# Notes 9.

# Turbulence and Fluid Inertia Effects in Fluid Film Bearings

Fluid inertia and flow turbulence affect the performance of (modern or state of the art) fluid film bearing and seals. Prevailing trend towards light and compact turbomachinery operating at higher speeds and the use of process liquids (of low viscosity) and gases determine turbulent flow operating conditions with dominance of fluid inertia effects. Current applications of importance include operation with water and lubricant mixtures, liquid metals in the nuclear industry, and cryogenic fluids in space turbopumps. Large clearance elements such as squeeze film dampers and annular seals, for example, show large fluid inertia effects in the form of large magnitude added mass coefficients. Notes 9 continues the analysis of turbulent flow bearings to derive the equations for bulk-flow transport in regions without strong recirculation zones; to specify equations for the modeling of turbulent flow in short length journal bearings with details on the conditions for which fluid inertia effects are lesser than those from turbulent flow (flow turbulence, modeled as a diffusive component, dominates over (advection) fluid inertia); to provide a simple model for evaluation of short length journal bearing performance with flow turbulence. The Notes also give MATHCAD codes for prediction of pressure profiles and journal bearing forced performance, static and dynamic.

## Analysis

The continuity and momentum equations for the motion of an incompressible fluid in a thin film flow geometry are, in dimensionless form (see Notes 1):

$$\frac{\partial \tilde{u}_j}{\partial x_j} = 0; \quad Re_S \frac{\partial \tilde{u}_i}{\partial \tau} + Re_* \, u_j \frac{\partial \tilde{u}_i}{\partial x_j} = -\frac{\partial \tilde{p}}{\partial x_i} + \frac{\partial}{\partial y}\tilde{\tau}_{iy}; \quad {\scriptstyle i=x,z;\; j=x,y,z} \tag{1}$$

where

$$\tilde{x} = \frac{x}{R}; \quad \tilde{z} = \frac{z}{R}; \quad \tilde{y} = \frac{y}{c}; \; \tau = \omega t;$$

$$\tilde{u} = \frac{u}{U_*}; \quad \tilde{w} = \frac{w}{U_*}; \quad \tilde{v} = \frac{v}{\left(U_* \frac{c}{R}\right)}; \; \tilde{p} = \frac{p}{\left(\frac{\mu U_* R}{c^2}\right)}, \; \tilde{\tau}_{iy} = \frac{\tau_{iy}}{\left(\frac{\mu U_*}{c}\right)} \tag{2}$$

with $U_*$ as a characteristic surface speed and $\omega$ as a typical frequency for squeeze film motions. ($U_* = \Omega R$ in journal bearings). The importance of fluid inertia effects relates to the magnitude of the **modified Reynolds number** ($Re_*$) for shear flow and the **squeeze film Reynolds number** ($Re_s$) for unsteady or periodic whirl motions. These Reynolds numbers are:

$$Re_* = \frac{\rho U_* c}{\mu}\frac{c}{R}; \quad Re_s = \frac{\rho \omega c^2}{\mu} >>> 1 \tag{3}$$

Most thin film flows in lubrication configurations handling mineral oils show (modified) Reynolds numbers of small magnitude ($Re_* <<1$). However, high speed (or high frequency) bearing and seal applications with large clearances and low viscosity fluids bring a dominance of fluid inertia effects in the thin film lands and at the inlet and discharge sections of annular regions and bearing pads.

Consider now the **turbulent flow** of an incompressible and isoviscous fluid in a thin film region. The equations of motion including fluid inertia effects for the (time-averaged) mean velocities $\left(\overline{u},\overline{v},\overline{w}\right)$ and pressure $\overline{p}$ are (Szeri, 1981):

$$\frac{\partial \overline{u}}{\partial x}+\frac{\partial \overline{v}}{\partial y}+\frac{\partial \overline{w}}{\partial z}=0 \tag{4}$$

$$-\frac{\partial \overline{p}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}=-\frac{\partial \overline{p}}{\partial x}+\frac{\partial}{\partial y}\left\{\mu\frac{\partial \overline{u}}{\partial y}-\overline{\rho u'v'}\right\}=\rho\left\{\frac{\partial \overline{u}}{\partial t}+\overline{u}\frac{\partial \overline{u}}{\partial x}+\overline{v}\frac{\partial \overline{u}}{\partial y}+\overline{w}\frac{\partial \overline{u}}{\partial z}\right\} \tag{5}$$

$$-\frac{\partial \overline{p}}{\partial z}+\frac{\partial \tau_{zy}}{\partial y}=-\frac{\partial \overline{p}}{\partial z}+\frac{\partial}{\partial y}\left\{\mu\frac{\partial \overline{w}}{\partial y}-\overline{\rho w'v'}\right\}=\rho\left\{\frac{\partial \overline{w}}{\partial t}+\overline{u}\frac{\partial \overline{w}}{\partial x}+\overline{v}\frac{\partial \overline{w}}{\partial y}+\overline{w}\frac{\partial \overline{w}}{\partial z}\right\} \tag{6}$$

where ($\tau_{xy}$, $\tau_{zy}$) are the shear stress components combining the viscous and turbulent flow effects. The boundary conditions for the velocities at the bottom and top surfaces of the flow region with film thickness $h$ are:

$$\begin{aligned} &at\ y=0, && \overline{u}=\overline{v}=\overline{w}=0 \\ &at\ y=h, && \overline{u}=\Omega R=U,\ \ \overline{w}=0,\ \ \overline{v}=\frac{\partial h}{\partial t}+U\frac{\partial h}{\partial x} \end{aligned} \tag{7}$$

**Bulk-flow velocities** are the mean or average fluid velocities across the film thickness[1], i.e.

$$V_x=\frac{1}{h}\int_0^h \overline{u}\,dy;\quad V_z=\frac{1}{h}\int_0^h \overline{w}\,dy \tag{8}$$

Integration of the governing equations (4-6) across the film thickness ($y$ direction) leads to

$$\frac{\partial}{\partial x}\left(hV_x\right)+\frac{\partial}{\partial z}\left(hV_z\right)+\frac{\partial h}{\partial t}=0 \tag{9}$$

[1] As shown in Notes 8, for turbulent flows these bulk-flow velocities represent both time and space (across the film) averages.

$$-h\frac{\partial P}{\partial x}+\Delta\tau_{xy}=\rho\left\{\frac{\partial(V_x h)}{\partial t}+\frac{\partial I_{xx}}{\partial x}+\frac{\partial I_{xz}}{\partial z}\right\} \quad (10)$$

$$-h\frac{\partial P}{\partial z}+\Delta\tau_{xz}=\rho\left\{\frac{\partial(V_z h)}{\partial t}+\frac{\partial I_{xz}}{\partial x}+\frac{\partial I_{zz}}{\partial z}\right\} \quad (11)$$

Above $P=\bar{p}$ for simplicity in notation.

Eqs. (10) and (11) establish a balance among the pressure forces, the wall shear stress differences and the fluid inertia (temporal and advective) forces exerted on the fluid.

The wall shear stress differences $(\Delta\tau_{xy}, \Delta\tau_{zy})$ are regarded as similar to those for the inertialess fluid flow condition[2]:

$$\Delta\tau_{xy}=\tau_{xy}|_0^h=-\frac{\mu}{h}\left\{\kappa_x V_x-\kappa_J\frac{U}{2}\right\};\ \Delta\tau_{zy}=\tau_{zy}|_0^h=-\frac{\mu}{h}\kappa_z V_z \quad (12)$$

$\kappa_x$, $\kappa_y$, $\kappa_J$ are turbulence shear parameters which depend on the structure of the turbulent flow, i.e., the bulk-flow velocities $(V_x, V_z)$ and the surface velocity $U$, and the condition (rough or smooth) of the bearing and journal surfaces. Note that $\kappa_x=\kappa_y=\kappa_J=12$ denotes the laminar flow condition.

The **momentum-flux integrals** $(I_{ij})$ in Eqns. (10) and (11) are

$$I_{xx}=\int_0^h \bar{u}^2 dy,\quad I_{zz}=\int_0^h \bar{w}^2 dy,\quad I_{xz}=I_{zx}=\int_0^h \bar{u}\cdot\bar{w}\,dy \quad (13)$$

which are functions of the bulk-flow velocities. However, the exact velocity profiles across the film thickness are needed to evaluate these flux-integrals. This implies a-priori knowledge of the velocity fields via solution of the whole system of fluid flow equations of motion. This is clearly a major undertaking. In practice, fluid inertia is thought not to affect greatly the shape of the fluid velocities, and hence the inertialess (laminar flow) fluid velocity profiles are used for evaluation of the integrals above[3].

