X + \frac{\Omega}{2} e_Y$; $-V_S \sin\eta = \dot{e}_Y - \frac{\Omega}{2} e_X$. And,

$$V_S^2 = \left\{\dot{e}_X + \left(\frac{\Omega}{2}\right)e_Y\right\}^2 + \left\{\dot{e}_Y - \left(\frac{\Omega}{2}\right)e_X\right\}^2; \ \tan\eta = -\frac{\left\{\dot{e}_Y - \frac{\Omega}{2} e_X\right\}}{\left\{\dot{e}_X + \frac{\Omega}{2} e_Y\right\}} \tag{3.32}$$

Thus ***Reynolds equation*** in terms of the pure squeeze velocity V_S is

$$\frac{1}{R^2}\frac{\partial}{\partial\theta}\left\{\frac{h^3}{12\mu}\frac{\partial P}{\partial\theta}\right\} + \frac{\partial}{\partial z}\left\{\frac{h^3}{12\mu}\frac{\partial P}{\partial z}\right\} = V_S \cos(\theta + \alpha) = \dot{e}\cos\theta + e\left\{\dot{\phi} - \frac{\Omega}{2}\right\}\sin\theta \tag{3.33}$$

in the *(r, t)* system with $h = C + e \cos\theta$; **or**

$$\frac{1}{R^2}\frac{\partial}{\partial\Theta}\left\{\frac{h^3}{12\mu}\frac{\partial P}{\partial\Theta}\right\} + \frac{\partial}{\partial z}\left\{\frac{h^3}{12\mu}\frac{\partial P}{\partial z}\right\} = V_S \cos(\Theta + \eta) = \left\{\dot{e}_X + e_Y \frac{\Omega}{2}\right\}\cos\Theta + \left\{\dot{e}_Y - \frac{\Omega}{2} e_X\right\}\sin\Theta \tag{3.34}$$

in the *(X, Y)* coordinate system with $h = C + e_X \cos\Theta + e_Y \sin\Theta$.