NOTES 4
STATIC LOAD PERFORMANCE OF PLAIN JOURNAL BEARINGS

Lecture 4 introduces the fundaments of journal bearing analysis. The long and short length bearing models are introduced. The pressure field in a short length bearing is obtained and examples follow for the pressure profiles generated under various operating conditions, namely journal rotation w/o whirling, pure circular centered whirl, and radial squeeze film motion. Next, the analysis focuses on determining the equilibrium journal eccentricity for an applied static load. The Sommerfeld number is a single parameter that permits quick bearing design since, for example, a large load or a low journal angular speed or a low viscosity produce the same operating (large) journal eccentricity. The journal eccentricity and attitude angle defining the static performance of the journal bearing are shown as functions of operating speed, lubricant viscosity, and applied load.

Nomenclature

C	Bearing radial clearance. $= R_B - R_J$ [m]
e	$\sqrt{e_X^2 + e_Y^2}$. Journal center eccentricity [m]
F_X, F_Y	Fluid film reaction forces along X,Y axes, $F = \sqrt{F_X^2 + F_Y^2} = \sqrt{F_r^2 + F_r^2}$ [N]
F_r, F_t	Fluid film reaction forces along r,t axes [N]
h	$C + e\cos\theta = C + e_X\cos\Theta + e_Y\sin\Theta$. Film thickness [m], $H=h/C$
$J_i^{j,k}$	Booker's journal bearing integral
L	Bearing axial length [m]
P	Hydrodynamic pressure [N/m^2]
P_{amb}	Ambient pressure = 0 (for simplicity of analysis) [N/m^2]
Q_z	Axial flow rate (per unit circumferential length) [m^2/s]
R_B , $R_J=R$	Bearing Radius ~ Journal Radius [m]
S	Sommerfeld number (bearing design parameter) [rev]
t	Time [s]
V_X, V_Y	$\dot{e}_X, \dot{e}_Y$. Components of journal velocity along X,Y axes[m/s]
V_r, V_t	$\dot{e}, e\dot{\phi}$. Components of journal velocity along r,t axes [m/s]
V_S	(pure) squeeze film velocity [m/s]
W	Applied (external) static load (along X axis) [N]
(X,Y) & (r,t)	Coordinate systems
α	Angle of squeeze velocity vector with axis r
ε	e/C. Journal eccentricity ratio
$\Theta=x/R$, y, z	Coordinate system on plane of bearing
ϕ	$\text{tang}\,\varphi = -\left(F_t / F_r \right)$. Journal attitude angle
ρ	Fluid density [kg/m^3]
μ	Fluid absolute viscosity [N.s/m^2]
σ	$\sigma = \frac{\mu\,\Omega L R}{4W}\left(\frac{L}{C}\right)^2$ Modified Sommerfeld number (short length bearing)
Ω	Journal angular speed (rad/s)

For incompressible and isoviscous fluids, and in terms of the pure squeeze velocity V_S, Reynolds equation for generation of the hydrodynamic pressure P is

$$\frac{1}{R^2}\frac{\partial}{\partial\theta}\left\{\frac{h^3}{12\mu}\frac{\partial P}{\partial\theta}\right\}+\frac{\partial}{\partial z}\left\{\frac{h^3}{12\mu}\frac{\partial P}{\partial z}\right\}=V_S\cos(\theta+\alpha)=\dot{e}\cos\theta+e\left\{\dot{\phi}-\frac{\Omega}{2}\right\}\sin\theta \tag{4.1}$$

where $h=(C+e\cos\theta)$ in the (r,t) system, and

$$V_S\cos\alpha=\dot{e};\quad -V_S\sin\alpha=e\left\{\dot{\phi}-\frac{\Omega}{2}\right\}$$
$$V_S^2=\dot{e}^2+e^2\left(\dot{\phi}-\frac{\Omega}{2}\right)^2;\quad \tan(\alpha)=-\frac{e\left\{\dot{\phi}-\frac{\Omega}{2}\right\}}{\dot{e}} \tag{4.2}$$

Recall that (e) is the journal center eccentricity, Ω is the journal rotational speed, $\dot{e}=V_r$ is the journal radial velocity, and $e\dot{\phi}=V_t$ is the journal tangential velocity.

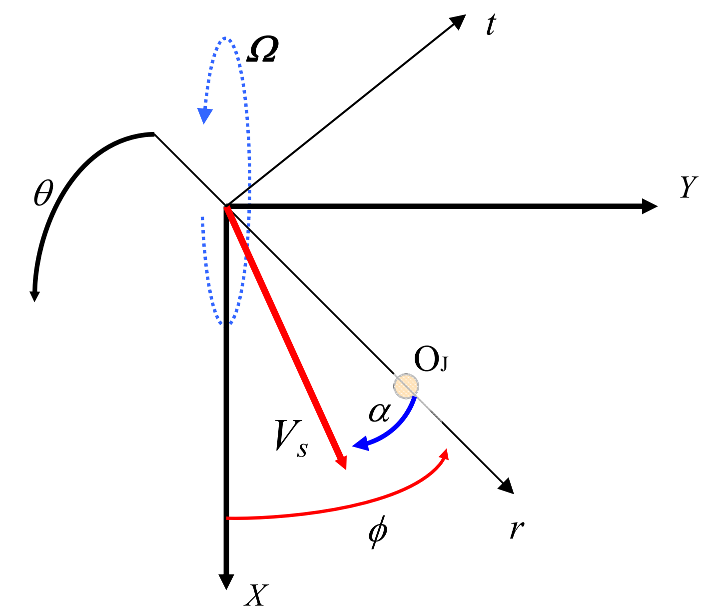

Figure 4.1. Pure squeeze film velocity in rotating coordinate system

The boundary conditions for the pressure field P in a plain cylindrical journal bearing are:
a) The hydrodynamic pressure and its gradients are continuous and single valued in the circumferential direction, i.e.

$$P(\theta,z,t)=P(\theta+2\pi,z,t) \tag{4.3}$$

b) At the bearing axial ends, the pressure is ambient (P_a)

$$P\left(\theta,\frac{L}{2},t\right)=P\left(\theta,-\frac{L}{2},t\right)=P_a \quad (4.4)$$

c) and as a constraint, the pressure is always equal or larger than the liquid cavitation pressure,

$$P(\theta,z,t)\geq P_{cav} \quad (4.5)$$

More physically sound and appropriate boundary conditions at the onset of the lubricant cavitation region are given later (**see Notes 6**). Boundary conditions at the film reformation boundary follow later.

An analytical solution to equation (4.1) for arbitrary geometry cylindrical bearings is unknown. Most frequently, numerical methods are employed to solve Reynolds equation and then to obtain the performance characteristics of bearing configurations of particular interest.

The bearing performance characteristics as a function of the applied external load are the journal eccentricity and attitude angle, bearing flow rate, drag power loss or friction coefficient, (temperature rise), and the dynamic force coefficients (stiffness and damping) at the operating rotational speed Ω.

There are analytical solutions to Reynolds equation applicable to two limiting geometries of journal bearings. These are known as the ***infinitely long*** and ***infinitely short*** length journal bearing models.

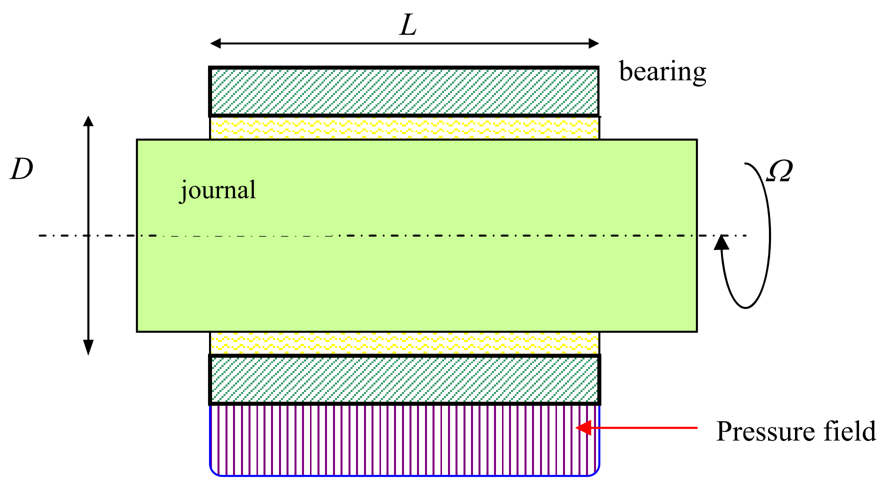

Figure 4.2. The long bearing model

In the **LONG BEARING MODEL**, the length of the bearing is regarded as very large, $L/D \rightarrow \infty$, and consequently the axial flow is effectively very small, $(\partial P/\partial z) = 0$. The pressure profile does not vary along the bearing length (except at its edges), as shown in Figure 4.2.