The laminar flow inertialess fluid velocity components are the superposition of pressure (Poiseuille) and shear (Couette) flow driven components,

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[2] The rationale for this is based more on experience and simplicity rather than solid theoretical foundation As a note aside, it can be shown from a first-order regular perturbation scheme on the Reynolds number that the wall shear stress differences also have inertial components

[3] The assumption does not work well for velocity fields which have strongly curved streamlines; for example, a labyrinth seal shows a strong recirculation zone in its cavities.

$$u = \frac{1}{2\mu}\frac{\partial P}{\partial x}\left\{y^2 - yh\right\} + \frac{y}{h}U; \qquad w = -\frac{1}{2\mu}\frac{\partial P}{\partial z}\left\{y^2 - yh\right\} \tag{14}$$

The pressure gradients are, in terms of the mean (bulk)-flow components,

$$\frac{\partial P}{\partial x} = -\frac{12\mu}{h^2}\left\{V_x - \frac{U}{2}\right\}; \quad \frac{\partial P}{\partial z} = -\frac{12\mu}{h^2}V_z \tag{15}$$

Hence, the laminar flow velocity components are expressed in the form

$$w = -6V_z\left(\eta^2 - \eta\right); \qquad u = -6\left(V_x - \frac{U}{2}\right)\left(\eta^2 - \eta\right) + \eta\, U \tag{16}$$

where $\eta = y/h$. Note that at $\eta = ½$, i.e. at half the film thickness,

$$w_{\left(y=\frac{h}{2}\right)} = \frac{3}{2}V_z; \quad u_{\left(y=\frac{h}{2}\right)} = \frac{3}{2}V_x - \frac{U}{4}$$

i.e, the velocities at the middle of the film thickness are greater than (or at least equal to) the bulk-flow velocities. Substitution of Eq. (16) into the momentum-flux integrals, Eqs. (13), gives the following:

$$\begin{gathered} I_{xx} = \alpha\, V_x^2 \cdot h + \beta\, U^2 \cdot h - \gamma\, U \cdot V_x \cdot h \\ I_{zz} = \alpha V_z^2 \cdot h \\ I_{xz} = \alpha\, V_x V_z \cdot h - \gamma\, V_z \cdot \left(\frac{U}{2}\right) \cdot h \end{gathered} \tag{17}$$

where $$\alpha = \frac{12}{10} = 1.2; \ \beta = \frac{2}{15} = 0.1333; \ \gamma = \frac{1}{5} = 0.20; \tag{18}$$

In **fully developed** turbulent flows, the velocity profiles do not have the typical parabolic profiles, being almost uniform across the film thickness and with steep gradients close to the wall boundaries, thus forming thin boundary layers.

Under turbulent flow conditions (without strong recirculation regions), the coefficients for the momentum flux integrals are

$\alpha = 1.0; \quad \beta = \gamma = 0$ as $Re \rightarrow \infty$ (ideal or inviscid fluid).

Then $$I_{xx} = V_x^2 \cdot h; \quad I_{zz} = V_z^2 \cdot h; \quad I_{xz} = V_x V_z \cdot h \tag{19}$$

Simon and Frene (1992), from extensive numerical solutions, obtain values of the $\alpha$, $\beta$, $\gamma$ coefficients for turbulent flows dominated by shear (*Couette*) flow effects as:

$$\alpha = 1.2 + \gamma; \quad \beta = 1.624 \cdot R_{e_h}^{-0.458}; \quad \gamma = 0.0932 \cdot R_{e_h}^{0.065} \tag{20}$$

in the range $Re_h = (\rho U h/\mu) \in [3\ 10^3 - 10^5\ ]$. Note again that the form of the flux-integrals stated by Eq. (19) is not adequate for flows with strong local recirculations across the film thickness.

## Bulk-flow equations for fully-developed turbulent thin film flows

The generally accepted bulk-flow equations for fully developed turbulent flows at high Reynolds numbers ($Re >>> 1$) are, with coefficients $\alpha = 1$, $\beta = \gamma = 0$:

$$\frac{\partial}{\partial x}(hV_x) + \frac{\partial}{\partial z}(hV_z) + \frac{\partial h}{\partial t} = 0 \tag{21}$$

$$-h\frac{\partial P}{\partial x} = \frac{\mu}{h}\left(\kappa_x V_x - \kappa_J \frac{U}{2}\right) + \rho h\left\{\frac{\partial V_x}{\partial t} + \frac{\partial V_x^2}{\partial x} + \frac{\partial V_x V_z}{\partial z}\right\} \tag{22}$$

$$-h\frac{\partial P}{\partial z} = \frac{\mu}{h}\kappa_z V_z + \rho h\left\{\frac{\partial V_z}{\partial t} + \frac{\partial V_x V_z}{\partial x} + \frac{\partial V_z^2}{\partial z}\right\} \tag{23}$$

These equations are strictly valid for flows without local recirculation zones. That is, these equations are of limited applicability in labyrinth seals or deep grooved bearings, for example.

For **compressible fluids** undergoing isothermal processes, the momentum equations above are still applicable. The equation of mass conservation for the bulk-flow velocities takes the form

$$\frac{\partial}{\partial x}(\rho h V_x) + \frac{\partial}{\partial z}(\rho h V_z) + \frac{\partial}{\partial t}(\rho\ h) = 0 \tag{24}$$

where the density of the fluid is a function of the pressure, $\rho = \rho(P)$, i.e. a **barotropic** liquid or and ideal gas.

The general equations of motion for turbulent flow fluid film bearings with fluid inertia effects are highly non-linear and not amenable of direct integration except in limiting geometry cases (i.e., the long and short length journal bearing models). Recall that the equations derived assume the velocity profiles not to deviate much from their inertialess values.

## Turbulent and inertial fluid flow in short length journal bearings

In journal bearings with small *L/D* ratios and for small to moderate journal eccentricity operation ($e/C$ < 0.7), the circumferential pressure gradient is small compared to the axial pressure gradient, i.e.

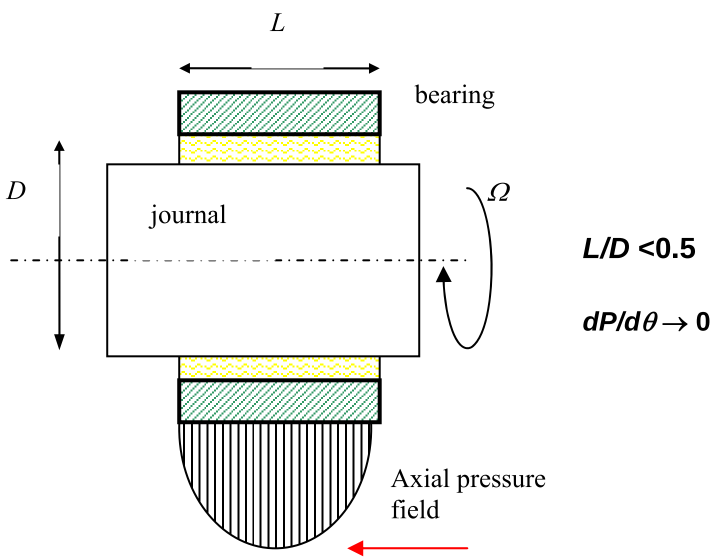
$$\frac{\partial P}{R\,\partial\theta} <<< \frac{\partial P}{\partial z} \tag{25}$$

and the bulk-flow velocity in the circumferential direction is just

$$V_x \simeq \frac{U}{2} = \frac{\Omega R}{2}$$

That is, the short length bearing condition regards the circumferential flow as fully developed.

Thus, the following analysis cannot be applied to predict performance of annular pressure seals or journal bearings with an inlet-swirl development.




**The short length bearing model**

The equation of momentum transport in the circumferential direction is neglected for the analysis of turbulent flow in short-length journal bearings ($L/D$<0.50). The remaining equations, flow continuity and axial momentum transport, are

$$\frac{U}{2}\frac{\partial h}{\partial x} + \frac{\partial}{\partial z}\left(hV_z\right) + \frac{\partial h}{\partial t} = 0 \tag{27}$$

$$-h\frac{\partial P}{\partial z} = \frac{\mu}{h}\kappa_z V_z + \rho\left\{\frac{\partial(hV_z)}{\partial t} + \frac{U}{2}(\alpha-\gamma)\frac{\partial(hV_z)}{\partial x} + \alpha\frac{\partial(hV_z^2)}{\partial z}\right\} \tag{28}$$

Above, the momentum-flux integrals, Eqs. (17), are approximately equal to

$$I_{xz} = (\alpha-\gamma)V_z\frac{U}{2}h; \qquad I_{zz} = \alpha V_z^2\, h \tag{29}$$

The turbulent shear stress factor ($\kappa_z$) is a function of the bulk-flow components ($V_z$, ½$U$). Recall that $\kappa_z$ = 12 for laminar flows. The first term on the right hand side of equation (28) denotes the viscous shear stresses, the second term shows the temporal inertia effect, and the last two (non-linear) terms denote fluid inertia advection effects.