For the long bearing, Reynolds equation reduces to:

$$\frac{1}{R^2}\frac{\partial}{\partial\theta}\left\{\frac{h^3}{12\,\mu}\frac{\partial P}{\partial\theta}\right\} = V_S\cos(\theta+\alpha) \quad (4.6)$$

The *long* bearing model gives accurate results for journal bearings with slenderness ratios *(L/D) > 2*. Most modern bearings in high performance turbomachinery applications have a small *L/D* ratio, rarely exceeding one. Thus, the *infinitely long* journal bearing model is of limited current interest.

This is not the case for squeeze film dampers (SFDs), however. The long bearing model provides a very good approximation for tightly sealed dampers even for small *L/D* ratios. Another application is long bearings supporting ship propellers, for example.

The Short Length Journal Bearing Model

In this model, the length of the bearing is regarded as very small, $L/D \rightarrow 0$, and consequently the circumferential flow is effectively very small, i.e. $(\partial P/\partial\theta) \cong 0$. For this limiting bearing configuration, the Reynolds equation reduces to

$$\frac{\partial}{\partial z}\left\{\frac{h^3}{12\,\mu}\frac{\partial P}{\partial z}\right\} = V_S\cos(\theta+\alpha) \quad (4.7)$$

The *short length* bearing model provides (surprisingly) accurate results for plain cylindrical bearings of slenderness ratios $L/D \leq 0.50$ and for small to moderate values of the journal eccentricity, $e \leq 0.75\ C$. The *short length* bearing model is widely used for quick estimations of journal bearing static and dynamic force performance characteristics.

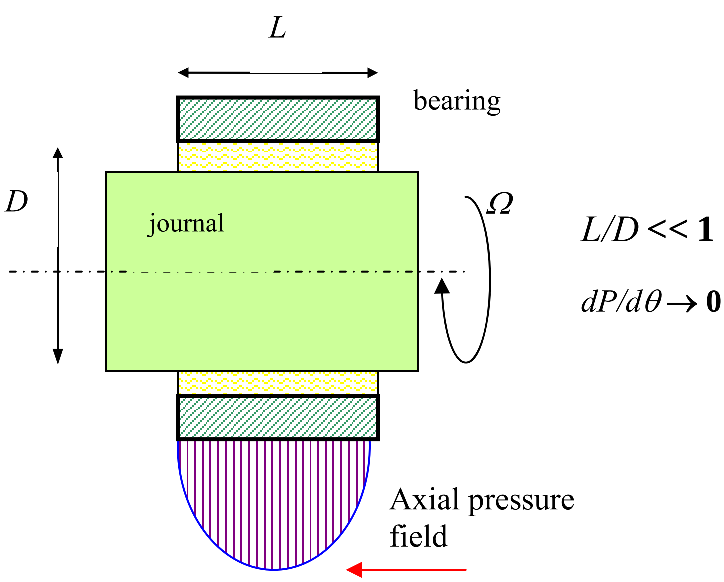

Fig. 4.3. The short length bearing

In the short length bearing model, the circumferential pressure gradient is taken as very small, i.e. $(\partial P/\partial\theta) \cong 0$, and hence, the mean fluid flow in the circumferential direction is

$$M_x = \rho\frac{U}{2}h = \rho\frac{\Omega}{2}R\,h \quad (4.8)$$

For film thickness, $h = C + e\cos\theta$, direct integration of Equation (4.7) is straightforward. For an aligned journal, the film h is not a function of the axial coordinate.

The integration renders the axial flow per unit circumference:

$$Q_z = -\frac{h^3}{12\mu}\frac{\partial P}{\partial z} = -V_S\, z\, \cos(\theta+\alpha) \tag{4.9}$$

Note that at the bearing middle plane, z=0, the flow rate is nil. Further integration of Equation (4.9) and applying the ambient pressure boundary condition at the bearing sides leads to the following parabolic pressure field,

$$P(\theta,z,t) - P_a = \frac{6\,\mu V_S}{C^3}\frac{\cos(\theta+\alpha)}{H^3}\left\{z^2 - \left(\frac{L}{2}\right)^2\right\}$$

$$or \tag{4.10}$$

$$P(\theta,z,t) - P_a = \frac{6\,\mu\left[\dot{e}\cos\theta + e\left(\dot{\varphi} - \frac{\Omega}{2}\right)\sin\theta\right]}{C^3 \qquad H^3}\left\{z^2 - \left(\frac{L}{2}\right)^2\right\}$$

with $$H = \frac{h}{C} = 1 + \varepsilon\, \cos\theta \tag{4.11}$$

as a dimensionless film thickness. $\varepsilon = e/C$ is a journal **eccentricity ratio**; $0 \le \varepsilon \le 1$, ε=0.0 means centered operation (typically a condition of no load support), and ε = 1.0 evidences solid contact of the journal with its bearing.

Note that the pressure field in Eqn. (4.10) is parabolic in the axial direction, with symmetry about the bearing middle plane z=0. Hence the maximum hydrodynamic pressure occurs at this middle plane and equals:

$$P(\theta,\ z=0,t) = P_a - \frac{6}{4}\frac{\mu V_S \cos(\theta+\alpha) L^2}{h^3} \tag{4.12}$$

In the *short length* bearing model, the circumferential coordinate (θ) and the time *(t)* are not variables but parameters. Consequently, imposing pressure boundary conditions in the circumferential direction is not possible. Fortunately, Eqn. (4.10) shows the hydrodynamic pressure is continuous and single valued in the circumferential direction θ

No lubricant cavitation will ever occur within the bearing if the magnitude of the side (exit or discharge) pressures P_a is well above the liquid cavitation pressure. However, if P_a is low, typically ambient at 14.7 psi (1 bar), it is almost certain that the bearing will show either liquid or gas cavitation, and under dynamic load conditions, air entrainment (ingestion and entrapment). The *cavitation model* in the short length bearing model sets P_a=0 and disregards any predicted negative pressures, it just equates them to *zero.* This "*chop*" procedure although not theoretically justified seems to grasp with some degree of accuracy the physics of the thin film flow.

Hence, if $P_a = 0$, and from Eqn. (4.12), the pressure field is positive, $P>0$, when

$$\cos(\theta+\alpha) < 0 \tag{4.13}$$

Thus, $P>0$ in the circumferential region delimited by

$$\frac{\pi}{2} \leq \theta + \alpha \leq \frac{3\pi}{2} \quad \rightarrow \frac{\pi}{2} - \alpha = \theta_1 \leq \theta \leq \theta_2 = \frac{3\pi}{2} - \alpha \tag{4.14}$$

That is, regardless of the type of journal motion, the region of positive pressure has an extent of π *(180°)* and it is centered (or aligned) with the pure squeeze vector (V_S). This important observation is the basis for the infamous ***π film cavitation model*** widely used in the engineering literature.

The study of a few simple cases for journal off centered motions ($e>0$) is of interest. Recall from Eqn. (4.2)

$$V_S \cos\alpha = \dot{e}; \quad -V_S \sin\alpha = e\left\{\dot{\phi} - \frac{\Omega}{2}\right\}$$
$$V_S = \sqrt{\dot{e}^2 + e^2\left(\dot{\phi} - \frac{\Omega}{2}\right)^2}; \quad \tan(\alpha) = -\frac{e\left\{\dot{\phi} - \frac{\Omega}{2}\right\}}{\dot{e}} \tag{4.2}$$

1) **Pure rotational (spinning) journal motion** $(\Omega > 0)$, $\dot{e}=0, \dot{\phi}=0$

Then $\alpha = ½\,\pi$, $V_S = ½\, e\Omega$

and $P > 0$ extends from $\theta_1 = 0$ to $\theta_2 = \pi$

2) **Pure radial squeeze motion** without journal rotation, $(\Omega = 0)$, $\dot{e}\neq 0, \dot{\phi}=0$

Then $\alpha = 0$, if $\dot{e} > 0$, $V_s = \dot{e} > 0$

and $P > 0$ from $\theta_1 = \frac{\pi}{2}$ to $\theta_2 = \frac{3\pi}{2}$;

while $\alpha = \pi$, if $\dot{e} < 0$, $V_s = |\dot{e}|$

and $P > 0$ extends from $\theta_1 = -\frac{\pi}{2}$ to $\theta_2 = \frac{\pi}{2}$;

3) **Pure tangential squeeze motion** (circular whirl) without journal rotation, $\dot{e} = \Omega = 0$

Then $\alpha = \frac{3\pi}{2}$ if $\dot{\phi} > 0$, $V_s = e\dot{\phi}$,

And $P > 0$ from $\theta_1 = \pi$ to $\theta_2 = 2\pi$.

Figure 4.4 depicts predicted centerline(at z=0) pressure profiles for a short length journal bearing with the following dimensions and operating characteristics. Length L=50 mm; clearance, C=100 μm, rotational speed at 3,000 rpm (Ω=314 rad/s), and lubricant viscosity μ=19 centipoise (19 10^{-3} N.s/m^2). The oil is an ISO VG 22 lubricant with a specific gravity of 0.86.

Note that the midplane pressure field P in Eqn (4.12) does NOT show the bearing radius or diameter in it.

The results shown correspond to three journal eccentricity ratios ε=e/C, equal to 25%, 50% and 75% of the radial clearance (C). The operating conditions are

a) **Journal spinning motion,** (Ω=314.2 rad/s), $\dot{e} = 0, \dot{\phi} = 0$,
Maximum V_S = ½ CΩ=0.016 m/s

b) **Journal circular whirl**, $\dot{\phi}$ =314.2 rad/s, Ω= $\dot{e}$ =0, Maximum $V_S = e\dot{\phi}$ =0.031 m/s

c) **Journal radial motion (pure squeeze),**
$\dot{e}$ =0.031 m/s, $\Omega = \dot{\phi}$ =0, $V_S = \dot{e}$ = 0.016 m/s

The graphs show the regions for positive pressure *(P>0)* for each particular case of journal motion. The hydrodynamic pressure increases rapidly as the journal eccentricity e increases and as the whirl orbit radius e increase. Note that circular whirl without journal spinning produces pressure magnitudes twice as large as for the case of steady journal rotation without journal whirling. Note also that the pressure regions in the two cases are shifted by 180°. It is noteworthy to show that pure squeeze film motions ($\dot{e}$ >0) generate pressures much larger than those due to the other two type of motions, since the squeeze film velocity (V_S) is larger.

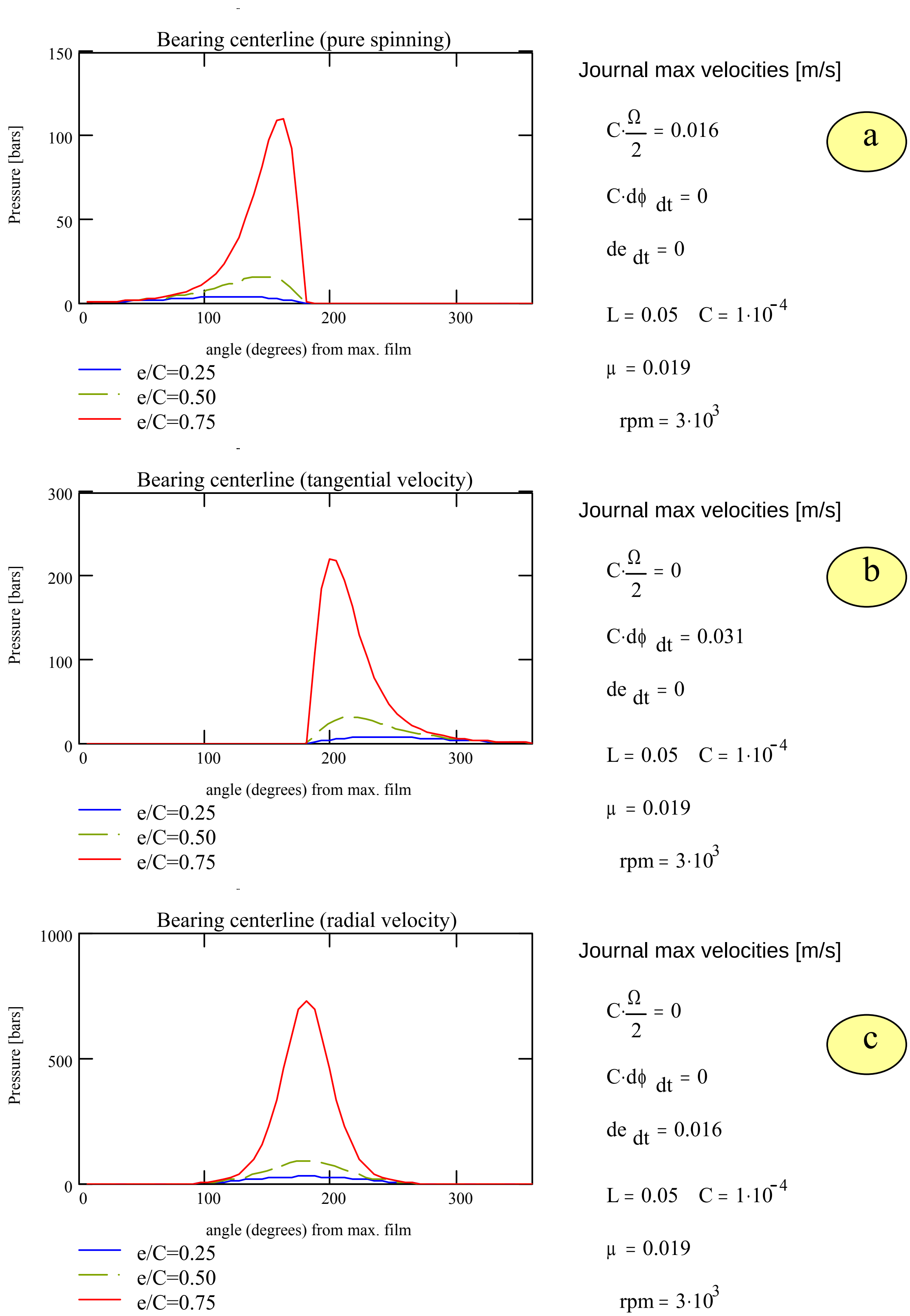

Figure 4.4. Centerline pressure profile for short length bearing. Journal motions are (a) pure spinning, (b) circular whirl, (c) pure radial squeeze motions

Fluid film forces for the short length journal bearing

Integration of the pressure field acting on the journal surface produces a fluid film force with radial and tangential components *(F_r, F_t)* defined by

$$\begin{bmatrix} F_r \\ F_t \end{bmatrix} = 2\int_0^{L/2}\int_{\theta_1}^{\theta_2} P(\theta,z,t)\begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix} R\, d\theta\, dz \tag{4.15}$$

where $\theta_1 = \dfrac{\pi}{2} - \alpha$; $\theta_2 = \dfrac{3\pi}{2} - \alpha$. Note that if the lubricant does not cavitate, then $\theta_1=0$, $\theta_2=2\pi$. Substitution of the pressure field, Eqn. (4.10) into the expression above, gives, for $P_a = 0$,

$$\begin{bmatrix} F_r \\ F_t \end{bmatrix} = \frac{12\,\mu V_S\, R}{C^3}\int_{\theta_1}^{\theta_2} \frac{\cos(\theta+\alpha)}{H^3}\begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix} d\theta \int_0^{L/2}\left(z^2 - \frac{L^2}{4}\right) dz \tag{4.16a}$$

$$\begin{bmatrix} F_r \\ F_t \end{bmatrix} = -\frac{\mu\, V_S\, R\, L^3}{C^3}\int_{\theta_1}^{\theta_2} \frac{\cos(\theta+\alpha)}{H^3}\begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix} d\theta \tag{4.16b}$$

where $H=(1+\varepsilon\ \cos\theta)$, $V_S \cos\alpha = \dot{e}$; $-V_S \sin\alpha = e\left(\dot{\phi} - \dfrac{\Omega}{2}\right)$. The bearing force is typically a reaction type, i.e. it balances an applied external force or load that acts on the journal. This external load is a fraction of the rotor weight. Recall that rotors are supported on a pair of bearings, in general.