The film thickness $h=h(\theta ,t)$ in an aligned journal bearing. For this condition, the continuity Eq. (27) is easily integrated along the axial direction to render the axial flow rate ($Q_z$) per unit circumferential length, i.e.,

$$Q_z = h V_z = -z\left\{\frac{U}{2}\frac{\partial h}{\partial x}+\frac{\partial h}{\partial t}\right\} = -z\cdot G(x,t) \tag{30}$$

Note that $Q_z$ is not affected directly by fluid inertia. Substitute $V_z$, Eq. (30), into the axial momentum transport Eq. (28) to obtain

$$h\frac{\partial P}{\partial z} = \frac{\mu}{h^2}\kappa_z G\, z + \rho\, z\left\{\frac{\partial G}{\partial t}+\frac{U}{2}(\alpha-\gamma)\frac{\partial G}{\partial x}-2\alpha\frac{G^2}{h}\right\} \tag{31}$$

$$\text{where } G=\left\{\frac{U}{2}\frac{\partial h}{\partial x}+\frac{\partial h}{\partial t}\right\};\quad \frac{\partial G}{\partial x}=\left\{\frac{U}{2}\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial t\partial x}\right\};\ \frac{\partial G}{\partial t}=\left\{\frac{U}{2}\frac{\partial^2 h}{\partial x\partial t}+\frac{\partial^2 h}{\partial t^2}\right\} \tag{32}$$

Eq. (32) can be integrated along the axial direction if the turbulent shear stress factor $\kappa_z$ is regarded as constant; i.e., $\neq f(Vz)$. This condition happens for operation at small to moderate journal eccentricities where shear *Couette flow* effects dominate. Thus, let $\kappa_z = f\left(\frac{\rho U h}{\mu}\right)$, or assume a (yet unknown) average turbulent shear stress parameter in the axial direction and denoted by (~).

These considerations lead to an expression for the hydrodynamic pressure as

$$P(x,z,t)=\frac{1}{2}\left(z^2-\frac{L^2}{4}\right)\left\{\frac{\mu\,\tilde{\kappa}_z}{h^3}G\right\}+\frac{1}{2}\left(z^2-\frac{L^2}{4}\right)\frac{\rho}{h}\left\{\frac{\partial G}{\partial t}+\frac{U}{2}(\alpha-\gamma)\frac{\partial G}{\partial x}-2\alpha\frac{G^2}{h}\right\} \tag{33}$$

with ambient pressure, $P$=0 (gauge), at the bearing sides $z=\{-½\ L,\ +½\ L\}$. Note that there are two distinct pressure fields, one due solely to turbulent-viscous effects (first term on the right hand side), and the other solely due to fluid inertia effects. Note that both pressure fields have the characteristic parabolic shape in the axial direction.

Define the average pressure ($P_{ave}$) in the axial direction,

$$P_{ave}(x,t)=\frac{1}{L}\int_{-L/2}^{L/2}P(x,z,t)\,dz=-\frac{L^2}{12}\left[\frac{\mu\,\tilde{\kappa}_z}{h^3}G+\frac{\rho}{h}\left\{\frac{\partial G}{\partial t}+\frac{U}{2}(\alpha-\gamma)\frac{\partial G}{\partial x}-2\alpha\frac{G^2}{h}\right\}\right] \tag{34}$$

Before proceeding further, define also dimensionless pressure and film thickness as:

$$\overline{P}_{ave} = P_{ave} \cdot 12 \cdot \frac{c^2}{\{\mu \Omega_* L^2\}}$$

$$H = \frac{h}{c} = 1 + \varepsilon \cos\theta = 1 + \varepsilon_X \cos\Theta + \varepsilon_Y \sin\Theta \tag{35}$$

where $\theta = \frac{x}{R}$, $\tau = \omega t$, and $\Omega_*$ is a characteristic rotational speed. *($\varepsilon_X$, $\varepsilon_Y$)* are the components of the journal center eccentricity ratio *($\varepsilon$)*.

In journal bearings, $\Omega_* = \Omega$ is the rotational speed of the journal; while in squeeze film dampers, $\Omega_* = \omega$ is a characteristic whirl frequency. Then, equation (34) is rewritten as

$$\overline{P}_{ave}(\theta,\tau) = -\frac{\widetilde{\kappa}_z}{H^3} g + \frac{1}{H}\left\{ \frac{\rho\omega c^2}{\mu}\frac{\partial g}{\partial \tau} + \frac{\rho\Omega c^2}{2\mu}(\alpha-\gamma)\frac{\partial g}{\partial \theta} - 2\alpha \frac{\rho\Omega_* c^2}{\mu}\frac{g^2}{H} \right\} \tag{36a}$$

or
$$\overline{P}_{ave}(\theta,\tau) = -\frac{\widetilde{\kappa}_z}{H^3} g + \frac{1}{H}\left\{ \mathrm{Re}_S \frac{\partial g}{\partial \tau} + \frac{\mathrm{Re}_*}{2}(\alpha-\gamma)\frac{\partial g}{\partial \theta} - 2\alpha\, \mathrm{Re}_{\Omega*} \frac{g^2}{H} \right\} \tag{36b}$$

where
$$g = \left\{ \frac{\Lambda}{2}\frac{\partial H}{\partial \theta} + \sigma \frac{\partial H}{\partial \tau} \right\} \text{ with } \Lambda = \frac{\Omega}{\Omega_*} \text{ and } \sigma = \frac{\omega}{\Omega_*} \tag{37}$$

$\mathrm{Re}_S = \frac{\rho\omega c^2}{\mu}$, $\mathrm{Re}_* = \frac{\rho\Omega c^2}{\mu}$ and $\mathrm{Re}_{\Omega*} = \frac{\rho\Omega_* c^2}{\mu}$ are known as the squeeze film Reynolds number; the modified shear flow Reynolds number, and the characteristic speed Reynolds number, respectively.

The influence of inertia effects on the pressure field can be determined from the relative magnitude of the Reynolds numbers defined above. Fluid inertia effects are important when

$$\frac{\text{Reynolds number}}{\widetilde{\kappa}_z} \geq 1 \tag{38}$$

Note that this condition establishes a more accurate ratio than that given in Eq. (3).

## Turbulent-inertial fluid flow model for hydrodynamic journal bearings

In journal bearings, the characteristic speed of motion corresponds to the journal rotational speed ($\Omega_* = \Omega$), i.e.

$$\Omega_* = \Omega = \omega, \text{ and hence } \mathrm{Re}_S = \mathrm{Re}_* = R_{\Omega*} = \frac{\rho\,\Omega R c}{\mu}\frac{c}{R} = \mathrm{Re}\frac{c}{R}$$

and Eq. (36) for the axially average pressure field is rewritten as

$$\overline{P}_{ave}(\theta,\tau) = -\frac{\tilde{\kappa}_z}{H^3}\left\{ g + \frac{H^2\,\mathrm{Re}_*}{\tilde{\kappa}_z}\left( \frac{\partial g}{\partial \tau} + \frac{1}{2}(\alpha-\gamma)\frac{\partial g}{\partial \theta} - 2\alpha\frac{g^2}{H} \right) \right\} \tag{39}$$

where $g = \left\{ \frac{1}{2}\frac{\partial H}{\partial \theta} + \frac{\partial H}{\partial \tau} \right\}$ since $\Lambda = \sigma = 1$. All the $g$ terms and their derivatives above are of the same order. Thus fluid inertia effects are of important in the generation of hydrodynamic pressures if

$$\frac{\mathrm{Re}_*}{\tilde{\kappa}_z} \geq 1. \;\text{ or }\; \mathrm{Re} = \frac{\rho\Omega R c}{\mu} \geq \tilde{\kappa}_z\left(\frac{R}{c}\right) \tag{40}$$

In laminar flows this condition occurs for $Re_* > 12$ since $\tilde{\kappa}_z = 12$. For example, consider two journal bearings with $c/R$ ratios equal to 0.005 and 0.001, respectively. Fluid inertia effects are important for Reynolds numbers (Re) larger than

$Re > 12\,(R/c) = 12/0.005 = 2{,}400$ and $Re > 12/0.001 = 12{,}000$, respectively.

However, the large magnitude of the Reynolds numbers implies that the flows in each of these bearings is likely to be fully turbulent!