Booker (1965)[1] defines the following bearing integrals

$$J_i^{jk} = \int_{\theta_1}^{\theta_2} \frac{\sin^k(\theta)\cos^j(\theta)}{H^i} d\theta \tag{4.17}$$

and provides recursive analytical formulas for their evaluation. Using the definition above, the fluid film reaction forces become

$$\begin{bmatrix} F_r \\ F_t \end{bmatrix} = -\frac{\mu V_S\, R\, L^3}{C^3}\begin{bmatrix} J_3^{02} & J_3^{11} \\ J_3^{11} & J_3^{20}\end{bmatrix}\begin{pmatrix}\cos\alpha \\ -\sin\alpha\end{pmatrix} \tag{4.18a}$$

or in terms of the journal velocity components

1 *Booker, J. F., 1965, "A Table of the Journal Bearing Integral," ASME Journal of Basic Engineering, pp. 533-535.*

$$\begin{bmatrix} F_r \\ F_t \end{bmatrix} = -\frac{\mu\, R\, L^3}{C^3}\begin{bmatrix} J_3^{02} & J_3^{11} \\ J_3^{11} & J_3^{20} \end{bmatrix}\begin{pmatrix} \dot{e} \\ e\left\{\dot{\phi} - \frac{\Omega}{2}\right\} \end{pmatrix} \tag{4.18b}$$

Note that the fluid film forces are proportional to the journal center translation velocities $\left(V_r = \dot{e},\, V_t = e\dot{\phi}\right)$ as well as the journal rotational speed Ω. The reaction forces depend linearly on the lubricant viscosity (μ), the bearing radius *R;* growing very rapidly with the ratio $(L/C)^3$.

The linear transformation between the *(r, t)* and *(X,Y)* coordinate systems gives a relationship for the evaluation of the fluid film forces (F_X, F_Y) in the Cartesian coordinate system

$$\begin{bmatrix} F_X \\ FY \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}\begin{bmatrix} F_r \\ F_t \end{bmatrix} \tag{4.19}$$

Equilibrium condition for a short length journal bearing

Hydrodynamic journal bearings are designed and built to support a static load *W,* hereafter aligned with the *X* axis (for convenience). At the equilibrium condition, denoted by journal center eccentric displacement (*e)* with attitude angle *(ϕ)*, the fluid film journal bearing generates a reaction force balancing the applied external load *W* at the rated rotational speed (Ω*)*. The equations of ***static equilibrium*** are

$$\begin{aligned} W + F_X = 0 \Rightarrow -W = F_X &= F_r\cos\phi - F_t\sin\phi \\ F_Y = 0 \quad \Rightarrow \quad 0 = F_Y &= F_r\sin\phi + F_t\cos\phi \end{aligned} \tag{4.20}$$

For static equilibrium, $\dot{e}=0, \dot{\phi}=0$, so then $\alpha = \frac{1}{2}\pi$, $V_S = \frac{1}{2}e\Omega$; and $\theta_1 = 0$ to $\theta_2 = \pi$. From Eqn. (4.18b), the radial and tangential film forces reduce to

$$\begin{bmatrix} F_r \\ F_t \end{bmatrix} = +\frac{\mu R L^3}{C^3}\begin{vmatrix} J_3^{11} \\ J_3^{20} \end{vmatrix}\frac{e\Omega}{2} \tag{4.21}$$

Using the formulas from Booker's Journal Bearing Integral Tables, and after some algebraic manipulations,

$$J_3^{11} = \left(\frac{1}{\varepsilon}\right)\left\{-J_3^{10} + J_2^{10}\right\} = \frac{-2\cdot\varepsilon}{\left(1-\varepsilon^2\right)^2} \tag{4.22a}$$

$$J_3^{20} = \left(\frac{1}{\varepsilon^2}\right)\left\{-\left(1-\varepsilon^2\right)J_3^{00} + 2J_2^{00} - J_1^{00}\right\} = \frac{\pi}{2\left(1-\varepsilon^2\right)^{3/2}} \tag{4.22b}$$

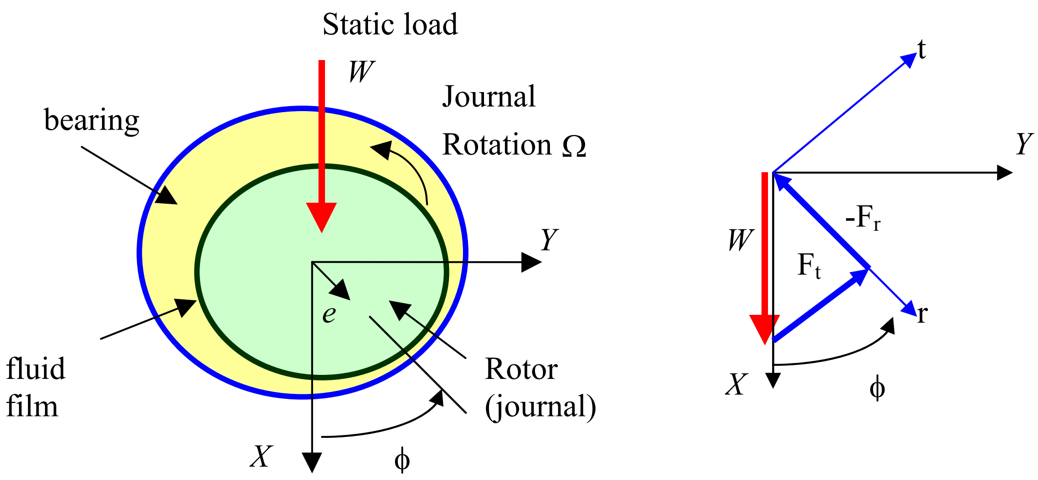

Figure 4.5. Force equilibrium for static load *W*

Hence, the radial and tangential fluid film forces for the short length bearing are

$$F_r = -\frac{\mu\, R L^3\, \Omega}{C^2}\frac{\varepsilon^2}{\left(1-\varepsilon^2\right)^2}$$
$$F_t = +\frac{\mu\, R L^3\, \Omega}{C^2}\frac{\pi\cdot\varepsilon}{4\left(1-\varepsilon^2\right)^{3/2}} \tag{4.23}$$

Note that the bearing reaction force is proportional to the journal speed Ω, the lubricant viscosity μ, and the bearing radius R. The forces are strong nonlinear functions of the bearing length L and the radial clearance C, *i.e.* $\sim L^3/C^2$.

The fluid film bearing reaction force balances the applied external load W. Thus,

$$W = \left(F_r^2 + F_t^2\right)^{1/2} = \mu\,\Omega\, R\, L\left(\frac{L}{C}\right)^2 \frac{\varepsilon}{4}\frac{\sqrt{16\varepsilon^2 + \pi^2\left(1-\varepsilon^2\right)}}{\left(1-\varepsilon^2\right)^2} \tag{4.24}$$

and the journal attitude angle ϕ (angle between the load and the journal eccentricity vector ***e***) is

$$\text{tang}\,\phi = -\frac{F_t}{F_r} = \frac{\pi\sqrt{\left(1-\varepsilon^2\right)}}{4\,\varepsilon} \tag{4.25}$$

Note that as the journal eccentricity $\varepsilon \to 0,\ \phi \to \pi/2$, while as $\varepsilon \to 1,\ \phi \to 0$.

In the design of hydrodynamic journal bearings, the bearing static performance characteristics relate to a **single** dimensionless parameter known as the **Sommerfeld Number (*S*)**

$$S = \frac{\mu\, N\, L\, D}{W}\left(\frac{R}{C}\right)^2 \tag{4.26}$$

where $N = (\Omega/2\pi)$ is the rotational speed in revolutions/s. In Eqn. (4.26), the ratio *(W/LD),* load divided by the bearing projected area, is known as the **bearing specific load** or specific pressure. Various journal bearing configurations are rated by their peak specific pressure, for example, up to 300 psi for tilting pad bearings and ~1,000 psi for a cylindrical journal bearing. The specific pressure is a relatively good indicator of the maximum (peak) film pressure within the thin film bearing.

In short length journal bearings, a **modified Sommerfeld number** σ**)** is more physically adequate. The parameter is defined as

$$\sigma = \pi\, S\,(L/D)^2 = \frac{\mu\, \Omega\, L\, R}{4W}\left(\frac{L}{C}\right)^2 \tag{4.27}$$

Substitution of Eqn. (4.24) into Eqn. (4.27) relates σ to the equilibrium journal eccentricity ε, i.e.

$$\frac{\mu\, \Omega\, L\, R}{4W}\left(\frac{L}{C}\right)^2 = \sigma = \frac{\left(1-\varepsilon^2\right)^2}{\varepsilon\sqrt{\left\{16\varepsilon^2 + \pi^2\left(1-\varepsilon^2\right)\right\}}} \tag{4.28}$$

At a rated operating condition, σ is a known magnitude or value since the bearing geometry *(R, L, C)*, rotational speed (Ω), fluid viscosity (μ) and applied load *(W)* are specified. Then Eqn. (4.28) provides a relationship to determine (iteratively) the equilibrium journal eccentricity ratio $\varepsilon=(e/C)$ required to generate the hydrodynamic pressure field that produces a fluid film reaction force equal and opposite to the applied load *W*.

The following figures depict the modified Sommerfeld number σ and attitude angle ϕ versus the journal eccentricity $\varepsilon=(e/C)$. Large Sommerfeld numbers σ, denoted by either a small load *W*, a high speed Ω rotor, or a lubricant of large lubricant viscosity μ, determine small operating journal eccentricities or nearly a centered operation, i.e. $\varepsilon \rightarrow 0$ and $\phi \rightarrow \pi/2$ (90°). That is, the journal eccentricity vector ***e*** is nearly orthogonal or perpendicular to the applied load vector ***W***.

Small Sommerfeld numbers σ (large load W, low speed Ω, or low lubricant viscosity μ) determine large journal eccentricities, i.e. $\varepsilon \rightarrow 1.0$, $\phi \rightarrow 0$ (0°). Note that the journal eccentricity vector $\boldsymbol{e}$ is nearly parallel to or aligned with the applied load $\boldsymbol{W}$.

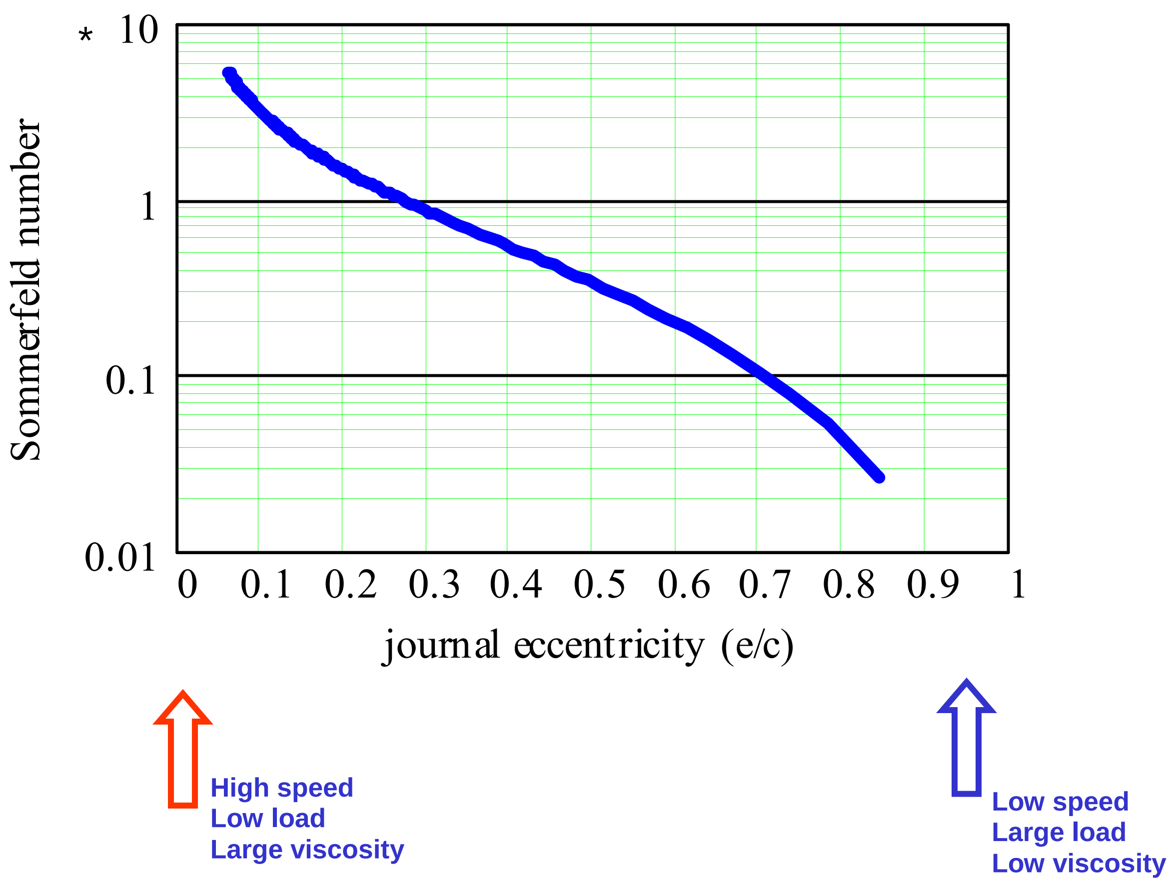

Figure 4.6. Modified Sommerfeld number σ versus journal eccentricity ε. Short length journal bearing

$$\sigma = \frac{\mu\,\Omega\,L\,R}{4\,W}\left(\frac{L}{C}\right)^2$$

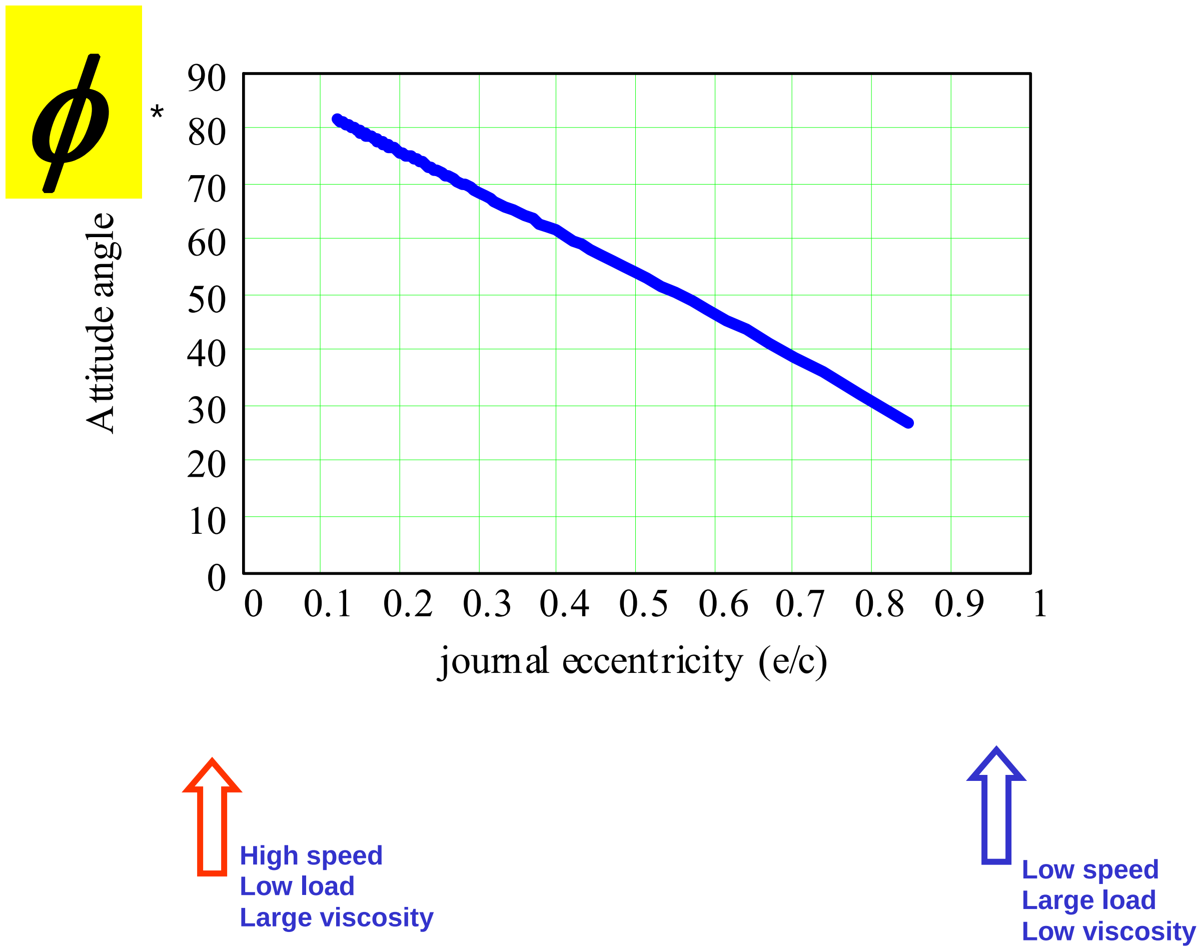

Figure 4.7. Equilibrium attitude angle φ versus journal eccentricity ε. Short length journal bearing