In **turbulent flows** ($Re > 2{,}000$) with dominance of Couette (shear driven) flow effects, i.e. , operating with small to moderate journal eccentricities $(h \simeq c)$, the turbulent shear stress parameter ($\kappa_z$) based on Hirs' bulk-flow model is

$$\tilde{\kappa}_z = n\left(\frac{\mathrm{Re}}{2}\right)^{m+1} \quad ; \quad n = 0.066,\; m = -0.25 \tag{41}$$

Combining Eqs. (40) and (41) shows that fluid inertia effects are important in turbulent flow journal bearings if

$$\mathrm{Re} \geq \left(0.03932\frac{R}{c}\right)^4 \tag{42}$$

The following table shows the shear flow Reynolds numbers (Re) required to bring fluid inertia effects to be on par (same order of magnitude) with the turbulent flow-viscous dissipation effect.

| $R/c$ | $Re$ | $Re_*$ |
|---|---|---|
| 150 | 1,210 | 8.06 |
| 200 | 3,824 | 19.12 |
| 500 | 149,394 | 298.80 |
| 1000 | 2,390,308 | 2390.30 |

The results show that fluid inertia effects are important in fluid film bearings with small *(R/c)* ratios (large clearances , for example). **That is, in most practical applications of**

**hydrodynamic journal bearings, fluid inertia effects are negligible compared to those derived from viscous and turbulent flow effects since *c/R*~1/1000.**

Hashimoto et al. (1988) present a complete analysis on the effects of fluid inertia and flow turbulence on short length journal bearings. The authors also discuss the effects of fluid inertia and turbulence on the onset speed of instability for a rigid rotor supported on short length journal bearings. In general, the results show that turbulence tends to deteriorate the stability characteristics of the rotor-bearing system. However, fluid inertia tends to ameliorate this condition.

The accompanying Attachment (MATHCAD file) depicts dimensionless pressure profiles and fluid film reaction forces for a laminar flow short journal bearing including fluid inertia effects. The graphical results demonstrate the paramount influence of fluid inertia on the performance of superlaminar journal bearings, squeeze film dampers undergoing circular centered orbits and pure radial squeeze motions.

Use the program to observe the effects of fluid inertia in the pressure field (shifting and increase/decrease) and the resulting forces. In addition, derive conclusions from the effects of the Gumbel cavitation condition on the fluid film forces.

**Question to ponder: Does the physical modeling of liquid cavitation in superlaminar thin film flows must be revised?**

## (Inertialess) Turbulent flow model for short length journal bearings

Fluid inertia effects are not that important in a hydrodynamic journal bearing application (based on the rationale given above). Hence, the axially averaged film pressure reduces to:

$$-P_{ave}(\theta,\tau)=\frac{\tilde{\kappa}_z}{H^3}g=\frac{\tilde{\kappa}_z}{H^3}\left\{\frac{1}{2}\frac{\partial H}{\partial\theta}+\frac{\partial H}{\partial\tau}\right\} \tag{43}$$

where $\kappa_Z$=12 for laminar flow and $\tilde{\kappa}_z=\tilde{\kappa}_{z(\mathrm{Re}_h)}=\tilde{\kappa}_{z\left(\frac{\rho\Omega Rh}{\mu}\right)}$ for turbulent flows. Hirs' and Constaninescu's turbulence flow models give

$$\tilde{\kappa}_z=n\left(\frac{\mathrm{Re}_h}{2}\right)^{m+1} \text{ and } \tilde{\kappa}_z=12+0.296\left(\kappa^2\,\mathrm{Re}_h\right)^{0.65},\ \kappa\text{=0.3 or 0.4} \tag{44}$$

respectively. Note that the variation of the shear stress parameter in the axial direction is not accounted for. Furthermore, $\tilde{\kappa}_z$ is a function that varies on the circumferential direction, i.e., $f(\theta)$.

Integration of the hydrodynamic pressure on the journal surface gives the radial and tangential components *(*$F_r$*, Ft)* of the bearing reaction force,

$$\begin{pmatrix} F_r \\ F_t \end{pmatrix} = 2\int_0^{\frac{L}{2}}\int_0^{2\pi} P\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} R d\theta\, dz = RL\int_0^{2\pi} P_{ave}\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} d\theta \tag{45}$$

The integral is taken over the range of circumferential coordinates *($\theta$)* where the pressure is above the cavitation pressure *($P_{cav}$* = 0 for simplicity*)*. With

$$H = 1+\varepsilon\cos\theta,\ \frac{\partial H}{\partial\theta} = -\varepsilon\sin\theta;\ \frac{\partial H}{\partial\tau} = \frac{d\varepsilon}{d\tau}\cos\theta + \varepsilon\frac{d\phi}{d\tau}\sin\theta$$

Then $g = \left\{\frac{1}{2}\frac{\partial H}{\partial\theta} + \frac{\partial H}{\partial\tau}\right\} = g = \varepsilon\left(\phi' - \frac{1}{2}\right)\sin\theta + \varepsilon'\cos\theta$, where $\left[(') = \frac{d}{d\tau}\right]$. Substitution of Eq. (43) into (45) gives the journal bearing reaction forces as

$$\begin{pmatrix} F_r \\ F_t \end{pmatrix} = -\frac{\mu\Omega R L^3}{c^2}\int \frac{\kappa_z}{12H^3}\left[\varepsilon\left(\phi' - \frac{1}{2}\right)\sin\theta + \varepsilon'\cos\theta\right]\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} d\theta \tag{46a}$$

or

$$\begin{pmatrix} F_r \\ F_t \end{pmatrix} = -\frac{\mu\Omega R L^3}{c^2}\begin{bmatrix} J_{t3}^{02} & J_{t3}^{11} \\ J_{t3}^{11} & J_{t3}^{20}\end{bmatrix}\begin{bmatrix}\varepsilon' \\ \varepsilon\left(\phi' - \frac{1}{2}\right)\end{bmatrix} \tag{46b}$$

and Booker's integrals are redefined for turbulent flows as

$$J_{ti}^{kj} = \int \frac{\tilde{\kappa}_z}{12}\frac{\sin^k(\theta)\cdot\cos^j(\theta)}{H^i}\, d\theta \tag{47}$$

In laminar flow $\tilde{\kappa}_z = 12$, and the integrals above reduce to the ones in Notes 4. The integrals in Eq. (47) could be evaluated numerically. However, for the sake of simplicity use a shear parameter $\tilde{\kappa}_z$ based on the nominal circumferential flow Reynolds number,

$$\tilde{\tilde{\kappa}}_z = f(\text{Re}) = f\left(\frac{\rho\Omega R c}{\mu}\right) \tag{48}$$

Then, the fluid film force components are, in approximate form:

$$\begin{pmatrix} F_r \\ F_t \end{pmatrix} = -\frac{\mu\Omega R L^3}{c^2}\frac{\tilde{\tilde{\kappa}}_z}{12}\begin{bmatrix} J_3^{02} & J_3^{11} \\ J_3^{11} & J_3^{20}\end{bmatrix}\begin{bmatrix}\varepsilon' \\ \varepsilon\left(\phi' - \frac{1}{2}\right)\end{bmatrix} \tag{49}$$

The simplification introduced intends to show the major effect of flow turbulence to the fluid film reaction forces of a short length journal bearing. That is, turbulence increases the fluid film forces by a ratio equal to ($\tilde{\kappa}_z/12$) as compared to the forces determined in a laminar flow bearing.

At the steady-state or equilibrium condition, the journal bearing supports a static load ($W$). At this condition $d\varepsilon/d\tau = d\phi/d\tau = 0$, and the fluid film bearing static forces are

$$\begin{bmatrix} F_r \\ F_t \end{bmatrix} = -\frac{\mu R L^3}{C^3} \begin{bmatrix} J_3^{11} \\ J_3^{20} \end{bmatrix} \frac{e\Omega}{2} \frac{\tilde{\tilde{\kappa}}_z}{12} \tag{50}$$

For π-film cavitation extent, the components of the journal bearing reaction force are

$$F_r = -\frac{\mu R L^3 \Omega}{C^2} \frac{\varepsilon^2}{\left(1-\varepsilon^2\right)^2}\left(\frac{\tilde{\tilde{\kappa}}_z}{12}\right); \quad F_t = +\frac{\mu R L^3 \Omega}{C^2} \frac{\pi \cdot \varepsilon}{4\left(1-\varepsilon^2\right)^{\frac{3}{2}}}\left(\frac{\tilde{\tilde{\kappa}}_z}{12}\right) \tag{51}$$