Figure 4.8 shows the locus of journal center displacement or journal eccentricity within the bearing clearance for various operating conditions. The journal eccentricity *(e)* approaches the clearance (*C)* for operation with either large loads, or low rotor speeds, or light lubricant viscosity, and it is aligned with the load vector. For either small loads, or high shaft speeds, or large fluid viscosity (large Sommerfeld number), the journal travels towards the bearing center and its position is orthogonal to the applied load. This peculiar behavior is the source of **rotordynamic instability** as will be shown shortly.

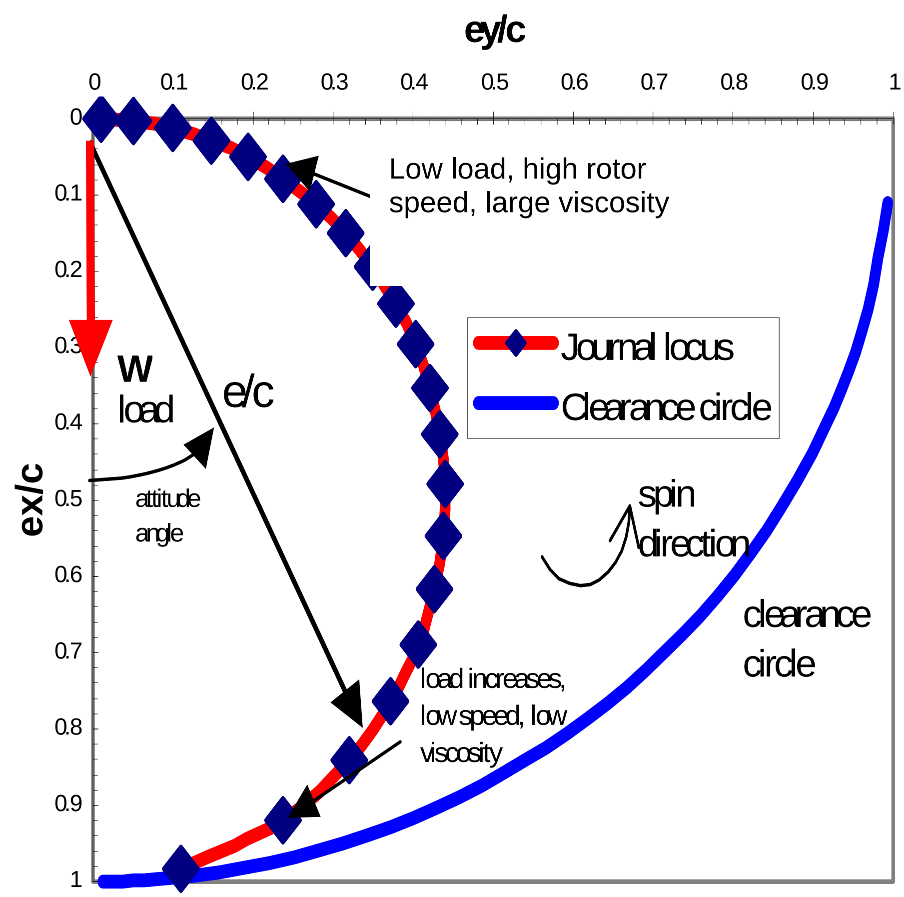

Figure 4.8. Locus of journal center for short length bearing

Use the accompanying MATHCAD program to determine the journal eccentricity for a journal bearing configuration and specified load, fluid properties and speed operating conditions. The program implements a simple thermal model and also predicts the exit film temperature, bearing drag power and flow rate.

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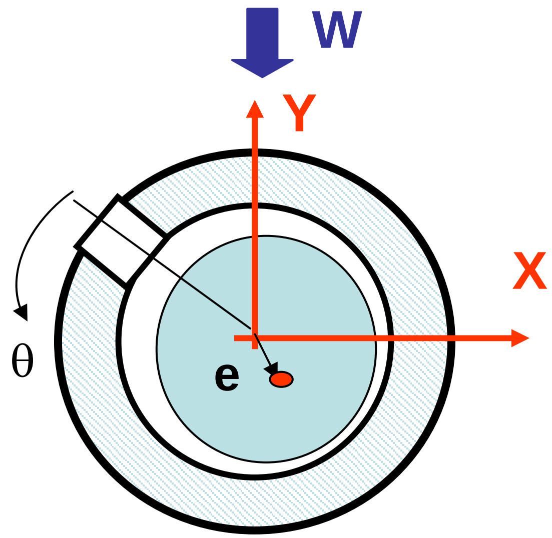


All journal bearings have a supply port (axial groove or hole) to feed cold lubricant into the film separating the rotating journal and its the bearing. The lubricant gets hotter (increase in temperature) as it flows down thru the hydrodynamic wedge. Some hot lubricant leaves the bearing through its sides. The spinning journal draws some hot lubricant around towards the inlet port where it mixes with the cold stream of lubricant. The temperature of the lubricant at the inlet of the film land is higher than the oil supply temperature.

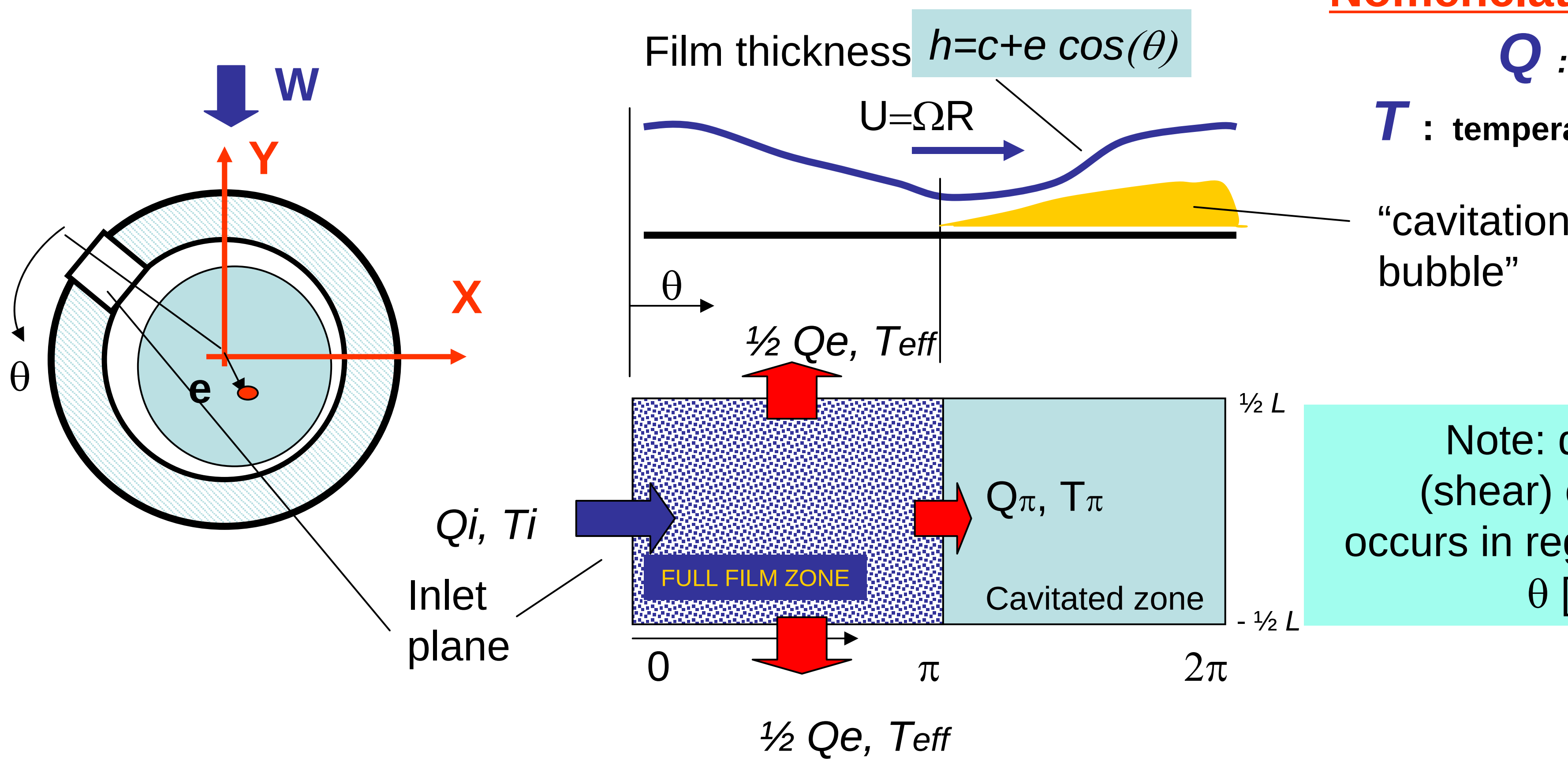
Appendix to Notes 4 : Static load performance of journal bearing

Simple lumped parameter thermal analysis for predicting the exit temperature and effective viscosity in a short length journal bearing

Nomenclature

Q **: flow**

T **: temperature**

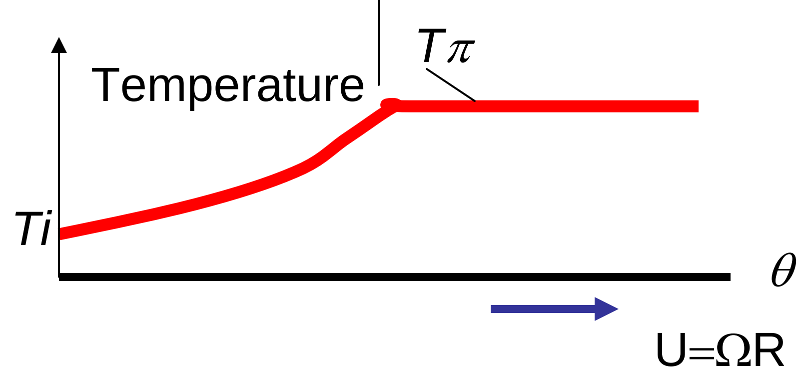


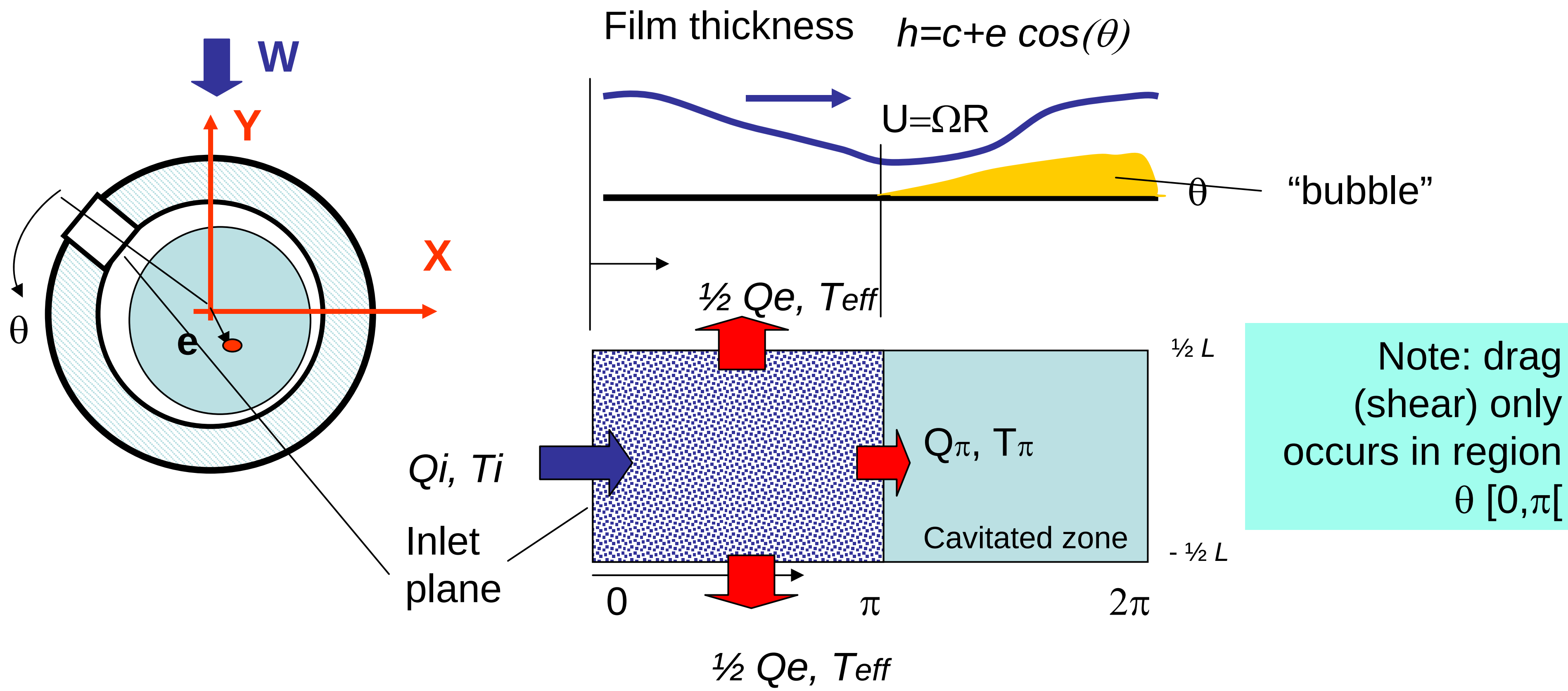
Note: drag (shear) only occurs in region θ [0,π[

Let:

$$T_{eff} = \tfrac{1}{2}\,(T_i + T_\pi)$$

As a weighed average





GLOBAL FLOW CONTINUITY EQN

$$Qi = Qe + Q_\pi$$

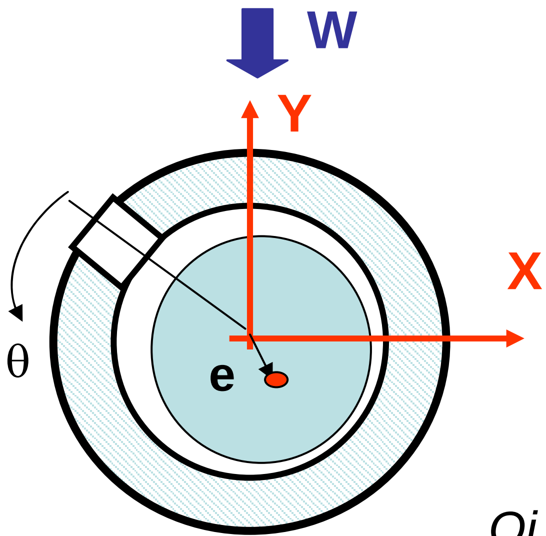
1D ENERGY TRANSPORT EQUATION

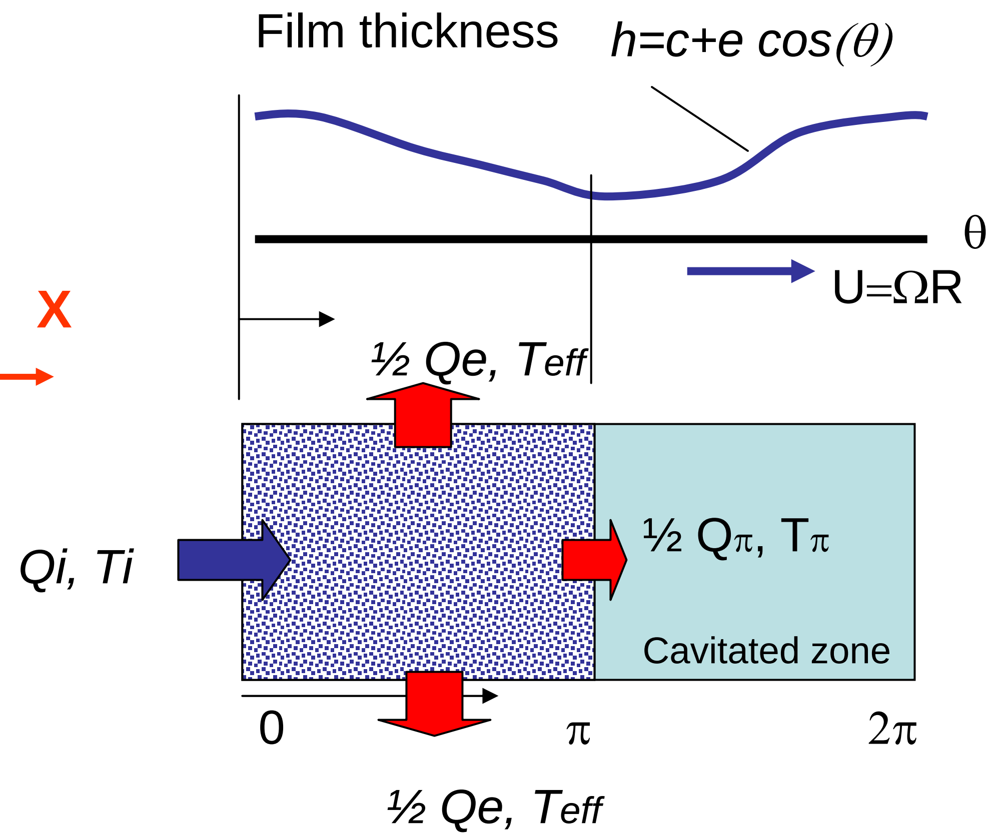
$$Eo - Ei = \kappa \, Power$$

κ is a fraction of the mechanical power is converted into heat and carried away by lubricant flow

$$Ei = \rho \, C_p \, Qi \, Ti$$

$$Eo = \rho \, C_p \, Qe \, T_{eff} + \rho \, C_p \, Q_\pi \, T_\pi$$





Note: drag (shear) only occurs in region θ [0,π[

Working with both eqns. leads to

$$\frac{\kappa \, Power}{\rho \, C_p} = (Q_i - \tfrac{1}{2} Q_e)\,(T_\pi - T_i)$$

Nomenclature

Q : flow

T : temperature

λ : thermal mixing coefficient

= 0.80 (TYP)

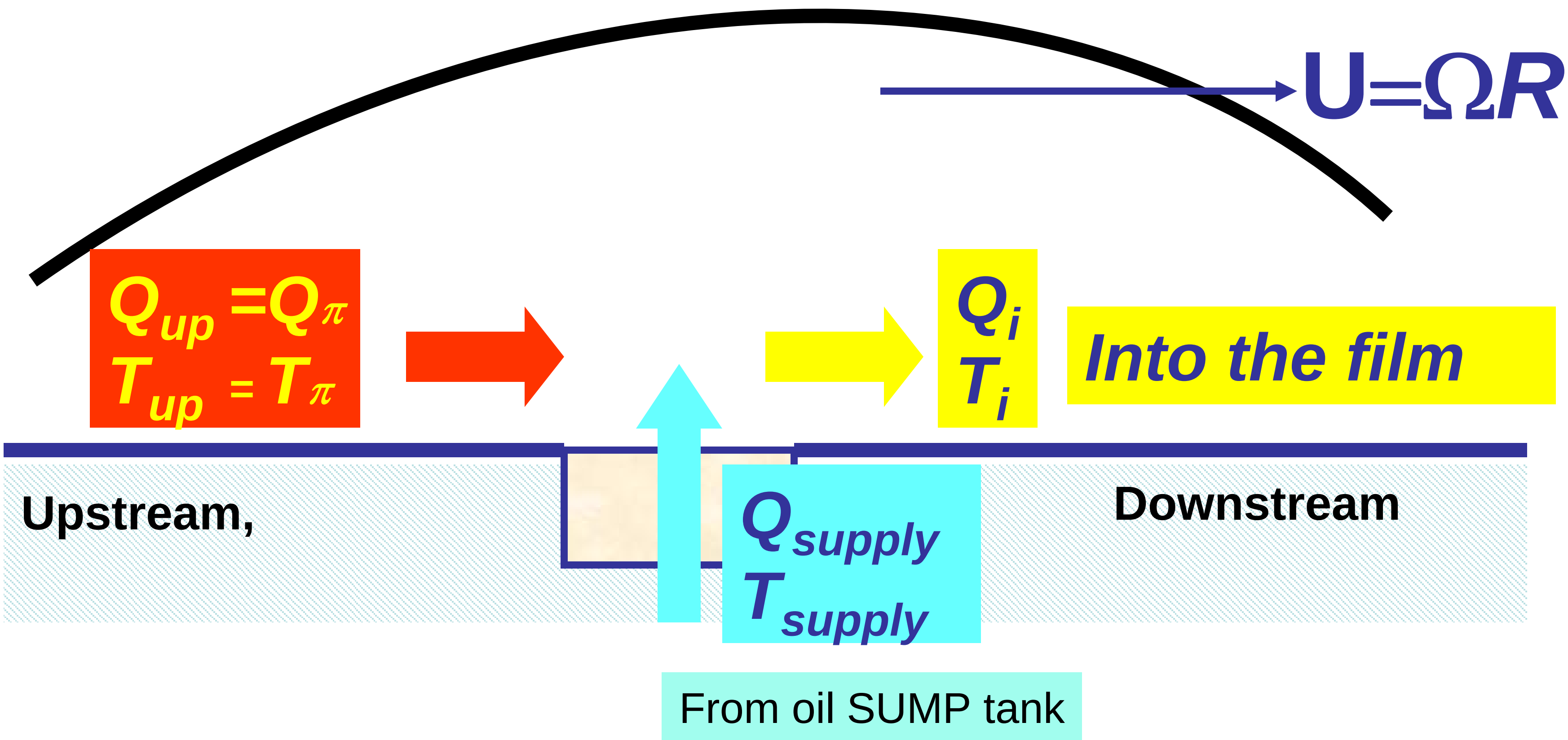
Heat carry-over coefficient

Flow balance

$$Q_i = Q_{supply} + \lambda Q_{up}$$

Energy balance

$$Q_i T_i = Q_{supply} T_{supply} + \lambda Q_{up} T_{up}$$



Thermal mixing at bearing inlet “groove”

Nomenclature

Q : flow

T : temperature

E : Energy

Flow & energy balance in feed groove

$$Q_i = Q_{supply} + \lambda\, Q_\pi$$

$$Q_i\, T_i = Q_{supply}\, T_{supply} + \lambda\, Q_\pi\, T_\pi$$

Flow & energy balance in film land

$$Qi = Qe + Q\pi$$

$$Eo - Ei = \kappa\, Power$$

$$Ei = \rho\, Cp\, Qi\, Ti$$

$$Eo = \rho\, Cp\, Qe\, Teff + \rho\, Cp\, Q\pi\, T\pi$$

$$Q_i \sim \frac{U}{2} h_{\theta=0} = L \cdot (c+e) \cdot \frac{R\Omega}{2};$$

$$Q_\pi \sim \frac{U}{2} h_{\theta=p} = L \cdot (c-e) \cdot \frac{R\Omega}{2}$$

$$Q_e = Q_i - Q_\pi = e\, L\; R\Omega$$

$$Power \sim Torque\; \Omega = \Omega \int_0^L \int_0^\pi \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} R^2\, d\theta\, dz; \;\; \text{recall}\;\; u = U\, {}^{y}\!/_{h}$$

$$\sim \Omega \int_0^L \int_0^\pi \mu \left(\frac{\Omega R}{h} \right)_{y=0} R^2\, d\theta\, dz;$$

$$Power \;\sim\; \mu_{eff} \frac{\Omega^2 R^3 L}{c} \int_0^\pi \frac{1}{1+\varepsilon \cos\theta} d\theta = \mu_{eff} \frac{\Omega^2 R^3 L}{c} \frac{\pi}{\sqrt{1-\varepsilon^2}}$$

Thermal energy transport – short journal bearing