These forces balance (or support) the external load W. Thus,

$$W = \left(F_r^2 + F_t^2\right)^{\frac{1}{2}} = \mu \cdot \Omega R \cdot L \left(\frac{L}{C}\right)^2 \left(\frac{\tilde{\tilde{\kappa}}_z}{12}\right) \frac{\varepsilon}{4} \frac{\sqrt{16\varepsilon^2 + \pi^2\left(1-\varepsilon^2\right)}}{\left(1-\varepsilon^2\right)^2} \tag{52}$$

and the journal center attitude angle $\phi$ is

$$\tan(\phi) = -\frac{F_t}{F_r} = \frac{\pi\sqrt{\left(1-\varepsilon^2\right)}}{4 \cdot \varepsilon} \tag{53}$$

which is identical to the result derived for the short length bearing operating in the laminar flow regime. Recall the modified Sommerfeld number $(\sigma)$,

$$\sigma = \pi S \left(\frac{L}{D}\right)^2 = \frac{\mu \Omega L R}{4W}\left(\frac{L}{C}\right)^2 \tag{54}$$

Substitution of Eq. (52) into (54) relates the Sommerfeld number $(\sigma)$ to the operating journal eccentricity ($\varepsilon$) in a turbulent flow short journal bearing,

$$\sigma = \frac{12}{\tilde{\tilde{\kappa}}_z} \frac{\left(1-\varepsilon^2\right)^2}{\varepsilon\sqrt{\left\{16\varepsilon^2 + \pi^2\left(1-\varepsilon^2\right)\right\}}} \tag{55}$$

where $\tilde{\tilde{\kappa}}_z = f\left(\mathrm{Re} = \frac{\rho \Omega R c}{\mu}\right)$. For a rated operating condition, the Sommerfeld number $(\sigma)$ is known since the bearing geometry, rotational speed, material fluid properties and applied load are known. Thus, Eq. (55) provides a relationship to determine (iteratively) the equilibrium eccentricity ratio $\varepsilon = (e/C)$ required to balance the applied load $W$.

Conversely, a turbulent flow Sommerfeld number $(\sigma_t)$ is defined as:

$$\sigma_t = \frac{\mu \Omega L R}{4 \cdot W} \left(\frac{L}{C}\right)^2 \frac{\tilde{\tilde{\kappa}}_z}{12} = \frac{\left(1-\varepsilon^2\right)^2}{\varepsilon \sqrt{\left\{16\,\varepsilon^2 + \pi^2\left(1-\varepsilon^2\right)\right\}}} \tag{56}$$

For laminar flows, $\kappa_z = 12$, and Eq. (56) reduces to the conventional form given in Notes 4. On the other hand, for turbulent flows $\tilde{\kappa}_z / 12 > 1$, and thus the turbulent flow Sommerfeld number $(\sigma_t)$ is larger than the conventional $(\sigma)$. Hence, a turbulent flow bearing will determine a smaller or lower operating journal eccentricity than if the bearing would have if operating in the laminar flow regime. This simple observation explains why inertialess-turbulent flow journal bearings are less stable than laminar flow journal bearings.

An analysis to determine the rotordynamic force coefficients in a turbulent flow journal bearing could follow. However, this is not necessary since the analysis for the laminar flow journal bearing is given in detail earlier (see Notes 5). The turbulent flow analysis just shows that turbulent force coefficients are equal to the laminar flow coefficients multiplied by the ratio $\left(\tilde{\kappa}_z / 12\right)$.

Incidentally, note that the figures given in Notes 4 and 5 can also be used in the design of turbulent flow journal bearings provided that the turbulent flow Sommerfeld number is used in the analysis.

# Closure

Fluid inertia effects are of importance in thin film flows which show large Reynolds numbers. However, in conventional journal bearing applications with small clearance to radius ratios, flow turbulence and viscous effects are most likely to dominate the performance of the fluid film bearing.

Turbulent flow journal bearings determine a smaller journal eccentricity than laminar flow bearings for the same applied load. This condition then renders turbulent flow bearings to be more prone to show hydrodynamic instability. This assertion needs to be taken with caution since the analysis considered an isoviscous (e.g. isothermal) lubricant.

That is, turbulence implies more dissipation of mechanical energy, which results in an increase of the thermal energy convected, i.e. lubricant heating and viscosity reduction if the through flow rate remains invariant. The drop in viscosity causes larger Reynolds numbers and reduced viscous effects which determine larger operating journal eccentricities, etc.

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## Nomenclature

| | |
|---|---|
| $c$ | Radial clearance |
| $D$ | Bearing diameter |
| $e$ | Journal eccentricity |
| $F_r$, $F_t$ | Radial and tangential components of bearing reaction force , $F=W$ |
| $h$ | $c + e\cos(\theta)$. Film thickness |
| $I_{xx}$, $I_{zz}$, $I_{xz}$ | Momentum flux integrals |
| $L$ | Bearing axial length |
| $P$ | Hydrodynamic pressure |
| $R$ | $\frac{1}{2}D$, Radius |
| Re | $\frac{\rho U c}{\mu}$ ; Shear flow Reynolds number . $Re_* = \frac{\rho U_* c}{\mu}\frac{c}{R}$; $Re_s = \frac{\rho \omega c^2}{\mu} >>> 1$ |
| $t$ | Time (s) |
| $U$ | $\Omega R$. Journal surface rotational speed |
| $u, v, w$ | Velocity components in $x, y, z$ directions |
| $V_x$, $V_z$ | Bulk-flow velocities along $x$ and $z$ directions |
| $x=R\theta, y, z$ | Circumferential, cross-film and axial coordinates |
| $\kappa_x, \kappa_y, \kappa_J$ | Turbulence flow shear parameters, in laminar flow=12 |
| $\eta$ | $y/h$ |
| $\rho$, $\mu$ | Fluid density and absolute viscosity |
| $\tau$ | $\Omega t$. Dimensionless time parameter |
| $\tau_{xy}$, $\tau_{xz}$ | Wall shear stresses |
| $\sigma_t$ | Sommerfeld number for turbulent flow – short length journal bearing |
| $\Lambda$, $\sigma$ | Shear flow and squeeze film flow parameters |
| $\Omega$, $\omega$ | Journal rotational speed, whirl frequency (rad/s) |

**Superscripts**

| | |
|---|---|
| * | Characteristic value |
| - | Time averaged value |
| ~ | Space averaged value |

**Appendix to Notes 9.**

# Laminar flow short journal bearing with fluid inertia effects

Luis San Andres - TAMU 08/2006

See Notes 9 for all definitions

The axially averaged (dimensionless) pressure in a short length journal bearing is given by:

$$-p_{ave} = \frac{\kappa_z}{H^3}\cdot g + \frac{1}{H}\cdot\left[Re_s\cdot\frac{dg}{d\tau} + Re_c\cdot(\alpha-\gamma)\cdot\frac{dg}{d\theta} - 2\cdot\alpha\cdot Re_T\cdot\frac{g^2}{H}\right] = \frac{\left(12\cdot c^2\cdot P_{ave}\right)}{\mu\cdot\Omega_T\cdot L^2}$$

ORIGIN := 1

where: $g = \frac{\Lambda}{2}\cdot\frac{dH}{d\theta} + \sigma\cdot\frac{dH}{d\tau}$ and $H = 1 + \varepsilon(\tau)\cdot\cos\theta$ $\quad \Lambda = \frac{\Omega}{\Omega_T}$ $\quad \sigma = \frac{\omega}{\Omega_T}$

**Definitions:** $\Omega_T$ is a characteristic speed in rad/s

Journal position: $\varepsilon(\tau)$

Journal radial and precessional velocities $\quad \varepsilon_{dot} = \frac{d\varepsilon}{d\tau} \quad \phi_{dot} = \frac{d\phi}{d\tau}$

Journal radial and precession accelerations $\quad \varepsilon_{ddot} = \frac{d^2\varepsilon}{d\tau^2} \quad \phi_{ddot} = \frac{d^2\phi}{d\tau^2}$

**Laminar flow** $\kappa_z := 12 \quad \alpha := 1.2 \quad \gamma := 0.2$

**Reynolds numbers**

$Re_s = \frac{\rho\cdot\omega\cdot c^2}{\mu}$ squeeze film Reynolds number

$Re_c = \frac{\rho\cdot\Omega\cdot c^2}{\mu}$ modified shear flow Reynolds number

$Re_T = \frac{\rho\cdot\Omega_T\cdot c^2}{\mu}$ advection flow Reynolds number

Set Pcav=0 for cavitation at 0 pressure

$P_{cav} := -1\cdot10^{10}$

Forces shown dimensionless

$$f = \frac{F}{\frac{\mu\cdot R\cdot L^3\cdot\Omega_T}{C^2}}$$

Code for average pressure field

Below: define operating parameters

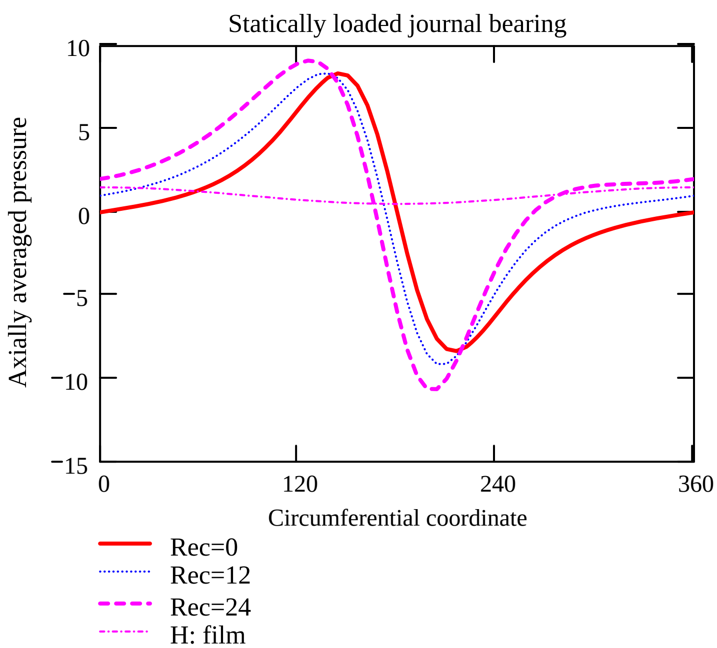
**set journal eccentricity** $\varepsilon := 0.5$

Calculations performed for 3 - Reynolds #

## 1. Static equilibrium journal bearing:

No journal center velocities or accelerations

$Re_T = Re_c$




$\varepsilon = 0.5$

$\Lambda = 1$

**Dimensionless (radial & tang) forces**

$$Re_c = \begin{pmatrix} 0 \\ 12 \\ 24 \end{pmatrix} \qquad f_r = \begin{pmatrix} 0 \\ 0.338 \\ 0.677 \end{pmatrix} \qquad f_t = \begin{pmatrix} 1.209 \\ 1.209 \\ 1.209 \end{pmatrix}$$

### Observations

Fluid inertia increases the (negative) peak pressure and displaces the positive peak pressure upstream of the minimum film thcikness.

Fluid inertia effects decrease in magnitude as journal eccentricity increases, i.e. at region where viscous effects dominate.

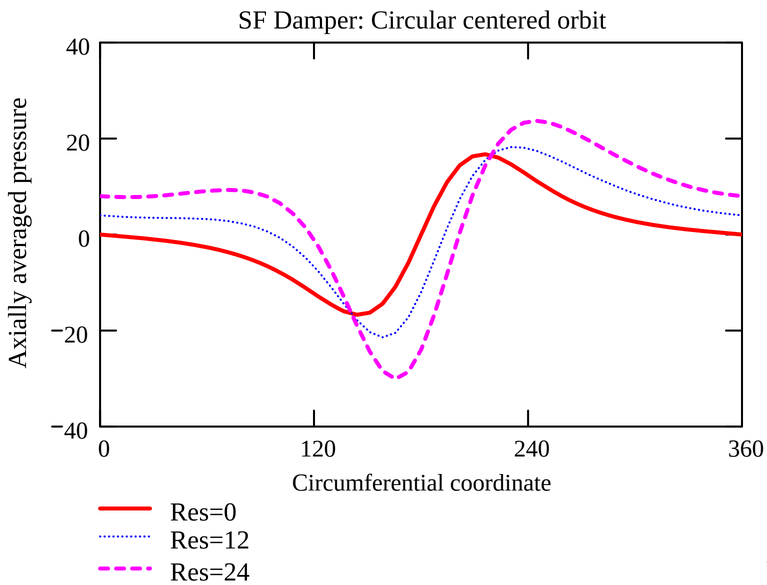
Fluid inertia introduces a radial (outward) force if no cavitation is present. Tangential force does not change as Rec increases.

If Pcav=0, then radial force increases and becomes positive, while tangential force raises steadily. Thus, the attitude angle can be larger than 90 deg indicating negative direct stiffness. **Less stability?**

## 2. Squeeze film damper: circular centered orbit:

Journal whirls about bearing center with constant orbit radius ($\varepsilon$) at fixed whirl frequency ($\omega$). There is no journal spinning ($\Omega$=0)

Set orbit radius $\varepsilon = 0.5$




$\varepsilon = 0.5$ orbit radius

$\phi_{dot} = 1$ whirl frequency

$\sigma = 1$

**Dimensionless (radial & tang) forces**

$Res_s = \begin{pmatrix} 0 \\ 12 \\ 24 \end{pmatrix}$ $f_r = \begin{pmatrix} 0 \\ 1.361 \\ 2.722 \end{pmatrix}$ $f_t = \begin{pmatrix} -2.418 \\ -2.418 \\ -2.418 \end{pmatrix}$

## Observations:

Fluid inertia increases the (negative) peak pressure and displaces the positive peak pressure downstream of the minimum film thcikness.

Fluid inertia effects decrease in magnitude as journal orbit increases, i.e. at region where viscous effects dominate.

Fluid inertia introduces a radial (outward) force if no cavitation is present. Tangential force does not change as Res increases.

If Pcav=0, then radial force increases and becomes positive, while tangential (damping) force raises steadily showing more damping.

**Check forces with predictions based on force coefficients** $Res := 24$

$a := 2$ 2 for full film, 1 for PI-film

$b := 2 - a$

$$c_{rt}(\varepsilon) := \frac{2\cdot\varepsilon\cdot b}{\left(1-\varepsilon^2\right)^2} \qquad c_{tt}(\varepsilon) := \frac{\pi\cdot a}{2\cdot\left(1-\varepsilon^2\right)^{\frac{3}{2}}} \qquad m_{rr}(\varepsilon) := \frac{a\cdot\pi\cdot\left[\left(1-\varepsilon^2\right)^{.5}-1\right]}{12\cdot\varepsilon^2\cdot\left(1-\varepsilon^2\right)^{.5}}\cdot\left[1-2\cdot\left(1-\varepsilon^2\right)^{.5}\right]$$

$$m_{tr}(\varepsilon) := \frac{27\cdot b}{70\cdot\varepsilon}\cdot\left(2+\frac{1}{\varepsilon}\cdot\ln\left(\frac{1-\varepsilon}{1+\varepsilon}\right)\right)$$

$$f_t(\varepsilon, Res) := -1\cdot\left(c_{tt}(\varepsilon) - m_{tr}(\varepsilon)\cdot Res\right)\cdot\varepsilon \qquad f_r(\varepsilon, Res) := -1\cdot\left(c_{rt}(\varepsilon) - m_{rr}(\varepsilon)\cdot Res\right)\cdot\varepsilon$$

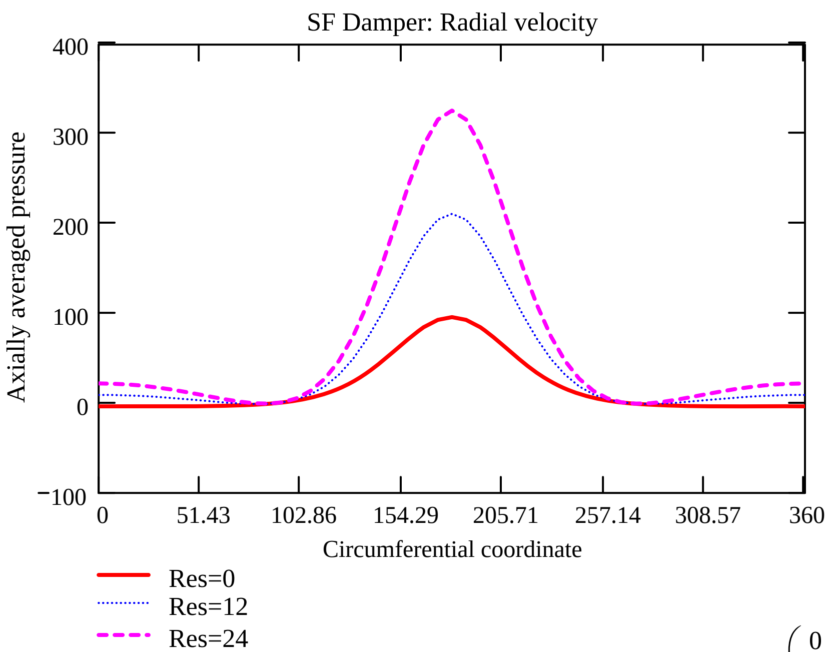
| | | |
|---|---|---|
| Res=0 | $f_r(\varepsilon, 0) = 0$ | $f_t(\varepsilon, 0) = -2.418$ |
| $Res = 24$ | $f_r(\varepsilon, Res) = 2.846$ | $f_t(\varepsilon, Res) = -2.418$ |

Why do these forces differ with the integrated ones as Re increases?

# 3. Squeeze film damper: radial velocity

No journal spinning or whirling. Assume journal is at eccentricity (ε) with instantaneous radial velocity (dε/dt).

Set: $\varepsilon_{dot} := 1$ $\varepsilon = 0.5$




$\varepsilon_{dot} := 1$

**Dimensionless (radial & tang) forces**

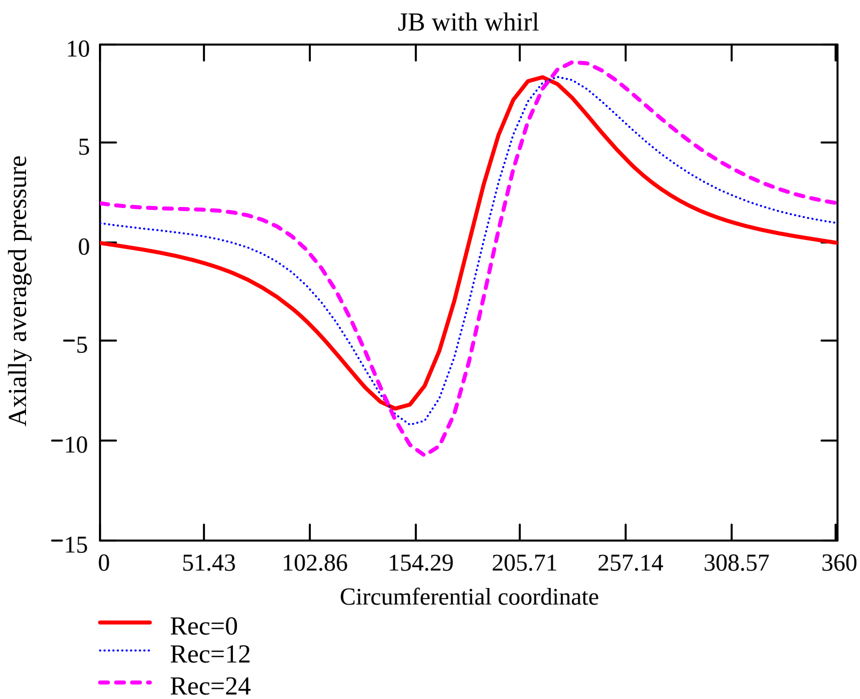
$$Re_s = \begin{pmatrix} 0 \\ 12 \\ 24 \end{pmatrix} \quad f_r = \begin{pmatrix} -9.711 \\ -18.685 \\ -27.659 \end{pmatrix} \quad f_t = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

## Observations:

Fluid inertia increases dramatically the peak pressure and radial force opposing the radial velocity.
Cavitation has little effect on pressure generation and reaction force.

# 4. Journal bearing with fractional synchronous whirl

$\Lambda := 1$ $\phi_{dot} := \Lambda$ $\sigma := 1$ $Re_T = Re_c = Re_s$




$\varepsilon = 0.5$

$\varepsilon_{dot} = 0$

$\varepsilon_{ddot} = 0$

synchonous whirl

**Appendix to Notes 9.**

# Laminar flow short journal bearing with fluid inertia effects

Luis San Andres - TAMU 08/2006

See Notes 9 for all definitions

The axially averaged (dimensionless) pressure in a short length journal bearing is given by:

$$-p_{ave} = \frac{\kappa_z}{H^3}\cdot g + \frac{1}{H}\cdot\left[Re_s\cdot\frac{dg}{d\tau} + Re_c\cdot(\alpha-\gamma)\cdot\frac{dg}{d\theta} - 2\cdot\alpha\cdot Re_T\cdot\frac{g^2}{H}\right] = \frac{\left(12\cdot c^2\cdot P_{ave}\right)}{\mu\cdot\Omega_T\cdot L^2}$$

ORIGIN := 1

where: $g = \frac{\Lambda}{2}\cdot\frac{dH}{d\theta} + \sigma\cdot\frac{dH}{d\tau}$ and $H = 1 + \varepsilon(\tau)\cdot\cos\theta$ $\quad \Lambda = \frac{\Omega}{\Omega_T}$ $\quad \sigma = \frac{\omega}{\Omega_T}$

**Definitions:** $\Omega_T$ is a characteristic speed in rad/s

Journal position: $\varepsilon(\tau)$

Journal radial and precessional velocities $\quad \varepsilon_{dot} = \frac{d\varepsilon}{d\tau}$ $\quad \phi_{dot} = \frac{d\phi}{d\tau}$

Journal radial and precession accelerations $\quad \varepsilon_{ddot} = \frac{d^2\varepsilon}{d\tau^2}$ $\quad \phi_{ddot} = \frac{d^2\phi}{d\tau^2}$

**Laminar flow** $\kappa_z := 12$ $\quad \alpha := 1.2$ $\quad \gamma := 0.2$

**Reynolds numbers**

**Set Pcav=0 for cavitation at 0 pressure**

$Re_s = \frac{\rho\cdot\omega\cdot c^2}{\mu}$ squeeze film Reynolds number

$P_{cav} := 0\cdot 10^{10}$

$Re_c = \frac{\rho\cdot\Omega\cdot c^2}{\mu}$ modified shear flow Reynolds number

Forces shown dimensionless

$Re_T = \frac{\rho\cdot\Omega_T\cdot c^2}{\mu}$ advection flow Reynolds number

$$f = \frac{F}{\frac{\mu\cdot R\cdot L^3\cdot\Omega_T}{C^2}}$$

Code for average pressure field

Below: define operating parameters

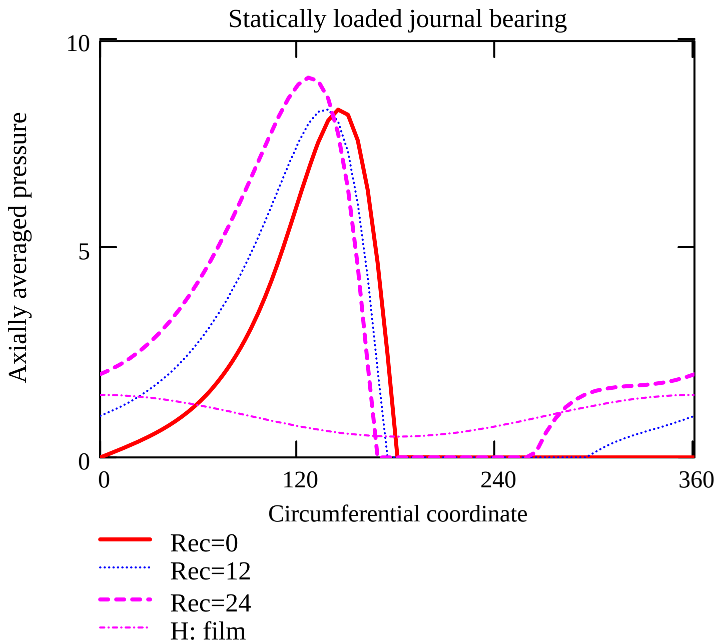
**set journal eccentricity** $\varepsilon := 0.5$

Calculations performed for 3 - Reynolds #

## 1. Static equilibrium journal bearing:

No journal center velocities or accelerations

$Re_T = Re_c$




$\varepsilon = 0.5$

$\Lambda = 1$

**Dimensionless (radial & tang) forces**

$$Re_c = \begin{pmatrix} 0 \\ 12 \\ 24 \end{pmatrix} \quad f_r = \begin{pmatrix} -0.443 \\ -0.239 \\ -0.01 \end{pmatrix} \quad f_t = \begin{pmatrix} 0.605 \\ 0.772 \\ 0.858 \end{pmatrix}$$

## Observations

Fluid inertia increases the (negative) peak pressure and displaces the positive peak pressure upstream of the minimum film thcikness.

Fluid inertia effects decrease in magnitude as journal eccentricity increases, i.e. at region where viscous effects dominate.

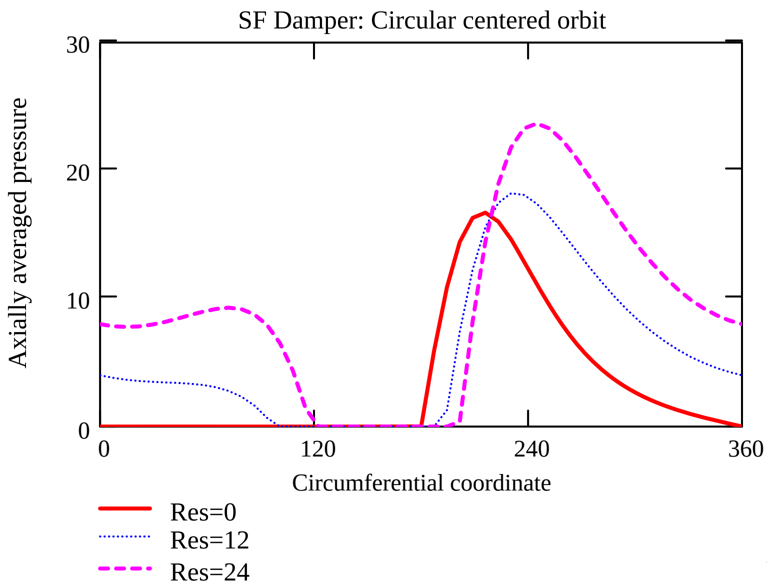
Fluid inertia introduces a radial (outward) force if no cavitation is present. Tangential force does not change as Rec increases.

If Pcav=0, then radial force increases and becomes positive, while tangential force raises steadily. Thus, the attitude angle can be larger than 90 deg indicating negative direct stiffness. **Less stability?**

## 2. Squeeze film damper: circular centered orbit:

Journal whirls about bearing center with constant orbit radius ($\varepsilon$) at fixed whirl frequency ($\omega$). There is no journal spinning ($\Omega$=0)

Set orbit radius $\varepsilon = 0.5$




$\varepsilon = 0.5$ orbit radius

$\phi_{dot} = 1$ whirl frequency

$\sigma = 1$

**Dimensionless (radial & tang) forces**

$$Res_s = \begin{pmatrix} 0 \\ 12 \\ 24 \end{pmatrix} \quad f_r = \begin{pmatrix} -0.884 \\ -0.014 \\ 0.783 \end{pmatrix} \quad f_t = \begin{pmatrix} -1.209 \\ -1.715 \\ -1.821 \end{pmatrix}$$

## Observations:

Fluid inertia increases the (negative) peak pressure and displaces the positive peak pressure downstream of the minimum film thcikness.

Fluid inertia effects decrease in magnitude as journal orbit increases, i.e. at region where viscous effects dominate.

Fluid inertia introduces a radial (outward) force if no cavitation is present. Tangential force does not change as Res increases.

If Pcav=0, then radial force increases and becomes positive, while tangential (damping) force raises steadily showing more damping.

**Check forces with predictions based on force coefficients** $Res := 24$

$a := 2$ 2 for full film, 1 for PI-film

$b := 2 - a$

$$c_{rt}(\varepsilon) := \frac{2\cdot\varepsilon\cdot b}{\left(1-\varepsilon^2\right)^2} \qquad c_{tt}(\varepsilon) := \frac{\pi\cdot a}{2\cdot\left(1-\varepsilon^2\right)^{\frac{3}{2}}} \qquad m_{rr}(\varepsilon) := \frac{a\cdot\pi\cdot\left[\left(1-\varepsilon^2\right)^{.5}-1\right]}{12\cdot\varepsilon^2\cdot\left(1-\varepsilon^2\right)^{.5}}\cdot\left[1-2\cdot\left(1-\varepsilon^2\right)^{.5}\right]$$

$$m_{tr}(\varepsilon) := \frac{27\cdot b}{70\cdot\varepsilon}\cdot\left(2+\frac{1}{\varepsilon}\cdot\ln\left(\frac{1-\varepsilon}{1+\varepsilon}\right)\right)$$

$$f_t(\varepsilon, Res) := -1\cdot\left(c_{tt}(\varepsilon) - m_{tr}(\varepsilon)\cdot Res\right)\cdot\varepsilon \qquad f_r(\varepsilon, Res) := -1\cdot\left(c_{rt}(\varepsilon) - m_{rr}(\varepsilon)\cdot Res\right)\cdot\varepsilon$$

Res=0 $\quad f_r(\varepsilon, 0) = 0 \quad f_t(\varepsilon, 0) = -2.418$

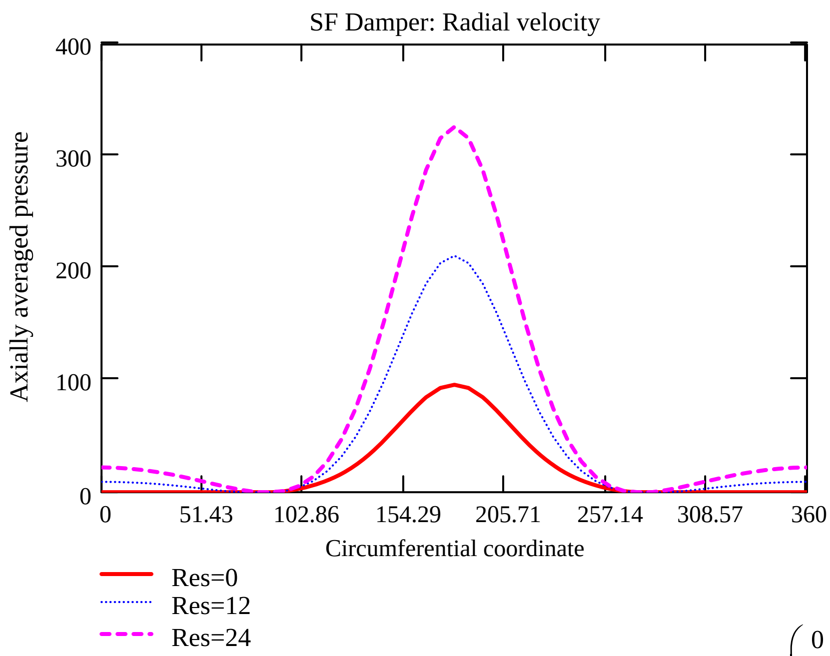
$Res = 24 \quad f_r(\varepsilon, Res) = 2.846 \quad f_t(\varepsilon, Res) = -2.418$

Why do these forces differ with the integrated ones as Re increases?

## 3. Squeeze film damper: radial velocity

No journal spinning or whirling. Assume journal is at eccentricity ($\varepsilon$) with instantaneous radial velocity ($d\varepsilon/dt$).

Set: $\varepsilon_{dot} := 1$ $\varepsilon = 0.5$




$\varepsilon_{dot} := 1$

**Dimensionless (radial & tang) forces**

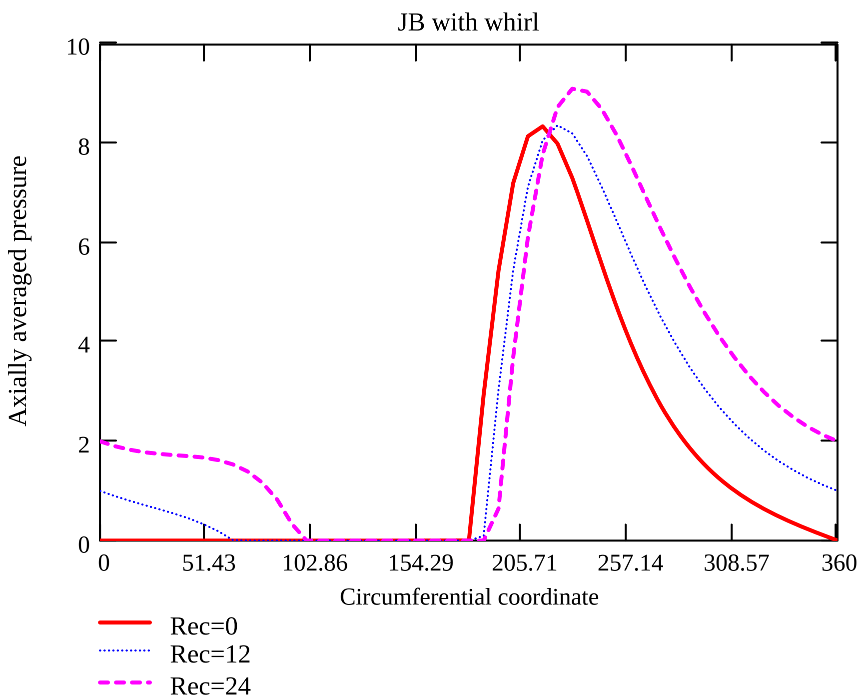
$$Re_s = \begin{pmatrix} 0 \\ 12 \\ 24 \end{pmatrix} \quad f_r = \begin{pmatrix} -9.116 \\ -18.678 \\ -27.658 \end{pmatrix} \quad f_t = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

## Observations:

Fluid inertia increases dramatically the peak pressure and radial force opposing the radial velocity.
Cavitation has little effect on pressure generation and reaction force.

## 4. Journal bearing with fractional synchronous whirl

$\Lambda := 1$ $\quad \phi_{dot} := \Lambda$ $\quad \sigma := 1$ $\qquad Re_T = Re_c = Re_s$




$\varepsilon = 0.5$

$\varepsilon_{dot} = 0$

$\varepsilon_{ddot} = 0$

synchonous whirl